### Genomic data confirms that mutation cannot restore genetic 1 diversity lost through population bottlenecks 2 3 Toby G. L. Kovacs<sup>1</sup>, Simon Y. W. Ho<sup>1</sup>, Carolyn J. Hogg<sup>1</sup> & Catherine E. Grueber<sup>1</sup> 4 5 6 <sup>1</sup>School of Life and Environmental Sciences, The University of Sydney, Sydney, New South Wales, 7 Australia 8 9 Corresponding Author: Toby G. L. Kovacs, toby.kovacs@sydney.edu.au, (+61) 431 775 796 Emails of Co-authors: simon.ho@sydney.edu.au, carolyn.hogg@sydney.edu.au and 10 11 catherine.grueber@sydney.edu.au 12 **Manuscript Information:** 13 • Type of article: Perspective 14 Word count: 2849 (plus a 198-word abstract) • Number of references: 33 15 • 16 • Figures: 2 Tables: 0 17 •

# 18 Key Words

- 19 allelic richness, conservation, genetic diversity, heterozygosity, koala, mutation rate,
- 20 population bottleneck, recovery, site frequency spectrum

# 21 ABSTRACT

- 22 Maintaining within-species genetic diversity is a critical goal of biodiversity conservation as it
- 23 determines a species' ability to adapt to environmental change. Without human intervention, isolated
- 24 populations can only recover genetic diversity post-bottleneck through the accumulation of new
- 25 mutations over evolutionary timescales. Using recent estimates of mutation rates from major genomic
- 26 datasets across the tree of life, we modelled the recovery of genetic diversity post-bottleneck. We
- 27 found that mutation rates do not affect the recovery rate of genetic diversity, which instead is
- determined by the effective population size  $(N_e)$ . For example, regardless of the mutation rate, an
- isolated population with  $N_e = 500$  that experiences a 50% reduction in heterozygosity would require  $\sim 2,300$  generations to return to 95% of its pre-disturbance level. In contrast, a bottleneck can lead to a
- 30 ~2,300 generations to return to 95% of its pre-disturbance level. In contrast, a bottleneck can lead to a 31 50% reduction in heterozygosity very quickly, taking just 30 generations for bottlenecks of  $N_e = 20$ .
- 31 30% reduction in heterozygosity very quickly, taking just 50 generations for bothenecks of  $N_e = 20$ . 32 We also demonstrate that allelic richness responds quickly following the recovery of  $N_e$  but argue that
- 33 this is unlikely to correspond to the recovery of evolutionary potential. Our results reinforce that
- 34 recovery via mutation alone is too slow to be effective within conservation timeframes, providing an
- 35 evolutionary context to genetic diversity loss.

#### 36 1 Determinants of genetic diversity

37 Maintaining global biodiversity is essential for the regular and reliable function of the world's 38 agricultural, economic and health systems (Scheffers et al. 2016). Despite this, anthropogenic influences continue to deteriorate ecosystems, leading to widespread losses in species (Cowie et al. 39 40 2022) and genetic diversity (Shaw et al. 2025). Fortunately, the importance of maintaining genetic diversity is becoming increasingly recognised in global conservation policy. The United Nations 41 42 Convention on Biological Diversity (CBD) Kunming-Montreal Global Biodiversity Framework 43 (GBF) marks a significant improvement in this regard, introducing its support for the preservation of 44 genetic diversity of wild animals (CBD 2022). This includes Target 4 of the agreement that aims to 45 maintain and restore genetic diversity in order to maintain adaptive potential. Maintaining genetic 46 diversity has long been the key focus of conservation genetic projects and there are a suite of tools 47 available to management practitioners (Shaw et al. 2025). However, options available for restoring 48 genetic diversity in wild populations are much more limited.

- 49 During a population bottleneck, genetic drift erodes genetic diversity, leading to the loss of
- 50 evolutionary potential (Franklin 1980). This lost genetic variation can be summarised in two
- 51 components: the number of segregating sites in a population (quantified as allelic richness or 52 nucleotide diversity), and the evenness of their frequencies (quantified by expected heterozygosity,
- 53 also known as gene diversity) (Hoban et al. 2022). These measurements can be characterised together
- using the site frequency spectrum (SFS), which plots the number of alleles at each frequency in a 54
- 55 population, to quantify the proportion of the genetic richness comprised by alleles at very low
- 56 frequency (rare alleles) versus those that are common in the population (Allendorf et al. 2012). For
- 57 example, both rare and common alleles contribute equally to genetic richness, but rare alleles have a
- 58 negligible impact on heterozygosity (Allendorf 1986). Maintaining both genetic richness and
- 59 evenness is required to maintain the evolutionary potential of a species (Allendorf et al. 2012), and
- 60 hence both should be measured to effectively describe the genetic composition of a population
- (Hoban et al. 2022). Decades of theoretical and empirical studies have demonstrated that genetic 61
- 62 diversity is a good predictor of population persistence (Kardos et al. 2021) especially under changing 63 environmental pressures (Sgrò et al. 2011). Thus, when recovering from a bottleneck, populations are
- 64 at an increased risk of extinction unless their genetic diversity can be restored.
- 65 In a closed population, the amount of neutral genetic diversity is determined by the rate of loss due to genetic drift, which is inversely proportional to the effective population size  $(N_e)$ , and by the rate of 66 gain of novel genetic variation through mutations, which occurs at  $\mu$  mutations per individual per site 67
- 68 per generation. Heterozygosity in a population can be estimated using the mutation-drift evolutionary
- 69 model (Malécot 1948, Kimura and Crow 1964), a well-established relationship that is described in
- 70 most textbooks on population and conservation genetics (e.g., Frankham et al. 2010; Hedrick 2011;
- Allendorf et al. 2012). This model is often used to calculate the equilibrium heterozygosity for neutral 71
- sites as  $H_{eq} = \frac{4N_e\mu}{4N_e\mu+1}$  (Kimura and Crow 1964). When population sizes change to a new  $N_e$ , as is the 72
- 73 case for threatened species experiencing population decline or post-decline recovery, genetic diversity
- 74 starts to move towards the corresponding new equilibrium level. The rate at which this diversity
- approaches this new equilibrium, is determined by the relative sizes of  $2N_e$  and  $\frac{1}{2u}$ . When the two 75
- values are sufficiently different, the rate of decay will be effectively controlled by only the smaller 76
- component. For example, if a species had a high mutation rate and/or a very large effective population 77
- size, such that  $2N_e \gg \frac{1}{2\mu}$ , the rate of genetic diversity decay and recovery will be inversely 78
- proportional to  $\mu$  (Nei et al. 1975). However, in a species with a low mutation rate and/or a small 79
- effective population size, such that  $2N_e \ll \frac{1}{2\mu}$ , the rate of genetic diversity decay and recovery will be 80
- 81 proportional to  $N_{\rho}$  (Nei et al. 1975).
- 82 After a population bottleneck, the recovery of heterozygosity by mutation has been estimated to take
- millions of generations (Nei et al. 1975; Lande & Barrowclough 1987). Based on these estimates, 83
- 84 many population and conservation genetic studies have considered mutation rates too low to
- 85 substantially contribute to the recovery of genetic diversity in threatened species, at least in the short

