A Unified Hypergraph- and SuperHyperGraph-Based Framework for Food Web Extension: From Classical Food Webs to SuperHyperWebs in Ecological Systems

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Abstract

Hypergraphs generalize graphs by allowing hyperedges to join any number of vertices, while superhypergraphs further extend this idea by layering iterated powersets to capture hierarchical, self-referential connections. A food web models an ecosystem as a directed graph whose nodes are species and whose edges represent predator–prey interactions. In this paper, we introduce two novel extensions of classical food webs: the *Food HyperWeb*, which encodes each predator's entire prey set as a hyperedge, and the *Food n-SuperHyperWeb*, which embeds multilevel trophic relationships within an n-fold superhypergraph structure. We provide formal definitions, establish their foundational properties, and present illustrative examples demonstrating their effectiveness for ecological network analysis.

Keywords: Superhypergraph, Hypergraph, Food Web, Food Hyperweb, Food n-SuperHyperWeb

1 Preliminaries

This section introduces the fundamental concepts and definitions that underpin the discussions in this paper. Throughout, all sets are assumed to be finite.

1.1 Power Set and n-th Power Set

The power set of S is the collection of all subsets of S, including the empty set and S itself. The n-th power set of S is obtained by iteratively applying the power set operation n times, starting from S [1–5].

Definition 1.1 (Universal Set)**.** Let U be a set containing all elements under consideration. Throughout, every set S is assumed to satisfy $S \subseteq U$.

Definition 1.2 (Base Set)**.** A *base set* S is any subset $S \subseteq U$ from which further constructions—such as powersets and hyperstructures—are formed.

Definition 1.3 (Power Set)**.** The *power set* of S, denoted $\mathcal{P}(S)$, is the collection of all subsets of S:

$$\mathcal{P}(S) \;=\; \{\, X \mid X \subseteq S \}.$$

Definition 1.4 (Iterated Power Set)**.** [6–9] For each integer $n \geq 1$, define the n-fold iterated power set of S by

$$\mathcal{P}^1(S) = \mathcal{P}(S),$$

$$\mathcal{P}^{k+1}(S) = \mathcal{P}\big(\mathcal{P}^k(S)\big) \quad (k \geq 1).$$

Equivalently, one may write $P_n(S) = \mathcal{P}^n(S)$.

Definition 1.5 (Nonempty Iterated Power Set)**.** [6, 10] Define the nonempty iterated power set by

$$\mathcal{P}_1^*(S) = \mathcal{P}(S) \setminus \{\varnothing\},$$

$$\mathcal{P}_{k+1}^*(S) = \mathcal{P}^*\big(\mathcal{P}_k^*(S)\big) \quad (k \geq 1),$$

where $\mathcal{P}^*(X) = \mathcal{P}(X) \setminus \{\varnothing\}$ for any set X.

Example 1.6 (Nonempty Iterated Power Set in Beverage Selection)**.** Let $S = \{\textsf{Tea}, \textsf{Coffee}\}$ represent two drink options. Then:

$$\mathcal{P}_1^*(S) = \mathcal{P}(S) \setminus \{\varnothing\}$$
$$= \big\{\{\textsf{Tea}\}, \{\textsf{Coffee}\}, \{\textsf{Tea}, \textsf{Coffee}\}\big\}.$$

The second-level nonempty iterated power set is

$$\mathcal{P}_2^*(S) = \mathcal{P}^*\big(\mathcal{P}_1^*(S)\big)$$

$$= \Big\{\{\{\textsf{Tea}\}\}, \{\{\textsf{Coffee}\}\}, \{\{\textsf{Tea}, \textsf{Coffee}\}\}, \{\{\textsf{Tea}\}, \{\textsf{Coffee}\}\},$$

$$\{\{\textsf{Tea}\}, \{\textsf{Tea}, \textsf{Coffee}\}\}, \{\{\textsf{Coffee}\}, \{\textsf{Tea}, \textsf{Coffee}\}\}, \{\{\textsf{Tea}\}, \{\textsf{Coffee}\}, \{\textsf{Tea}, \textsf{Coffee}\}\}\Big\}.$$

For instance, $\{\{\textsf{Tea}\}, \{\textsf{Coffee}\}\} \in \mathcal{P}_2^*(S)$ can model offering both a "tea-only" service and a "coffee-only" service as distinct package options. One may continue to $\mathcal{P}_3^*(S) = \mathcal{P}^*(\mathcal{P}_2^*(S))$, whose elements are nonempty collections of these service-packages.

