¹ Disordered and Partially Structured Models in Community Ecology: ² What are they, and how do we use them?

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Abstract

Community ecology describes how species interact with each other and with their environment. In 8 nature, processes can be very complex because they involve hundreds to thousands of species interacting 9 with each other in complex environmental landscapes. Classical approaches that have provided key 10 insights have largely focused on the study of tractable subsets of species and patches, but these do not 11 always adequately address the wider scope of natural complexity. Alternate approaches that use specific 12 parameters and/or that use simulations to study such highly diverse systems are problematic because they 13 can become very detailed, system-specific, and easily divorced from general principles. Finally, 'minimal' 14 models to explain data exist (e.g. null models, 'neutral theory' and 'entropy based' models), but they 15 often do not provide adequate connections to experimental or mechanistic studies and results. Here we 16 describe and discuss an alternate approach that seeks to link basic processes of community assembly 17 18 (environmental heterogeneity, species interactions, dispersal, and stochasticity) with each other using 'disordered systems models' to make robust predictions about community structure, albeit without the 19 detail of more system-specific approaches. We describe the logic of the approach, outline the methods 20 involved, and identify important limitations. We also describe how this approach can be expanded 21 to better incorporate additional nonrandom structure (such as correlated parameters) in these basic 22 processes and leading to 'partially structured models', and we introduce the idea that this could also 23 be applied to metacommunities. Although implementing this approach in empirical studies will still be 24

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quite challenging, these approaches reduce the complexity of the overall problem by orders of magnitude,
making it a promising approach to improve the study of biodiversity in realistic landscapes.
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1 Introduction

² Community ecology (and its relation to ecosystems ecology) is inherently a science about complexity. This
³ important idea is captured by Darwin's 'entangled bank' that describes the complex nature of interspecific
⁴ interactions and environmental factors (and, in Darwin's mind, their impacts on evolution). The nature of
⁵ this complexity is exemplified by starting with simple approaches (e.g. exponential population growth) and
⁶ gradually adding complexity (e.g. logistic growth, pairwise direct interactions, dispersal, etc.) that eventually
⁷ reaches arbitrary levels of complexity. To a substantial degree, this approach describes the historical legacy
⁸ of community ecology today.

An important step in illustrating the consequences of complexity involves the idea of indirect interaction 9 effects among species, as described by Darwin's metaphor. In models that involve a small number of species 10 (that is, 'modules' sensu Holt and Hochberg (2001), see also McPeek (2021)), the consequences of such indirect 11 effects can be resolved, so that the overall 'net effects' can be derived from direct effects, resulting in concepts 12 such as trophic cascades (Hairston et al., 1960) and apparent competition (Holt, 1977). However, building up 13 from this to many more species becomes increasingly problematic, both theoretically and empirically, since 14 the number of interactions increases as the square of the number of species. This approach effectively reaches 15 its limit when the number of species starts to exceed five or so (McPeek, 2021). 16

Under some conditions, it is possible to get some insight about these net effects in highly diverse sys-17 tems (Levine, 1976; Lawlor, 1979) but these insights can be strongly constrained because of the delicate 18 context dependence that determines these net effects even when the direct effects are fixed (Schaffer, 1981). 19 How can we realistically investigate the community ecology of ecosystems that involve many species (i.e., 20 dozens, hundreds, or more)? Here we describe an approach based first on 'disordered systems models' (DSM) 21 that can help us understand generic effects, and then on 'partially structured models' (PSMs) that further 22 resolve possible deviations from DSMs in ecologically interesting ways. These approaches seek to provide ro-23 bust insights about highly diverse communities and metacommunities without requiring completely detailed 24 knowledge of the species and ecosystems involved, and thus bypassing the need to understand the detailed 25 complexity of the 'entangled bank'. 26

To provide context for our claims, we imagine at least three plausible scenarios that can justify different possible approaches to the study of community ecology in the case of highly diverse communities and metacommunities:

One possibility is that much of this complexity is not important at all, and that there are only a few
 important components (e.g. species, traits or functional groups) to the dynamics. If so, we can justify focus ing on those few particular details. We can call this approach 'classical simplicity', since the study of such

key mechanisms is a traditional approach in community ecology, characterized by the 'modules' approach
described above.