86 term (Lacy 1987; Frankham 2022). However, these seminal papers estimated the recovery of diversity

87 in large populations using rough estimates of the genomic mutation rate, preventing robust estimation

88 of how long heterozygosity would take to recover for threatened species (Nei et al. 1975; Lande &

89 Barrowclough 1987).

90 Rates of germline single-nucleotide mutations have now been estimated for a diverse range of

91 eukaryotes in parent-offspring and mutation accumulation whole-genome sequencing studies (Wang

92 & Obbard 2023). These include estimates for 116 species, albeit with a skew towards mammals (48 species), with estimates of mutation rates falling between  $10^{-10}$  and  $10^{-7}$  mutations per site per 93

- 94 generation (Figure 1a). These precise estimates of mutation rates provide an opportunity to explicitly
- 95 model the recovery of genetic variation after population bottlenecks in a variety of non-model species
- 96 across the tree of life. Here we show that, across the known range of mutation rates, the rate of genetic

97 diversity recovery is solely driven by the population size, independent of species-specific mutation

98 rates. Hence, we provide the first universal estimates for the time required to recover genetic diversity

- after various bottleneck scenarios, in order to contextualise the loss of deep evolutionary histories 99 100 over short timeframes commensurate with current biodiversity declines. Our results reveal the
- 101 permanency of genetic diversity losses and highlight the critical need for intervention methods that
- 102 prevent further loss, or in some cases even increase genetic diversity, to ensure the long-term survival
- 103 of populations and species.

104

#### 105 2 Response of genetic diversity during population-size changes

106 Using the generalised formula for the inbreeding coefficient (the probability that two randomly

107 selected alleles in a population are identical) while considering genetic drift and mutation (Wright et

108 al. 1931; Malécot 1948; Kimura & Crow 1964) we can derive an equation for heterozygosity as a 109 function of time (generations) as a population moves towards equilibrium (Nei et al. 1975).

110 Heterozygosity, as a proportion (or multiple) of equilibrium heterozygosity ( $H_{t/eq}$ ), can be modelled in

111 terms of time (t, generations), the starting heterozygosity, also as a proportion (or multiple) of the

112 equilibrium heterozygosity ( $H_{0/eq}$ ), the effective population size ( $N_e$ ) and the mutation rate ( $\mu$ ).

113 
$$H_{t/eq} = 1 - \left( \left(1 - 2\mu\right) \left(1 - \frac{1}{2N_e}\right) \right)^t \left(1 - H_{0/eq}\right)$$
(1)

114 When calculating this equation in a population with  $N_e = 500$ , for a range of mutation rates, we

115 confirm that the time taken to recover heterozygosity in a population is largely unaffected by the mutation rate, except when mutation rate is very high (>  $10^{-4}$  mutations/site/generation; Figure 1b). 116 This means that for the estimated range of germline mutation rates  $(10^{-10}-10^{-7})$ 117

mutations/site/generation; Figure 1a), the mutation rate has no impact on the recovery of genetic 118

diversity in a population with  $N_e = 500$ . For this range of mutation rates,  $2N_e \gg \frac{1}{2\mu}$  only occurs when 119

the effective population size is  $> 10^6 - 10^8$ , meaning that the mutation rate will only affect the rate of 120

heterozygosity recovery in very large populations. For most species, and particularly those of 121

122 conservation concern, effective population sizes are smaller by several orders of magnitude; therefore,

123 the rate of genetic diversity recovery, and loss, is not governed by their species-specific mutation rate.