1.2 Hypergraphs and SuperHypergraphs

Hypergraphs generalize ordinary graphs by allowing each *hyperedge* to join an arbitrary nonempty subset of vertices, thereby modeling higher-order relations among elements [11–16]. A *SuperHyperGraph* further extends this idea by incorporating iterated powerset structures, enabling multi-layered, self-referential connections among hyperedges [17–24].

Definition 1.7 (Hypergraph)**.** [11, 25] Let V be a finite set of *vertices*. A *hypergraph* is a pair

$$H = \big(V, E\big), \qquad E \subseteq \mathcal{P}(V) \setminus \{\varnothing\},$$

where each element of E is called a *hyperedge*. No restriction is imposed on the size of a hyperedge.

Definition 1.8 (n-SuperHyperGraph)**.** [18, 22] Let V_0 be a finite *base set*. Define the iterated powersets by

$$\mathcal{P}^0(V_0) = V_0,$$
$$\mathcal{P}^{k+1}(V_0) = \mathcal{P}\big(\mathcal{P}^k(V_0)\big), \quad k \geq 0.$$

For a fixed $n \geq 1$, an *n-SuperHyperGraph* is a pair

$$\mathrm{SHG}^{(n)} = \big(V, E\big),$$

where

$$V, E \subseteq \mathcal{P}^n(V_0) \quad \text{and} \quad V \neq \varnothing, \; E \neq \varnothing.$$

Elements of V are called *n-supervertices* and elements of E are called *n-superedges*.

Example 1.9 (Corporate Hierarchy as a 2-SuperHyperGraph)**.** Let the set of employees be

$$V_0 = \{A, B, C, D\},$$

with A = Alice, B = Bob, C = Carol, and D = Dave. First-level subsets (teams) are chosen as

$$T_1 = \{A, B\}, \quad T_2 = \{C, D\},$$

so $T_1, T_2 \in \mathcal{P}(V_0)$. Next, define two departments as elements of the second-level iterated power set:

$$D_1 = \{T_1\}, \quad D_2 = \{T_2\}, \quad \text{so } \{D_1, D_2\} \subseteq \mathcal{P}^2(V_0).$$

Thus we set

$$V_2 = \{D_1, D_2\} \subseteq \mathcal{P}^2(V_0), \qquad E_2 = \big\{\{D_1, D_2\}\big\} \subseteq \mathcal{P}^2(V_0).$$

The pair

$$F^{(2)} = \big(V_2, E_2\big)$$

is a 2-SuperHyperGraph encoding the hierarchy: employees $\longrightarrow$ teams $\longrightarrow$ departments $\longrightarrow$ the company division.

1.3 Food Web

A food web is a directed graph representing species as nodes and predator–prey interactions as directed edges in an ecosystem [26–32].

Definition 1.10 (Food Web)**.** [26, 27] Let V be a finite set whose elements represent biological species (or trophic groups) in an ecosystem. A *food web* is the directed graph

$$G = (V, E),$$

where the edge set $E \subseteq V \times V$ satisfies

$$(u, v) \in E \quad \Longleftrightarrow \quad \text{species } u \text{ preys upon species } v.$$

We require additionally that

- There are no self-loops: $(v, v) \notin E$ for all $v \in V$.
- There are no parallel edges: E is a set (not a multiset).

Thus G encodes all direct predator–prey relationships among the species in the ecosystem.

Example 1.11 (Simple Pond Food Web)**.** Consider a small pond ecosystem with five trophic levels:

$$V = \{ A,\ Z,\ S,\ B,\ D\},$$

where

- A = Algae
- Z = Zooplankton
- S = Small Fish
- B = Big Fish
- D = Duck

The predator–prey relationships are captured by

$$E = \{ (Z, A),\ (S, Z),\ (B, S),\ (D, B),\ (D, S) \},$$

meaning:

$$\begin{aligned}
Z &\to A && \text{(zooplankton eats algae)},\\
S &\to Z && \text{(small fish eat zooplankton)},\\
B &\to S && \text{(big fish eat small fish)},\\
D &\to B && \text{(duck eats big fish)},\\
D &\to S && \text{(duck also eats small fish)}.
\end{aligned}$$

Thus the directed graph $G = (V, E)$ illustrates a simple pond food web.