2. Another possibility is that many, or all, details affect community dynamics in a deeply inter-correlated 35 and interconnected way, which cannot be easily dissected (or even dissected at all!). We posit that such a 36 situation is fragile by definition: altering any single component of the community can impact all the others 37 in major ways, somewhat like a computer program where removing one character in the code will cause the 38 whole program to stop working. We can call this 'true complexity'. We acknowledge that it is possible that 39 communities in nature are, in fact, characterized by such complexity. If so, the study of community ecology 40 would be enormously challenging (Lawton, 1999). We question, however, if such ecosystems are likely to 41 be sufficiently closed and co-evolved to allow the emergence and persistence of such fragile structures and, 42 consequently, whether these structures are likely to be leading forces shaping the aggregate behavior of many 43 entire ecosystems. 44

3. Finally, it is possible that many (or all) of the factors involved matter roughly equally and influence com-45 munities in heterogeneous but weakly correlated directions. If so, their combined impacts will often cancel 46 each other out, so as to produce variation that resembles random variation, at least with respect to the ways 47 in which they alter many of the resulting patterns, especially those that represent aggregate features of the 48 ecosystem. Consequently, such aggregate properties can persist as important causes and consequences of 49 the statistical moments of these details (e.g. mean and variance of interactions) rather than being detail-50 dependent as in 2 above. This perspective is the one we base our arguments about disordered (and partially 51 ordered) models, and we call this "emergent simplicity". 52

If (meta-)communities actually show 'true complexity', we may expect most of the variation within metacommunities to appear to be highly context-dependent and have little resolvable structure. Recent developments in parsing metacommunity variation indicate that this is not the case (Leibold and Peres-Neto in prep), but rather that the observable variation can be related to some mixture of environmental, spatial, and among-species covariation. While resolving the effects of these factors is difficult, this provides evidence that 'true complexity' is not likely, so that some aspects of the complexity could well be understood as 'emergent simplicty.'

The study of emergent simplicity is made possible by the careful invocation of randomness and probabilistic modeling. More specifically, we can identify two kinds of "randomness" that affect how we think about these models: "Stochasticity", i.e. random fluctuations over time (like environmental perturbations or stochastic demographic effects), and "disorder" that describes the apparent and effective randomness that results from the statistical behavior of diffuse causal networks of otherwise complex but intermingled deterministic fixed processes. Of course, there is information that is lost when we lump these "disordered" deterministic processes by using statistical descriptions of the parameters instead of the details. The problem is that identifying and quantifying the detailed information can be very difficult when there are more than a handful of species. Additionally, doing so can also lead to errors when we don't have precise knowledge of the parameters and functions involved to make the detailed behavior of the ecosystem. Instead, here, we look to identify and focus on patterns that are likely to be robust to these precise details, while acknowledging that our models will not make predictions about other patterns that are not as robust.

We describe the general approach with models that characterize 'classical simplicity' in the form of pairwise 72 Lotka-Volterra models (we briefly discuss other options in the prospectus of this manuscript). Following May 73 (1972) (see also Novak et al. (2016)), we then imagine that a large number of such pairwise Lotka-Volterra 74 equations can be combined into a larger model that can be expressed in matrix form called the community 75 matrix (we note that the community matrix is typically used to describe local communities, but that this 76 matrix can be expanded to describe metacommunities as well (Gravel et al., 2016)). The key step in taking 77 a disordered modeling approach is then to replace the actual community matrix with a representation that 78 is only based on means and variances for the various components (figure 4). In practice this characterization 79 is only valid if there are a sufficient number of species so that the means and variances of the matrix entries 80 are sufficiently well characterized so as to be meaningful; e.g. one or several dozen species. We note that the 81 modeling of intermediate-size systems (say, 5-12 species) remains a daunting challenge. 82

Although we describe some of the mathematics involved, we aim for a more general understanding that does not require substantial mathematical expertise, and emphasize how this approach can be of interest to empirical ecologists. A rigorous mathematical overview, though more restricted in scope, is found in Akjouj et al. (2024). Technical presentations at a physicist level of rigor are found in Bunin (2017) for the random Lotka-Volterra model, and Cui et al. (2024) for random niche or consumer-resource models. Finally, a tutorial on how to implement these ideas in a concrete way, detailing the practicalities and challenges of using them in theoretical and data-based research, is provided in Barbier (2025).