Based on the mutation-drift evolutionary model, we propose a simplified equation for heterozygosity 124 when  $2N_e \ll \frac{1}{2u}$ , which applies to most populations of conservation concern. 125

126 
$$H_{t/eq} = 1 - \left(1 - \frac{1}{2N_e}\right)^t \left(1 - H_{0/eq}\right)$$
(2)

127

128

- The full derivation can be found in the <u>Appendix</u>. Over many generations,  $H_{t/eq}$  approaches 1 ( $H_t$  approaches  $H_{eq}$ ) as  $\left(1 \frac{1}{2N_e}\right)^t$  approaches zero. The rate of decay  $\left(1 \frac{1}{2N_e}\right)$  determines the rate at which the population approaches equilibrium, with the rate being higher for smaller effective 129
- 130

- 131 population sizes  $(N_e)$ . The difference between the starting heterozygosity and the equilibrium
- heterozygosity  $(1 H_{0/eq})$  also impacts how quickly the heterozygosity approaches equilibrium.
- 133 Using this model, we can estimate the recovery of heterozygosity after a bottleneck. A population
- 134 with  $N_e = 500$ , starting with 50% of the equilibrium heterozygosity, takes ~1600 generations to restore
- diversity to 90% of pre-disturbance level and ~2500 generations to increase diversity to 95% pre-
- 136 disturbance level (Figure 1b,d). We can also estimate the rate of loss of genetic diversity during a 137 population bottleneck. When starting with a population with  $N_e = 500$  at equilibrium heterozygosity, a
- population reduction to  $N_e = 20$  will result in the loss of 50% of heterozygosity in only 30 generations,
- 139 with 80% lost in 80 generations (Figure 1c). With an effective bottleneck size of 5, 50% of the
- 140 heterozygosity is lost in only 7 generations and 80% in 16 generations. The rate at which genetic
- 141 diversity approaches equilibrium is determined by the  $N_e$  of the population. This explains why genetic
- 142 diversity can be lost quickly when  $N_e$  is reduced during a bottleneck but is recovered slowly after  $N_e$ 143 increases. This also explains why more severe bottlenecks with smaller  $N_e$  lead to the faster loss of
- 144 genetic diversity, although they are also approaching a smaller equilibrium value (Figure 1c). These
- 145 patterns draw attention to the long-term evolutionary consequences of population bottlenecks on
- 146 heterozygosity.
- 147 At mutation–drift equilibrium, genetic richness (e.g., allelic richness) and evenness (e.g.,
- 148 heterozygosity) are correlated, but richness is expected to approach equilibrium more quickly after a
- 149 change in population size (Nei et al. 1975, Allendorf 1986). Modelling the gain and loss of genetic
- richness is more challenging than for heterozygosity (Greenbaum et al. 2014), but the effects of
- 151 bottlenecks on both measures can be observed from genetically explicit forward simulations (Haller &
- 152 Messer 2023). Results from simulations show that mutation rate does not impact the recovery of
- heterozygosity nor allelic richness (Figure 2a,b). Our simulations also confirm that allelic richness is
   generally recovered and lost at much faster rates than heterozygosity (Figure 2c). Following a
- population decrease, allelic richness is lost faster than heterozygosity (Figure 2c). Following a 155 population decrease, allelic richness is lost faster than heterozygosity for a bottleneck size of  $N_c = 50$
- 156 (census population size). In more extreme bottlenecks ( $N_c < 10$ ), however, heterozygosity can be lost
- 157 at a similar rate to allelic richness (Supplementary Figure 1). This initial change in richness is driven
- by a decrease in the number of alleles at very low frequencies (rare alleles), which contribute
   negligibly to heterozygosity (Figure 2d). Throughout the bottleneck, the SFS shows that the rapid loss
- negligibly to heterozygosity (Figure 2d). Throughout the bottleneck, the SFS shows that the rapid lossof genomic richness after a bottleneck is driven by the disproportionate loss of rare alleles (Figure 2d).
- 161 More common alleles are lost more slowly, as the SFS returns to its equilibrium distribution. A
- 162 similar pattern can be observed during the recovery of diversity following a bottleneck. Shortly after
- 163 the population size returns to  $N_e = 500$ , the rapid increase in allelic richness is predominantly driven
- by an increase in the number of rare alleles. It is not until these new alleles drift to higher frequencies
- and the SFS returns to equilibrium, over a much longer timeframe, that heterozygosity starts torecover.
- 167 Although the recovery times estimated here are too long for conservation management programs to
- 168 consider, they also represent minimum bounds for the recovery of sufficient diversity for population
- 169 persistence. Here we have used  $N_e = 500$  in our calculations for the recovery of diversity following a
- population bottleneck, because it is widely regarded as a benchmark for the minimum effective
- 171 population size in conservation management (Franklin 1980). However, it has been noted that  $N_e =$
- 172 500 does not guarantee sufficient genetic diversity for persistence and that larger effective population
- 173 sizes (1,000-5,000) are preferred for long-term genetic stability (Frankham et al. 2014). The recovery 174 time for heterozygosity scales directly with  $N_{\rm e}$ , such that recovering the genetic diversity of a
- 174 time for heterozygosity scales directly with  $N_e$ , such that recovering the genetic diversity of a population with  $N_e = 5,000$  would take 10 times longer than in a population with  $N_e = 500$ . Thus, our
- 176 estimates represent a lower bound for the recovery time of genetic diversity to healthy levels.
- 177

# 178 **3** The restoration of evolutionary potential

- 179 As the recovery of heterozygosity occurs over long timescales, genetically depleted populations will
- 180 show reduced evolutionary potential in the short term. We have also shown that allelic richness
- 181 recovers quickly following an expansion in population size, but also that this is unlikely to correspond

to an equivalent increase in evolutionary potential. Standing genetic variation is thought to be more

183 important for adaptation (Barrett & Schluter 2008), because *de novo* mutations start at a low

184 frequency (one copy), while older variants have had time to reach higher frequencies in a population,

185 either through drift or selection. Furthermore, older variants have reduced linkage disequilibrium (are

present on more diverse genetic backgrounds) softening selective sweeps (Barrett & Schluter 2008).
Standing genetic variation can also increase the likelihood that *de novo* mutations produce new

- Standing genetic variation can also increase the likelihood that *de novo* mutations produce new
   phenotypes, leading to increased evolvability (Tenaillon & Matic 2020). These results suggest that
- allelic richness is the better indicator of recent demographic processes, and that heterozygosity is the
- 190 better indicator of the evolutionary potential in a population.