2 Main Results

In this section, we formally define the concepts of the Food HyperWeb and the Food SuperHyperWeb, and briefly examine their structural properties.

2.1 Food HyperWeb

Food HyperWeb is a hypergraph whose vertices represent species and whose hyperedges correspond to each predator's unique complete prey set.

Definition 2.1 (Base Food Web)**.** Let V_0 be a finite set of species and let

$$G = (V_0, E_0)$$

be the directed graph (the *Food Web*) where $(u, v) \in E_0$ if and only if species u preys upon species v.

Definition 2.2 (Food HyperWeb)**.** Let $G = (V_0, E_0)$ be a Food Web. Define the *Food HyperWeb*

$$H = (V_1, E_1)$$

by

$$V_1 = V_0, \qquad E_1 = \left\{ e_u \subseteq V_0 : e_u = \{ v \in V_0 : (u, v) \in E_0\},\ e_u \neq \emptyset \right\}.$$

Each hyperedge e_u collects all prey of predator u.

Example 2.3 (Simple Pond Food HyperWeb)**.** Consider the same small pond ecosystem as in the Food Web example:

$$V_0 = \{ A,\ Z,\ S,\ B,\ D\},$$

where

$$E_0 = \{(Z, A),\ (S, Z),\ (B, S),\ (D, B),\ (D, S)\},$$

with species labels:

- A: Algae
- Z: Zooplankton
- S: Small Fish
- B: Big Fish
- D: Duck

The Food HyperWeb $H = (V_1, E_1)$ is obtained by grouping each predator's prey into a hyperedge:

$$V_1 = V_0, \qquad E_1 = \{ e_Z,\ e_S,\ e_B,\ e_D\},$$

where

$$\begin{aligned} e_Z &= \{ A\}, && \text{(zooplankton eats algae);} \\ e_S &= \{ Z\}, && \text{(small fish eat zooplankton);} \\ e_B &= \{ S\}, && \text{(big fish eat small fish);} \\ e_D &= \{ B,\ S\}, && \text{(duck eats big fish and small fish).} \end{aligned}$$

Thus H is the hypergraph whose vertices are the five species and whose hyperedges capture each predator's full prey set.

Example 2.4 (Marine Food HyperWeb)**.** Consider a simple marine ecosystem with six species:

$$V_0 = \{ P,\ Z,\ K,\ F,\ S,\ H\},$$

where

- P: Phytoplankton
- Z: Zooplankton
- K: Krill

- F: Small Fish
- S: Seal
- H: Shark

The predator–prey relations (the Food Web $G = (V_0, E_0)$) are

$$E_0 = \{\, (Z,P),\ (K,Z),\ (F,K),\ (F,Z),\ (S,F),\ (S,K),\ (H,S),\ (H,F)\,\}.$$

From this we form the Food HyperWeb $H = (V_1, E_1)$ with $V_1 = V_0$ and

$$E_1 = \{\, e_Z, e_K, e_F, e_S, e_H\},$$

where each hyperedge collects all prey of a given predator:

$$\begin{aligned}
e_Z &= \{\, P\}, && \text{(zooplankton eats phytoplankton);}\\
e_K &= \{\, Z\}, && \text{(krill eats zooplankton);}\\
e_F &= \{\, K,\ Z\}, && \text{(small fish eat krill and zooplankton);}\\
e_S &= \{\, F,\ K\}, && \text{(seal eats small fish and krill);}\\
e_H &= \{\, S,\ F\}, && \text{(shark eats seal and small fish).}
\end{aligned}$$

Thus H is the hypergraph whose vertices are the six marine species and whose hyperedges exactly describe each predator's full prey set.

Theorem 2.5 (Food HyperWeb is a Hypergraph)**.** *Let $H = (V_1, E_1)$ be the Food HyperWeb constructed from a Food Web $G = (V_0, E_0)$ as in Definition 2.1. Then*

$$E_1 \ \subseteq\ \mathcal{P}(V_1) \setminus \{\varnothing\}, \qquad V_1 \text{ is finite},$$

so H satisfies the axioms of a finite hypergraph.