90 1 Methods:

⁹¹ 1.1 The strategy of modeling using DSMs and PSMs:

Taking an approach that is based on DSMs (and subsequently on PSMs see below) involves a certain strategy to modeling that differs from the more conventional approaches in 'classical simplicity' or 'true complexity'. Rather than settling on a particular model structure and just 'filling in' parameters, we have to think carefully about which model components we want to aggregate and simplify using means and variances, and the types of patterns we hope to study. The strategy also differs from modeling approaches based on 'minimal' models, such as null models, neutral theory, or entropy-based approaches that start with data and ask what model structures can account for them, typically looking for minimally parameterized models. This is because we are interested in maintaining a strong connection to the fundamental dynamic processes and mechanisms of community dynamics.

Instead, the intermediate perspective we describe asks how we can identify and focus on patterns that can 101 be linked to processes and mechanisms in a robust way that minimally depends on detailed parameterization 102 of complex models. We try to structure our models and identify patterns that are insensitive to factors 103 other than those we chose to include in our approach based on 'emergent simplicity'. This means that 104 many differences in model selection, even striking ones such as being discrete or continuous, deterministic 105 or probabilistic, spatial or non-spatial, can be treated as irrelevant if/when they do not impact our selected 106 target outcomes. One way of articulating this idea is: all details matter, but they do not necessarily matter 107 for what we might care about. 108

Consequently, we focus on what might be called 'macroscopic' rather than detailed 'microscopic' model 109 components (and applying this perspective to patterns as well as processes). Macroscopic predictions are 110 thus likely to result from the aggregate behavior of model components, in the same way that the ideal gas law 111 is a robust outcome of the complex behavior of many colliding molecules. It turns out, in fact, that trying to 112 increase microscopic fidelity by adding more model ingredients does not always improve aggregate accuracy, 113 and can even detract from it if poorly estimated or biased parameters are added. Hence, we want to think 114 of models not in terms of how they fit the details of our microscopic intuition, but in terms of addressing the 115 following issues: 116

- What kind of macroscopic (i.e. aggregate) behaviors and patterns can they robustly generate?
- Are these interesting to us?
- Can we adjust model parameters so that they can quantitatively match observed patterns?
- Can parameters that are fit using some patterns also correctly predict other patterns? in which case the model is a workable description of macroscopic reality, even if its details turn out to be untrue to microscopic reality.

123 1.2 Why the Random Lotka-Volterra model?

To illustrate this approach, we focus on one basic model, the Random Lotka-Volterra (RLV) model (Bunin, 2017). This consists of using the well-known Lotka-Volterra equations for a diverse set of interacting species, and then assigning the parameters (i.e. intrinsic growth, carrying capacity, interaction coefficients, and dispersal) randomly among the species based on simple statistical features (e.g. means and variances). We do this because the RLV can be viewed as a highly simplified model that gives a first-order approximation to the most important and/or robust generic properties of more complex models. As such, it can serve as a 'baseline' model for understanding how arbitrarily complex systems might be regulated. We argue that they should be further elaborated, if need be, using Partially Structured Lotka-Volterra (PSLV) models as we describe later in section 2.2.

We also chose to build from the Lotka-Volterra model because it is a well-known generic model that has already been used in this way. However, there are alternate options that could be used for studying community and metacommunity dynamics (e.g. ecosystem-based models that describe stocks and fluxes of materials and energy, community assembly models based on colonization-extinction dynamics that ignore species abundances, etc.). At this point, we are unaware of published works that explicitly study such models with an explicit DSM formulation akin to the one we use here.

Although it seems relatively simple, the RLV model can span a very broad scope of behaviors, including single-point stable equilibrium, oscillations, alternate stable states, and more complex fluctuating dynamics such as high-dimensional and chaotic dynamics. This is because the entanglement of causal processes in the RLV can be extremely high and lead to behaviors that encompass the whole array of dynamics characterized by more complex non-linear models.