191 Although we have only demonstrated the timescales for the recovery of neutral diversity, our

192 estimates are likely to provide lower limits for the recovery of non-neutral diversity. The genetic

193 variation that is most likely to contribute to future adaptation are mutations that are neutral or nearly

194 neutral in the current environment and which may provide fitness benefits under future conditions

(Barrett & Schluter 2008). Weakly deleterious variants will behave as "effectively neutral" in small
 populations and hence will recover at rates similar to those of neutral alleles. Some proportion of non-

neutral diversity is selected against, so its recovery is expected to be slower than that of neutral

- 198 diversity (Willi et al. 2006). Therefore, we expect that the recovery of evolutionary potential occurs
- 199 over a timescale similar to or longer than that of neutral diversity, suggesting that our estimated
- 200 recovery times are lower bounds.
- 201

# 202 4 Case study: The demise of the iconic Australian koala

We illustrate the permanency of genetic diversity loss in a threatened species via empirical data for an iconic Australian marsupial, the koala (*Phascolarctos cinereus*). Land clearing across almost the entire east coast of Australia, along with fur trade in the 1900s and ongoing disease epidemics, have

205 entire east coast of Australia, along with fur trade in the 1900s and ongoing disease epidemics, have 206 driven large reductions in koala population sizes, especially in the southern parts of their range

206 ariven large reductions in koala population sizes, especially in the southern parts of their range 207 (Adams-Hosking et al. 2016). Recent genomic analyses have found that the Victorian population with

208  $N_e = 19$ , contains 58% of the heterozygosity present in the North Queensland population with  $N_e =$ 

209 250 (McLennan et al. 2024).

210 We can use the framework presented here to calculate the time that would be needed for the Victorian 211 population to recover to the equilibrium heterozygosity, if the  $N_e$  of this population were

- 212 hypothetically increased to 500. If we assume that the equilibrium value when  $N_e = 500$  is at least that
- 213 seen in the North Queensland population, then we can use  $H_{0/eq} = 0.58$  as an upper estimate of the
- starting heterozygosity in Victoria. We can then use a rearrangement of formula (2) to estimate the
- time needed for Victoria to regain 90% of the heterozygosity present in a population with  $N_e = 500$
- 216 (i.e., when  $H_{t/eq} = 0.9$ ). In isolation, the Victorian population would take 1435 generations, or ~
- 217 10,000 years (assuming a generation time of ~7 years) to recover to 90% of this equilibrium genetic

218 diversity from natural mutations alone (full working in Appendix). Our results highlight the need for

- 219 other measures, such as translocations, to improve the genetic diversity and hence the evolutionary
- 220 potential of this population.

We have also shown that the easiest way to maintain evolutionary potential is to limit how much is lost during population bottlenecks. Two koala populations in New South Wales have similar effective population sizes to Victoria ( $N_e = 27-28$ ) but have not yet experienced the same level of genetic diversity loss (McLennan et al. 2024). This suggests that population retractions in New South Wales occurred more recently, and recovering  $N_e$  in these populations has the potential to prevent the loss of diversity to Victorian levels.

227

# 228 5 Conclusion: so what can we do?

Genetic diversity is being lost by species on a global scale (Shaw et al. 2025); here we havedemonstrated that the natural evolutionary process of mutation is insufficient to *restore* genetic

- 231 diversity in threatened species. The permanent loss of historical genetic diversity and evolutionary
- potential is concerning, especially considering the speed at which selective pressures are currentlychanging. Human-mediated interventions are required to reduce the extinction risk of threatened
- 234 species and a number of current management practices are able to maximise the maintenance of
- evolutionary potential with the remaining diversity (Bolam et al. 2023). Recently threatened species,
- or those with longer generations, may have experienced bottlenecks for a short number of generations,
- 237 meaning that they have not yet lost a substantial amount of genetic diversity (Nei et al. 1975).
- 238 Increasing the effective population size for these populations is required to prevent the loss of
- diversity. Genetically depleted populations can be recovered by increasing the connectivity to, or
- direct supplementation from, other genetically diverse populations (Clarke et al. 2024) and concerns
- about outbreeding depression are often overstressed (Chan et al. 2019). Through this method, theeffective population size of the entire population can be increased, rather than genetic drift reducing
- the diversity in each population separately. Assisted gene flow from populations adapted to future
- climate conditions can further enhance resilience (Aitken & Whitlock 2013).
- 245 In cases where there is depleted genetic diversity in the entire species, the only remaining pathways
- for genetic diversity recovery are more drastic but less proven, and in some cases controversial.
- 247 Assisted introgression or interspecific hybridisation can introduce valuable genetic diversity to a
- threatened species and help preserve at least part of the original species' genome (Chan et al. 2019).
- Finally, introducing historical or novel diversity to the most severely depleted species through
- 250 synthetic biology may represent a last resort option (Kosch et al. 2022). Our results demonstrate the 251 effective permanency of genetic diversity losses, which highlights the critical need for rapid
- intervention methods that prevent further loss, or even increase genetic diversity, to ensure the long-
- 252 Intervention methods that prevent further loss, or even increase genetic divers253 term survival of species.
- 254

# 255 Acknowledgements and Data

- 256 The authors have no conflicts of interest. The full derivation of equations presented in this manuscript
- 257 can be found in the <u>Appendix</u>. Scripts used to simulate population bottlenecks, plot mathematical
- 258 formulas and summarise previous estimates of mutation rates can be found at
- 259 github.com/tobykovacs796/diversity\_recovery.git.