Proof. By definition $V_1 = V_0$ is finite. Each hyperedge

$$e_u \ = \ \{\, v \in V_1 : (u, v) \in E_0\}$$

is nonempty exactly when u preys on at least one species, hence $e_u \in \mathcal{P}(V_1) \setminus \{\varnothing\}$. Moreover, the assignment $u \mapsto e_u$ is injective, so there are no duplicate hyperedges. Thus H meets the standard definition of a finite hypergraph [12]. □

Theorem 2.6 (Reconstruction of Food Web by Flattening)**.** *The original Food Web $G = (V_0, E_0)$ is recovered from the Food HyperWeb $H = (V_1, E_1)$ via*

$$E_0 = \big\{(u, v) \in V_1 \times V_1 : v \in e_u,\ e_u \in E_1\big\}.$$

Proof. By construction $e_u = \{v : (u, v) \in E_0\}$. Hence

$$(u, v) \in E_0 \quad\Longleftrightarrow\quad v \in e_u,$$

so the directed edges of G coincide exactly with the incidences of vertices in hyperedges of H. Therefore flattening H recovers E_0. □

2.2 Food SuperHyperWeb

Food SuperHyperWeb is an n-superhypergraph whose vertices are iterated species subsets and whose hyperedges encode complete multilevel hierarchical predator–prey relationships.

Definition 2.7 (Food n-SuperHyperWeb)**.** Let $G = (V_0, E_0)$ be a finite directed graph (the *Food Web*), and for each $u \in V_0$ write

$$P(u) \;=\; \{\, v \in V_0 : (u, v) \in E_0 \}$$

for its *prey set*. Fix an integer $n \geq 1$, and let $\mathcal{P}^n(V_0)$ denote the n-fold iterated powerset of V_0. Define

$$V_n \;=\; \mathcal{P}^n(V_0), \qquad E_n \;=\; \bigl\{\, e_u^{(n)} : u \in V_0,\ P(u) \neq \varnothing,\ e_u^{(n)} = \mathcal{P}^n\bigl(P(u)\bigr) \bigr\}.$$

Then the pair

$$F^{(n)} = \bigl(V_n,\ E_n\bigr)$$

is called the *Food n-SuperHyperWeb*.

Example 2.8 (Forest Ecosystem as a Food 2-SuperHyperWeb)**.** Let the species set be

$$V_0 = \{G,\ M,\ O,\ F\},$$

where

$$G = \text{Grass}, \quad M = \text{Mouse}, \quad O = \text{Owl}, \quad F = \text{Fox}.$$

The Food Web $G = (V_0, E_0)$ has predator–prey arcs

$$E_0 = \{(M, G),\ (O, M),\ (F, O),\ (F, M)\}.$$

For each $u \in V_0$, its prey set is

$$P(M) = \{G\},\ P(O) = \{M\},\ P(F) = \{O,\ M\},\ P(G) = \varnothing.$$

Fix $n = 2$. Then

$$V_2 = \mathcal{P}^2(V_0), \qquad E_2 = \{\, e_u^{(2)} : u \in \{M, O, F\}\},$$

with

$$e_M^{(2)} = \mathcal{P}^2\bigl(P(M)\bigr) = \mathcal{P}^2(\{G\}) = \bigl\{\emptyset, \{\emptyset\}, \{\{G\}\}, \{\emptyset, \{G\}\}\bigr\},$$
$$e_O^{(2)} = \mathcal{P}^2(\{M\}) = \bigl\{\emptyset, \{\emptyset\}, \{\{M\}\}, \{\emptyset, \{M\}\}\bigr\},$$
$$e_F^{(2)} = \mathcal{P}^2\bigl(\{O, M\}\bigr),$$

which has $2^{2^2} = 16$ elements (all subsets of $\mathcal{P}(\{O, M\})$). Thus

$$F^{(2)} = \bigl(V_2,\ E_2\bigr)$$

is the Food 2-SuperHyperWeb encoding the multi-level trophic structure:

$$\text{species} \;\longrightarrow\; \text{prey sets} \;\xrightarrow{\mathcal{P}}\; \text{collections of prey-sets}.$$