We formulate the RLV in a way that captures the idea that community assembly and community dynamics 144 are driven by five basic processes (modified from Vellend (2010, 2016)). These consist of 'density-independent 145 selection' that favors different species at a given site as a function of their local intrinsic population growth 146 and carrying capacity, 'density-dependent selection' that accounts for feedback among component species, 147 'dispersal' that accounts for the movement of individual, 'stochasticity' that describes how chance events such 148 as demographic drift and disturbances affect each species, and 'novelty' that describes how de novo species 149 originate (here we however ignore this speciation process in our model since it is generally much slower than 150 the others). 151

At the most general level, we use a spatialized Lotka-Volterra model that separately accounts for these processes by describing each species as having growth rates r_i , interactions A_{ij} (in which the carrying capacity is thus given by $K_i = 1/A_{ii}$ for intra-specific interactions), stochastic perturbations ξ_i , and dispersal among patches x and y with rates D_i

$$\frac{dN_i(x,t)}{dt} = r_i N_i(x,t) \left(1 - \frac{\sum_j A_{ij} N_j(x,t)}{K_i(x)} \right) + \xi_i(x,t) + \sum_y D_i \left(N_i(y,t) - N_i(x,t) \right)$$
(1)

The most basic version of the Lotka-Volterra model, without stochasticity or spatial fluxes ($\xi_i = D_i = 0$), allows at most one stable state per set of surviving species: for a given set Ω , assuming they can coexist at equilibrium, their abundances must verify:

$$0 = 1 - \sum_{j \in \text{survivors}} A_{ij} N_j^* \qquad \text{for all } i.$$
(2)

Such a linear system of equations can only have one solution (a different system and a different solution for each possible set of species Ω); however, that solution need not be a stable equilibrium (it could be an unstable fixed point, e.g. in case of a limit cycle or bistability between other fixed points), as we discuss in Section 4.

Spatial fluxes can be approximated by simple "diffusion-like" dispersal between adjacent patches, which is enough to qualitatively capture many of the consequences of accounting for space. We may also assume that growth rates r_i or interactions A_{ij} depend on spatial location to introduce spatial heterogeneity. We also note that, here, interactions only happen within one locality, thus the community (or site) scale can be defined as the spatial scale over which species interactions can be modeled as well-mixed – we do not consider here systems where, for instance, different species might interact over vastly different scales e.g. (Zelnik et al., 2021).

¹⁷⁰ For the Random Lotka Volterra model, we proceed as follows:

1) We hypothesize a 'species pool' of size S (the number of species present that can potentially participate in community dynamics).

2) We assign parameters of the L-V equations at random to each of the species in the species pool; we describe
useful ways to do this so as to optimize the study of the RLV model below.

3) We try to use analytical methods to derive long-term expected outcomes of the solution to the RLV model.

¹⁷⁶ We focus on 'aggregated' properties that reflect community-wide attributes rather than predictions about

individual species because these are the most likely to be 'robust'. 4) If we cannot find analytical solutions,

- ¹⁷⁸ we use numerical or simulation approaches.
- ¹⁷⁹ 5) We compare the resulting outcomes to natural patterns (when possible). If we find that they match ¹⁸⁰ reasonably well, we claim that the RLV is sufficient to explain such patterns.
- ¹⁸¹ 6) If this comparison does not match adequately well, we work to develop modifications of the RLV model

¹⁸² by using PSLV approaches that we describe later in section 2.2.

We use the RLV to study how it can predict a number of (meta) community properties that may be robust to the details that are omitted in this approach. For example, Barbier et al. (2018) show that under some conditions (mostly that interspecific interactions aren't too strong), purely local RLVs can predict properties such as the following:

a) local species richness: the size of the subset of species that co-occur (or co-exist) in local patches. It is often more convenient to express this as the ratio of local to global species numbers.

189 b) local evenness in the relative abundances of these species.

c) total abundance (or total biomass, depending on how the RLV is formulated) of organisms d) mean turnover (or production/loss) of organisms (or biomass)

¹⁹² e) mean local CV of species in time.

¹⁹³ 1.3 Applicability of disorder

A simple objection to the use of randomness is that species typically seem to be much more different from each other than simple random draws from a distribution. To clarify this point, we can think about *degrees* of heterogeneity:

• "variance-like" heterogeneity: quantitative variation within a group of species that can be adequately quantified by statistical moments (e.g. a mean and a variance) of their parameters, recognizing that we will expect to occasionally see some species that lay at the extremes of the distribution.