# 261 FIGURE 1

260

262 (a) Frequency histogram of published mutation rates for multicellular organisms, from the database 263 assembled and updated by Wang and Obbard (2023). (b) Number of generations required to recover 264 90% of the equilibrium heterozygosity in a population with  $N_e = 500$ , starting with 50% of the 265 equilibrium heterozygosity. This graph uses a rearrangement of formula (1) which calculates 266 heterozygosity as a function of time, the effective population size and the mutation rate. This graph shows the effect that mutation rate has on the time taken to recover heterozygosity, with there being 267 no effect for low mutation rates (when  $2N_e \gg \frac{1}{2\mu}$ ), but a negative relationship for higher mutation 268 rates (when  $2N_e \ll \frac{1}{2\mu}$ ). Yellow shading indicates the range of estimated mutation rates for 269 multicellular organisms, as seen in panel a). The red dashed line indicates  $\mu = 0.0005$ , which is when 270  $2N_e = \frac{1}{2\mu}$  for  $N_e = 500$ . Around this value both  $N_e$  and  $\mu$  impact the recovery of genetic variation. (c) 271 272 Loss of heterozygosity during a population bottleneck, calculated using formula (2). The plot shows 273 the loss of genetic diversity in a population with various bottleneck sizes, starting at the equilibrium 274 diversity when  $N_e = 500$ . The y axis shows heterozygosity divided by the equilibrium heterozygosity for  $N_e = 500$ . (d) Recovery of heterozygosity after a population bottleneck, calculated using formula 275 276 (2). The plot shows the gain of genetic diversity in a population with  $N_e = 500$ , starting at 10%, 50% 277 and 80% of the equilibrium heterozygosity (blue, orange and green respectively). The y axis indicates 278 heterozygosity, divided by the equilibrium heterozygosity at  $N_e = 500$ . 279



## 281 FIGURE 2

280

282 Simulations of the loss and recovery of diversity during and after a population bottleneck. Simulations were run with an initial population size of  $N_c = 500$  before the population was reduced to  $N_c = 50$ 283 (bottleneck) for 100 generations, followed by a return to  $N_c = 500$ . Ten replicate simulations were run 284 for each scenario. Heterozygosity and allelic richness were averaged across replicates, with pale 285 ribbons indicating 95% confidence intervals. (a) Mean heterozygosity across replicates, for three 286 287 different mutation rates ( $10^{-7}$ ,  $10^{-8}$ , and  $10^{-9}$  mutations/site/generation) with a bottleneck size of N<sub>c</sub> = 50. (b) Mean allelic richness across replicates, for three different mutation rates  $(10^{-7}, 10^{-8}, \text{ and } 10^{-9})$ 288 mutations/site/generation) with a bottleneck size of  $N_c = 50$ . (c) Comparison of the loss and 289 recovery of allelic richness (dashed line) and heterozygosity (solid line) during and after a bottleneck. 290 Simulations were run with a mutation rate of  $10^{-8}$ . (d) Site frequency spectra at five time points 291 292 through the simulation. Pre-bottleneck, the population is in mutation-drift equilibrium and the SFS has an equilibrium distribution. At 20 and 90 generations after the population-size reduction, there has 293 294 been a large decrease in allelic richness which is predominantly driven by the loss of low frequency 295 alleles. At 150 generations, shortly after the population-size recovery, there has been a large increase 296 in allelic richness which is predominantly driven by the gain of low-frequency alleles. At 2000

- generations, drift has had enough time to allow the frequency of rare alleles to increase in the population and the SFS returns to its pre-bottleneck equilibrium distribution.
- 297 298

### 299 SUPPLEMENTARY FIGURES

300



301 302 Supplementary Figure 1. Population simulations of the loss and recovery of allelic richness 303 (dashed line) and heterozygosity (solid line) during and after a population bottleneck.

304

Simulations were run with an initial population size of  $N_c$  = 500 before the population size was 305 reduced to  $N_c = 2$  (red), 5 (olive), 10 (green), 50 (blue) and 200 (purple) for 100 generations,

306 followed by a return to  $N_c$  = 500. Simulations were run with a mutation rate of 10<sup>-8</sup>

307 mutations/site/generation.

308

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# Appendix to: Genomic data confirms that mutation cannot restore genetic diversity lost through population bottlenecks

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### Derivation of $H_t$ , $H_{t/eq}$ and t 1

### 1.1 Deriving a formula for Heterozygosity as a function of time as it moves towards equilibrium $(H_t)$

We want to derive an equation for  $H_t$  (the heterozygosity at generation t) in terms of t (generations) for constants  $\mu$  (mutation rate) and  $N_e$  (effective population size). First we will work with  $F_t$  (the inbreeding coefficient at generation t) because the equations are simpler, before converting to  $H_t$  using the equation:  $H_t = 1 - F_t$ .