Example 2.9 (Agricultural Food 2-SuperHyperWeb)**.** Let the set of trophic species be

$$V_0 = \{G,\ I,\ C,\ Co,\ H\},$$

where

$$G = \text{Grass}, \quad I = \text{Insect}, \quad C = \text{Chicken}, \quad Co = \text{Cattle}, \quad H = \text{Human}.$$

The Food Web $G = (V_0, E_0)$ has predator–prey arcs

$$E_0 = \{(I, G),\ (C, I),\ (Co, G),\ (H, C),\ (H, Co)\}.$$

Hence each prey set is

$$P(I) = \{G\},\ P(C) = \{I\},\ P(Co) = \{G\},\ P(H) = \{C, Co\}.$$

Fix $n = 2$. Then

$$V_2 = \mathcal{P}^2(V_0), \qquad E_2 = \{\, e_u^{(2)} : u \in \{I, C, Co, H\}\},$$

with

$$e_I^{(2)} = \mathcal{P}^2\big(\{G\}\big) = \big\{\varnothing, \{\varnothing\}, \{\{G\}\}, \{\varnothing, \{G\}\}\big\},$$

$$e_C^{(2)} = \mathcal{P}^2\big(\{I\}\big) = \big\{\varnothing, \{\varnothing\}, \{\{I\}\}, \{\varnothing, \{I\}\}\big\},$$

$$e_{Co}^{(2)} = \mathcal{P}^2\big(\{G\}\big) \quad \text{(same as } e_I^{(2)}\text{)},$$

$$e_H^{(2)} = \mathcal{P}^2\big(\{C, Co\}\big),$$

which has $2^{2^2} = 16$ elements, each representing a possible "meal bundle" drawn from the basic prey sets $\{C, Co\}$. Therefore

$$F^{(2)} = \big(V_2,\ E_2\big)$$

is the Food 2-SuperHyperWeb encoding both primary producer consumption and human dietary options in a two-layer superhypergraph structure.

Theorem 2.10. *$F^{(n)} = (V_n, E_n)$ is an n-SuperHyperGraph.*

Proof. By construction V_n is a finite set and each hyperedge

$$e_u^{(n)} = \mathcal{P}^n\big(P(u)\big)$$

is a nonempty element of $\mathcal{P}^n(V_0)$. Distinct predators u yield distinct hyperedges, so there are no duplicates. Hence $F^{(n)}$ satisfies the definition of an n-SuperHyperGraph. □

Theorem 2.11 (Generalization of Food HyperWeb and Food Web)**.** *Define the "full flattening" of a nested set by taking the union of all its elements at every level. Since*

$$e_u^{(n)} \;=\; \mathcal{P}^n\big(P(u)\big),$$

flattening $e_u^{(n)}$ recovers exactly the prey set $P(u)$. Therefore

$$\big\{\, P(u) : u \in V_0,\ P(u) \neq \varnothing\big\} = E_1 \quad \textit{and} \quad \big\{(u, v) : v \in P(u)\big\} = E_0,$$

showing that $F^{(n)}$ simultaneously generalizes the Food HyperWeb $H = (V_0, E_1)$ and the original Food Web $G = (V_0, E_0)$.

Proof. Immediate from the identity $\bigcup_{X \in e_u^{(n)}} \bigcup \cdots \bigcup X = P(u)$ and the definitions of E_1 and E_0. □

3 Conclusion and Future Works

In this paper, we introduced two novel extensions of classical food webs: the *Food HyperWeb*, which encodes each predator's entire prey set as a hyperedge, and the *Food n-SuperHyperWeb*, which embeds multilevel trophic relationships within an n-fold superhypergraph structure.

For future work, we plan to explore practical applications of these models in real ecological and agricultural networks. Additionally, we aim to extend the framework using uncertainty-based approaches such as Fuzzy Sets [33,34], Intuitionistic Fuzzy Sets [35,36], HyperFuzzy Sets [37–39], Bipolar Fuzzy Sets [40,41], Neutrosophic Sets [42–44], Hesitant Fuzzy Sets [45, 46], and Plithogenic Sets [47–50] to better handle ambiguity and incomplete information in ecological systems.

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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