• "systematic" heterogeneity: irreducible qualitative differences among such groups (e.g. autotrophs) versus heterotrophs).

Disorder is clearly applicable to "variance-like" heterogeneity¹hereas "systematic" variance is more naturally suited for partially structured models (described below). Even so, an important lesson from experience is that differences that appear irreducible and fundamental can **sometimes** be ignored, and entities that seem like they belong to different classes can **sometimes** be bunched together for practical purposes. For example, even species that are very different on a *per capita* basis, such as bacteria & whales, may be more comparable in terms of biomass (as demonstrated e.g. by allometric relations holding over many classes of organisms (Hatton et al., 2021) at least for the purpose of understanding certain macroecological patterns.

Consequently, whenever we think about using randomness in a model, we try to find the representation of the system that makes species as equivalent as possible so that their parameters can be represented by their 'variance like' heterogeneity.

 ^{1}w

To illustrate, we start with a simple theoretical example, using the non-spatial, unperturbed version of the equations (1) i.e.

$$\frac{dN_i}{dt} = r_i N_i \left(1 - \frac{N_i - \sum_{j \neq i}^S A_{ij} N_j}{K_i} \right)$$

The carrying capacity K_i is the equilibrium abundance in the absence of other species. Let us assume for now that we are looking at a competitive system where all species have a carrying capacity $K_i > 0$ (though everything that follows can be extended to e.g. trophic systems where predators cannot survive without prey).

We could then change variables to $\eta_i = N_i/K_i$, usually called the relative yield (see references and discussion in Barbier et al. (2021)), and still have a Lotka-Volterra model:

$$\frac{d\eta_i}{dt} = r_i \eta_i \left(1 - \eta_i - \sum_{j \neq i}^S \alpha_{ij} \eta_j \right)$$
(3)

with different interaction coefficients ($\alpha_{ij} = A_{ij}K_j/K_i$). If the A_{ij} are independent random parameters, then the α_{ij} are not independent (and conversely): they are correlated by row and by column due to the factors K_i . To apply disordered systems theory, the most appropriate choice is the one where parameters (α , or Kand A, or others) are as uncorrelated as possible. We note that this is not necessarily the most intuitive representation, and its variables might not be as interpretable as abundances, biomasses or relative yields (see e.g. Spaak and De Laender (2020)).

Hereafter we will use terminology suggesting that our variables N_i represent abundances, but it is crucial to recognize that this will not always be the right choice; instead, they should be understood as a general proxy for a species' importance in community dynamics, perhaps expresses in biomass units or consumption rates etc. When we see apparent inequalities between species, we should first ask how much is really due to some species being more or less important, or whether we can restructure out model to capitalize on expressing it in other units.

232 2 Results

²³³ 2.1 What can be predicted, inferred and tested empirically

Above, we emphasized that disordered systems theory is focused on deriving robust predictions from dynamical models that are parameterized using a statistical approach to describe their parameters. What kinds of data are likely to be robust in this way and what kinds of patterns can we predict about them from the disordered systems approach?

Communities can be described by an amazing variety of attributes (or 'observables'), that can be analyzed 238 for possibly regular arrangements; means, correlations, etc. that we can generally call 'patterns'. As we 239 pointed out earlier, many of these patterns can be very complicated in their details, for example, by often 240 being very species-specific in nature and these are unavailable to study by our approach since we ignore species 241 identities when we replace their attributes as drawn from statistical distributions. Many of these patterns, 242 especially the more detailed ones, are also likely to be 'fragile' in that they depend on specific detailed 243 assumptions that may or may not actually hold. For example, pairwise correlations in species abundances 244 seem like they could be linked to pairwise interactions between them, but this is generally not the case due to 245 the complex intertwined interactions involving multiple (and potentially all) species in the system. Schaffer 246 (1981) showed that even if they could be measured, these correlations did not have any clear link to the 247 pairwise interactions among the species, illustrating the fragility of relating patterns to the driving processes 248 in the complex community. 249