The generalised formula of the inbreeding coefficient based on genetic drift and mutations was derived by Crow and Kimura (1964) as:

$$F_t = \frac{(1-\mu)^2}{2N_e} + \frac{(1-\mu)^2(2N_e-1)}{2N_e}F_{t-1}$$

We recognise that his equation is in the form of the truncated geometric series:  $F_t = a + bF_{t-1}$ , where:

$$a = \frac{(1-\mu)^2}{2N_e}$$
 and  $b = \frac{(1-\mu)^2(2N_e-1)}{2N_e}$ 

Based on the geometric series formula, if an equation is in the form  $F_t = a + bF_{t-1}$  then we can make a formula for  $F_t$  in terms of time (t) and the inbreeding value at t = 0 ( $F_0$ ):  $F_t = \frac{a(b^t-1)}{b-1} + b^t F_0$ Substituting into the geometric series formula and then simplifying we get:

$$\begin{split} F_t &= \frac{\frac{(1-\mu)^2}{2N_e} \left( \left(\frac{(1-\mu)^2 (2N_e-1)}{2N_e} \right)^t - 1 \right)}{\frac{(1-\mu)^2 (2N_e-1)}{2N_e} - 1} + \left( \frac{(1-\mu)^2 (2N_e-1)}{2N_e} \right)^t F_0 \\ F_t &= \frac{\frac{(1-\mu)^2}{2N_e} \left( \left(\frac{(1-\mu)^2 (2N_e-1)}{2N_e} \right)^t - 1 \right)}{\frac{(1-\mu)^2}{2N_e} \left( 2N_e - 1 - \frac{2N_e}{(1-\mu)^2} \right)} + \left( \frac{(1-\mu)^2 (2N_e-1)}{2N_e} \right)^t F_0 \\ F_t &= \frac{\left(\frac{(1-\mu)^2 (2N_e-1)}{2N_e} \right)^t - 1}{2N_e - 1 - \frac{2N_e}{(1-\mu)^2}} + \left( \frac{(1-\mu)^2 (2N_e-1)}{2N_e} \right)^t F_0 \\ F_t &= \frac{\left(\frac{(1-\mu)^2 (2N_e-1)}{2N_e} \right)^t}{2N_e - 1 - \frac{2N_e}{(1-\mu)^2}} - \frac{1}{2N_e - 1 - \frac{2N_e}{(1-\mu)^2}} + \left( \frac{(1-\mu)^2 (2N_e-1)}{2N_e} \right)^t F_0 \\ F_t &= \left( \frac{(1-\mu)^2 (2N_e-1)}{2N_e} \right)^t \left( \frac{1}{2N_e - 1 - \frac{2N_e}{(1-\mu)^2}} + F_0 \right) - \frac{1}{2N_e - 1 - \frac{2N_e}{(1-\mu)^2}} \end{split}$$

We can calculate the Heterozygosity  $(H_t)$  from the inbreeding coefficient  $(F_t)$  using the equation:  $H_t = 1 - F_t$  (and  $H_0 = 1 - F_0$ ) which gives us:

$$H_t = 1 - \left(\left(\frac{(1-\mu)^2(2N_e-1)}{2N_e}\right)^t \left(\frac{1}{2N_e-1-\frac{2N_e}{(1-\mu)^2}} + 1 - H_0\right) - \frac{1}{(2N_e-1-\frac{2N_e}{(1-\mu)^2})}\right)$$

which simplifies to

$$H_t = \left(\frac{(1-\mu)^2(2N_e-1)}{2N_e}\right)^t \left(H_0 - \frac{1}{2N_e - 1 - \frac{2N_e}{(1-\mu)^2}} - 1\right) + 1 + \frac{1}{\left(2N_e - 1 - \frac{2N_e}{(1-\mu)^2}\right)}$$

In the main text we show that for all known metazoa, mutation rates are very small and therefore the square of this number is very very small. We can simplify this equation using approximations assuming that  $\mu$  is a very small number and  $\mu^2 \approx 0$ 

$$H_t = \left(\frac{(1-2\mu)(2N_e-1)}{2N_e}\right)^t \left(H_0 - \frac{1}{2N_e - 1 - \frac{2N_e}{1-2\mu}} - 1\right) + 1 + \frac{1}{\left(2N_e - 1 - \frac{2N_e}{1-2\mu}\right)^t}$$

which simplifies to

$$H_t = \left((1-2\mu)(1-\frac{1}{2N_e})\right)^t \left(H_0 - \frac{4N_e}{4N_e + 1 - 2\mu}\right) + \frac{4N_e}{4N_e + 1 - 2\mu}$$

When considering the gaining of heterozygosity towards the equilibrium, this could be better arranged as:

$$H_t = \frac{4N_e}{4N_e + 1 - 2\mu} - \left((1 - 2\mu)(1 - \frac{1}{2N_e})\right)^t \left(\frac{4N_e}{4N_e + 1 - 2\mu} - H_0\right) \tag{1}$$

This equation demonstrates that there is a exponential decay of the difference between the starting and equilibrium heterozygosity value. The rate of this decay is determined by  $(1 - 2\mu)(1 - \frac{1}{2N_e})$ where the impact of  $N_e$  and  $\mu$  depends on their relative sizes. When  $\frac{1}{2N_e} \gg 2\mu$  the recovery time is proportional to  $N_e$  (the time to recover 90% of equilibrium heterozygosity starting at zero  $(H_{0/eq} = 0)$ is  $\approx 4.6N_e$ ), but when  $\frac{1}{2N_e} \ll 2\mu$  the recovery time is proportional to  $\frac{1}{\mu}$  (the time to recover 90% of equilibrium heterozygosity starting at zero  $(H_{0/eq} = 0)$  is  $\approx \frac{1.15}{\mu}$ ). Note that in the main text, we rearrange these parameters so that both sides of the equation are larger than 1 and easier to think about  $(\frac{1}{2\mu} \gg 2N_e$  instead of  $\frac{1}{2N_e} \gg 2\mu$ ).