By taking a disordered systems approach to community ecology, we accept that many possible patterns 250 involving observable outcomes are likely to be fragile in this regard, and we try to focus on those that are 251 not; see Figure 3. As discussed in earlier sections of this paper, the models we create sacrifice the need 252 for detailed parameters (or predictors), which are often difficult to quantify anyway, and look for more 253 'macroscopic' predictors that can work equally well in making predictions about these more robust patterns. 254 Next, we discuss one of the most robust properties of RLV models, the existence of qualitatively distinct 255 dynamical regimes, which informs which patterns we can study. We then discuss more extensively how and 256 why we may select robust yet informative patterns to fit our model quantitatively. 257

258 2.1.1 Dynamical regimes

Contrary to simple statistical models (e.g. a linear relation or a normal distribution) where varying parameters 259 will only lead to quantitative changes in predictions, dynamical models like RLV can transition between 260 qualitatively different behaviors depending on their parameters, such as a stable equilibrium or limit cycles. 261 These regimes can be associated with entirely different patterns, rather than just quantitative differences: 262 for instance, an oscillatory regime affords new patterns such as cycle period. Different regimes of the same 263 model often entail different trends of pattern metrics with control parameters (e.g. diversity may increase 264 then decrease as we keep increasing dispersal), which has important consequences for fitting. This is typically 265 associated with a change in which terms dominate in the dynamical equation (for instance, dispersal rate may 266 go from being small compared to extinction rate, to being large compared to it (Barbier et al., 2022)). But 267 this is not always sufficient to distinguish meaningfully different regimes – for instance, strong competition 268 may lead to either chaotic fluctuations or multiple stable states depending on slightly more specific features 269

²⁷⁰ of that competition.

As shown in the Box 'Diagram of dynamical regimes', RLV models tend to have long-term behaviors that fall into a few broad classes; a single steady state point equilibrium, bounded fluctuations within a given domain or multiple outcomes that are "alternate attractors" (involving multiple possible combinations of either bounded fluctuations or steady-state points).

The existence of these distinct regimes, and how they relate to community-level parameters, is, in itself, a prediction amenable to empirical testing, as it is particularly robust to model details, and has recently received direct experimental support (Hu et al., 2022).

278 2.1.2 Selecting robust yet informative patterns

It is clear that we can only use our approach to try to predict 'macroecological' patterns that represent 279 aggregated features and cannot predict more specific ones, see e.g. McGill (2010); Grilli (2020). Nevertheless, 280 this still results in a wide array of possible features of the system that we can hope to study. The patterns of 281 greatest interest should have two properties. First, they should be robust to the details we have omitted. We 282 are thus not interested in patterns that are likely to be highly contingent on particular details, even if we can 283 consider them as highly aggregated properties. This has been a criticism leveled against any specific measure 284 of community stability, asymptotic resilience, where the rarest species tends to play a key role (Arnoldi et al., 285 2018). Secondly, we seek patterns that are likely to be sensitive to the driving variables we include (i.e. the 286 means and variances of the parameters). We imagine that some patterns are so robust that they are not 287 informative about anything. For example, McGill (2010) showed that properties such as the local Species 288 Abundance Distribution (SAD) and the Species Area Relationship (SAR) are qualitatively (and perhaps to 289 some degree quantitatively) robust to model formulations involving very different parameters. In this case, 290 these patterns are consequently rather uninformative. Instead, we seek patterns that have some intermediate 291 status so as to be robust to the details we ignore but sensitive to those we include. 292

Identifying these patterns can be an important step, although it can be challenging. In some cases, it is 293 possible to use analytical models to identify them. In this case, there is an explicit formulation of both the 294 robustness and characterization of the sensitivity (which patterns are sensitive to which parameter types). 295 For example, Bunin (2017) used a technique called 'cavity modeling' to derive explicit equations relating the 296 total biomass, total biomass turnover, species richness, species eveness, and population stability (figure 4), 297 from a disordered model driven by variation in the carrying capacities of the species, the mean, variance, and 298 mean asymmetry of interaction coefficients, and the total diversity of the species pool. This allowed them to 299 specify just how the various patterns should relate to each of the driving variables. 300

Although optimal for insights into the causal effects in a system, such analytical solutions are often

difficult or impossible to obtain. Instead, the search for relevant patterns will probably require simulations. Additionally, or alternatively, these patterns can be inspired by the interests of the researcher, perhaps based on previous knowledge such as might be obtained from previous work using e.g. 'classical simplicity' or previously documented macroecological patterns.

An important use of DSMs is to predict the generic effects of biodiversity. We may thus think about 306 patterns observed when biodiversity is experimentally controlled. Among those, the relationship between 307 diversity and total biomass provides an estimate of average competition (Fort, 2018; Wang et al., 2021), 308 while a positive relationship between biodiversity and temporal variability would be indicative of a fluctuating 309 dynamical regime (Roy et al., 2020; Hu et al., 2022). However, the informativeness of these patterns may 310 break down when they are not experimental but correlative, i.e. when diversity is not directly manipulated. 311 So, why we should try to identify such 'robust-but-informative' patterns anyway? We can think of at 312 least three reasons to seek to make such predictions: 313

1) We seek to make these predictions for their own sake. This might be useful in forecasting where we can 314 ask questions about what could happen under different conditions and might be useful in predicting effects 315 of e.g. climate change, eutrophication, repeated invasions, fragmentation etc. Under this scenario we wish 316 to work from left to right in figure 2. We might be particularly interested in predictions that are robust to 317 possible details since there are more likely to be reliable, see Figure 3. This approach may also, at least to 318 some extent, allow us to extrapolate predictions for situations that lie outside the range of current variation 319 since the robustness implies that they can do so unless there are fundamental changes to the structure of the 320 models and when they might apply. 321

22 2) We seek to infer information about the predictor variables that we have modelled. In particular, measuring the carrying capacities of species, and even more so, the pairwise interactions coefficients is increasingly tedious as the number of species increases. Here we know we won't be able to obtain detailed information on these parameters, but there may be reasons to want to know how their means and variances could affect patterns. Under this scenario we would be working from right to left in Figure 2. Barbier et al. (2022) show that this can be done using a disordered systems model and further show that this is robust to the amount of dispersal in the system.

3) We seek use this approach in a hypothetico-deductive context to test or derive hypotheses about community dynamics (and metacommunity dynamics described below) by evaluating if predictor variables are consistent with observable patterns. Under this scenario we would be looking to see if the two approaches above are internally consistent (jointly working in both directions in figure 2). This is a substantially more ambitious question, especially in the absence of additional information (e.g. manipulative experiments, structured use of trait variation etc.), but at least it would allow us to apply the approach within a hypothetico335 deductive research framework.

Barbier et al. (2018) found numerically that various straightforward aggregate predictions of DSMs were robust to a number of variations in fine-grained details of network structure, but not always to variations that could be interpreted as large-scale structures, e.g. when the network of interactions was either nested, directed, or bipartite. This suggests that it might be important to incorporate some general features of the networks as additional constraints on the purely disordered systems model, something that one might call 'partially structured' models; we describe and discuss these in a following section.

³⁴² 2.2 Partially structured models

Above we have focused on 'fully' disordered models, but of course this often seems like a strong constraint, 343 perhaps applicable only to interactions within a guild of similar species (e.g. grasses, or soil bacteria (Barbier 344 et al., 2021; Hu et al., 2022)). Often we can identify important aspects of community variation that we 345 think might be important but find it difficult to formulate our model to have narrowly constrained random 346 distributions for our parameters as described above. For example, if we wish to model food webs with 347 distinct trophic levels, it seems highly problematic to imagine that interactions are adequately characterized 348 by a simple mean and variance across all trophic levels. Instead we would be inclined to think that the 349 predator-prev interactions between trophic levels would have very distinct parameter distributions (means, 350 variances, asymmetries) than the interactions within any of the trophic levels. We can imagine the same 351 sort of problem for any so-called set of 'functional groups' such as plant-pollinator systems. And one can 352 also hypothesize other ways that interactions might be partitioned into different components (e.g. within 353 and between habitats or in relation to the size of organisms). It seems extremely unlikely that we can 354 entirely dismiss such empirically ubiquitous structures and often these seem particularly apparent to empirical 355 ecologists who have documented the effects of such factors. 356