We can then graph this formula over a range of mutation rates, to see the influence of this parameter (Fig 1b in main text). You can use this equation to test the influence of different values of  $\mu$  and  $N_e$  and plot how neutral genetic diversity changes as a population moves towards equilibrium.

For the known range of genome-wide metazoan mutation rates  $(10^{-9} - 10^{-7})$ , most populations, and especially those of conservation concern, will have small enough populations such that  $\frac{1}{2N_e} \gg 2\mu$  and the time that is required to approach equilibrium heterozygosity is independent of the mutation rate. Hence we can further simplify this equation to:

$$H_t = \frac{4N_e\mu}{4N_e\mu + 1} - (1 - \frac{1}{2N_e})^t (\frac{4N_e\mu}{4N_e\mu + 1} - H_0)$$
(2)

# 1.2 Deriving a formula for the change in Heterozygosity as a proportion of the equilibrium heterozygosity as it moves towards equilibrium $(H_{t/eq})$

We have shown that the mutation rate does not impact the time it takes for a population to approach equilibrium for most species, however the mutation rate does impact the equilibrium heterozygosity value. It could then be useful to consider heterozygosity in terms of the proportion (or multiple) of the equilibrium heterozygosity, which allows us to predict a universal recovery of heterozygosity across all species. To do this, we convert  $H_0$  and  $H_t$  into  $H_{0/eq}$  and  $H_{t/eq}$  by dividing by the formula for equilibrium heterozygosity given in Crow and Kimura (1964).

$$\begin{split} H_{0/eq} &= \frac{H_0}{H_{eq}} \\ H_{eq} &= \frac{4N_e\mu}{4N_e\mu+1} \text{ (from Crow and Kimura 1964)} \\ H_{0/eq} &= \frac{H_0}{\left(\frac{4N_e\mu}{4N_e\mu+1}\right)} \\ H_0 &= H_{0/eq} \left(\frac{4N_e\mu}{4N_e\mu+1}\right) \end{split}$$

We can then substitute this into equation 2

$$\begin{split} H_t &= \frac{4N_e\mu}{4N_e\mu + 1} - \left(1 - \frac{1}{2N_e}\right)^t \left(\frac{4N_e\mu}{4N_e\mu + 1} - H_{0/eq}\left(\frac{4N_e\mu}{4N_e\mu + 1}\right)\right) \\ H_t &= \frac{4N_e\mu}{4N_e\mu + 1} \left(1 - \left(1 - \frac{1}{2N_e}\right)^t (1 - H_{0/eq})\right) \end{split}$$

$$H_{t/eq} = \frac{H_t}{H_{eq}}$$

$$H_{eq} = \frac{4N_e\mu}{4N_e\mu + 1} \quad \text{from Crow and Kimura (1964)},$$

$$H_{t/eq} = \frac{\frac{4N_e\mu}{4N_e\mu + 1} \left(1 - (1 - \frac{1}{2N_e})^t (1 - H_{0/eq})\right)}{\frac{4N_e\mu}{4N_e\mu + 1}}$$

$$H_{t/eq} = 1 - (1 - \frac{1}{2N_e})^t (1 - H_{0/eq}) \quad (3)$$

You can use this equation to plot how neutral genetic diversity changes as a population moves towards equilibrium.

We can rearrange this formula to easily calculate the time taken to gain or lose a certain amount of heterozygosity

$$\begin{split} H_{t/eq} &= 1 - (1 - \frac{1}{2N_e})^t (1 - H_{0/eq}) \\ (1 - \frac{1}{2N_e})^t &= \frac{1 - H_{t/eq}}{1 - H_{0/eq}} \\ \ln\left((1 - \frac{1}{2N_e})^t\right) &= \ln\left(|\frac{1 - H_{t/eq}}{1 - H_{0/eq}}|\right) \\ t \ln\left(1 - \frac{1}{2N_e}\right) &= \ln\left(|\frac{1 - H_{t/eq}}{1 - H_{0/eq}}|\right) \end{split}$$

$$t = \frac{\ln\left(|\frac{1-H_{t/eq}}{1-H_{0/eq}}|\right)}{\ln\left(1-\frac{1}{2N_e}\right)}$$
(4)

### Calculating how long heterozygosity takes to recover following a population bottleneck: We can use 4 to calculate the time required for the Victorian population of koalas to recover to 90% of the equilibrium diversity ( $H_{t/eq} = 0.9$ ) if the effective population size was increased to 500 ( $N_e = 500$ ). In this example, the Victorian koalas are starting with 58% that estimated in the Northern Queensland population which is likely a conservative lower limit for the equilibrium diversity ( $H_{0/eq} = 0.58$ ).

$$t = \frac{\ln\left(\left|\frac{1-0.9}{1-0.58}\right|\right)}{\ln\left(1-\frac{1}{2\times 500}\right)}$$
  
\$\approx 1435 generations

**DOUBLECHECKING**: To double check that we haven't messed up the algebra, see if after the assumptions, formula 2 can be used to derive the same equilibrium formula as Crow and Kimura (1964).

$$\begin{split} &((1-2\mu)(1-\frac{1}{2N_e}))^t \to 0 \text{ because } |(1-2\mu)(1-\frac{1}{2N_e})| < 1 \\ \Rightarrow &\lim_{t \to \infty} H(t) = (0)(H_0 - \frac{4N_e\mu}{4N_e\mu + 1}) + \frac{4N_e\mu}{4N_e\mu + 1} \\ \Rightarrow &\lim_{t \to \infty} H(t) = \frac{4N_e\mu}{4N_e\mu + 1} \end{split}$$

This is the same formula derived by Crow and Kimura (1964).

IF WE ONLY USE THE APPROX FOR u2 EQUALS 0: Then we get the following

$$\begin{split} H_t &= 1 - \left( \left( \frac{(1-\mu)^2 (2N_e - 1)}{2N_e} \right)^t \left( \frac{1}{2N_e - 1 - \frac{2N_e}{(1-\mu)^2}} + 1 - H_0 \right) - \frac{1}{(2N_e - 1 - \frac{2N_e}{(1-\mu)^2})} \right) \\ H_t &= 1 - \left( \left( (1-2\mu+\mu^2) \left( 1 - \frac{1}{2N_e} \right) \right)^t \left( \frac{1}{2N_e - 1 - \frac{2N_e}{(1-\mu)^2}} + 1 - H_0 \right) - \frac{1}{(2N_e - 1 - \frac{2N_e}{(1-\mu)^2})} \right) \\ H_t &= 1 - \left( \left( (1-2\mu+\mu^2) \left( 1 - \frac{1}{2N_e} \right) \right)^t \left( \frac{(1-\mu)^2}{(1-\mu)^2 (2N_e - 1) - 2N_e} + 1 - H_0 \right) - \frac{(1-\mu)^2}{(1-\mu)^2 (2N_e - 1) - 2N_e} \right) \\ H_t &= 1 - \left( \left( (1-2\mu+\mu^2) \left( 1 - \frac{1}{2N_e} \right) \right)^t \left( \frac{1-2\mu+\mu^2}{(1-2\mu+\mu^2) (2N_e - 1) - 2N_e} + 1 - H_0 \right) - \frac{1-2\mu+\mu^2}{(1-2\mu+\mu^2) (2N_e - 1) - 2N_e} \right) \\ H_t &= \left( (1-2\mu+\mu^2) \left( 1 - \frac{1}{2N_e} \right) \right)^t \left( H_0 - \frac{1-2\mu+\mu^2 + (1-2\mu+\mu^2) (2N_e - 1) - 2N_e}{(1-2\mu+\mu^2) (2N_e - 1) - 2N_e} \right) \\ &+ \frac{1-2\mu+\mu^2 + (1-2\mu+\mu^2) (2N_e - 1) - 2N_e}{(1-2\mu+\mu^2) (2N_e - 1) - 2N_e} \\ H_t &= \left( (1-2\mu+\mu^2) \left( 1 - \frac{1}{2N_e} \right) \right)^t \left( H_0 - \frac{2N_e\mu^2 - 4N_e\mu}{(1-2\mu+\mu^2) (2N_e - 1) - 2N_e} \right) - \frac{2N_e\mu^2 - 4N_e\mu}{(1-2\mu+\mu^2) (2N_e - 1) - 2N_e} \\ H_t &= \left( (1-2\mu+\mu^2) \left( 1 - \frac{1}{2N_e} \right) \right)^t \left( H_0 - \frac{2N_e\mu^2 - 4N_e\mu}{2N_e\mu^2 - 4N_e\mu} - 1 - 2N_e\mu^2 - 4N_e\mu} \right) \\ H_t &= \left( (1-2\mu+\mu^2) \left( 1 - \frac{1}{2N_e} \right) \right)^t \left( H_0 - \frac{4N_e\mu - 2N_e\mu^2}{2N_e\mu^2 - 4N_e\mu} - 1 - 2N_e\mu^2} \right) \\ H_t &= \left( (1-2\mu+\mu^2) \left( 1 - \frac{1}{2N_e} \right) \right)^t \left( H_0 - \frac{4N_e\mu - 2N_e\mu^2}{2N_e\mu^2 - 4N_e\mu} - 1 + 2N_e\mu^2} \right) \\ H_t &= \left( (1-2\mu+\mu^2) \left( 1 - \frac{1}{2N_e} \right) \right)^t \left( H_0 - \frac{4N_e\mu - 2N_e\mu^2}{2N_e\mu^2 - 4N_e\mu} - 1 + 2N_e\mu^2} \right) \\ H_t &= \left( (1-2\mu+\mu^2) \left( 1 - \frac{1}{2N_e} \right) \right)^t \left( H_0 - \frac{4N_e\mu - 2N_e\mu^2}{2N_e\mu^2 - 4N_e\mu} - 1 + 2N_e\mu^2} \right) \\ H_t &= \left( (1-2\mu+\mu^2) \left( 1 - \frac{1}{2N_e} \right) \right)^t \left( H_0 - \frac{4N_e\mu - 2N_e\mu^2}{2N_e\mu^2 - 4N_e\mu^2} - \frac{4N_e\mu - 2N_e\mu^2}{4N_e\mu^2 - 4N_e\mu^2} - \frac{4N_e\mu^2 - 2N_e\mu^2}{4N_e\mu^2 - 4N_e\mu^2 - 2N_e\mu^2} \right) \\ H_t &= \left( (1-2\mu+\mu^2) \left( 1 - \frac{1}{2N_e} \right) \right)^t \left( H_0 - \frac{4N_e\mu - 2N_e\mu^2}{2N_e\mu^2 - 2\mu_e\mu^2} \right) \\ - \frac{4N_e\mu^2 - 2N_e\mu^2}{4N_e\mu^2 - 2N_e\mu^2 - 2\mu_e\mu^2} \right) \\ H_t &= \left( (1-2\mu+\mu^2) \left( 1 - \frac{1}{2N_e} \right) \right)^t \left( H_0 - \frac{4N_e\mu^2 - 2N_e\mu^2}{4N_e\mu^2 - 2\mu_e\mu^2} - \frac{4N_e\mu^2 - 2N_e\mu^2}{4N_e\mu^2 - 2\mu_e\mu^2} \right) \\ + \frac{4N_$$