Is there any way to extend the insights we describe above to account for such 'functional groups' (for 357 example, trophic levels)? This issue is closely related to the way food webs have classically been defined 358 since such food webs often (perhaps always!) lump at least some species together based on functional 359 considerations. For example, many food webs use the concept of 'trophospecies', defined as groups of species 360 that qualitatively share the same resources and predators. Although this is done largely for practical reasons 361 (the detailed differences among species in trophospecies is generally unknown, and formidable to study) this 362 is also often justified by the idea that interactions within such groups are relatively uniform and that shared 363 interactions with other groups are also relatively uniform even if they are distinct from each other and from the within group interactions. Extending disordered models in this way (within vs among groups), is what 365

is meant by 'partially structured' (and thus also 'partially disordered') models. 366

Barbier et al. (2018) suggested that this could be done and illustrated how it might modify predictions 367 from those based on the completely disordered approach. The idea is essentially to partition interactions into 368 within and among functional groups and characterize each of these separately by their means and variances. 369 If this approach can be justified and validated, it would provide a useful intermediate perspective between 370 disordered systems models and fully specified ones. This seems like a conceptually straightforward, and 371 to most community ecologists, meaningfully useful extension of disordered systems models. In fact, in the 372 absence of such structure, many ecologists would feel that they may not even have a job! 373

Adding structure means adding parameters, typically ending up with as many parameters as classical 374 few-species ecological models (e.g. tritrophic chains), or even more (since the presence of disorder means that 375 we care not only about average interaction between e.g. trophic levels, but also about the (co-)variance of 376 these interactions). Consequently, deciding just how interactions should be approximated by a combination 377 of disorder and structure involves careful thinking. We discuss below how and when we expect a gain from 378 this approach. 379

2.2.1Types of structures 380

We can start by thinking of three possible ways to structure interactions with some degree of heterogeneity: 381 keystone species or links, groups or modules, and continuous axes. All of these situations can be theoretically 382 understood in simple settings with a slight increase in complexity from DSMs. 383

The basic intuition for the importance of the first possibility is that ecology is often a combination of a 384 small numbers game and a large numbers one: for instance, it will often occur that a single species has a 385 disproportionate and idiosyncratic role in the dynamics (e.g. Starfish in intertidal food webs (Paine, 1966), 386 Daphnia in aquatic systems, (Mazumder, 1994) or wolves in temperate forests (Ripple et al., 2014), whereas 387 many other species can modelled through their aggregate impact on the community. These scenarios can 388 be theoretically understood with a 'focal species' perspective, often associated with studying the impacts of 389 invasion and extinction (Arnoldi et al., 2022). 390

One of the most common types of structure that can be added to a random network is blocks or modules. 391 such as functional groups, trophic levels, core and peripheral species, etc. This has been investigated theo-392 retically, notably in the context of bitrophic systems (Yoshino et al., 2007; Advani et al., 2018; Feng et al., 393 2024). Stochastic Block Models (Vaca-Castano et al., 2022) are a favored inference tool to estimate these 394 matrices of parameters. It might also be possible to detect the existence of groups more easily than we can 395 infer their precise parameter values. 396



Another important type of structure is quantitative axes along which species are positioned, e.g. traits

³⁹⁸ such as body size or degree of generalism, or trade-off surfaces such as the colonization-competition trade-off.
³⁹⁹ They are in some ways more complex than groups since we can imagine arbitrary dependencies of parameters
⁴⁰⁰ in the position of species along the axis, and in other ways simpler, since we expect continuity between
⁴⁰¹ adjacent values, allowing us to simply ask about, e.g., the slope of variation of parameters with species traits,
⁴⁰² as in models based on metabolic allometry (Brose et al., 2006; Barbier et al., 2018).

At this time, the use of partially structured models is rather scarce. However, it is a method that has tremendous potential. If we look at figure 4, we see that it is likely that reality is not very likely to resemble fully disordered systems models and that partially structured models represent the path leading to greater realism. It is also the most important element that allows us to interrogate the variety of possible constraints that might act on natural ecosystems. Just exactly how to proceed with the selection of different possible structuring elements is not clear, but it is an exciting possible way to add realism to the study of highly diverse communities without having to resort to the 'true complexity' approach described above.

410 2.2.2 Additional patterns

Introducing structure opens up new measurable properties, typically the same observables as before now 411 resolved by positions in the structure. For instance, resolving a food web into a few trophic levels opens up 412 many new properties to measure. Instead of total community biomass or abundance, we can have a more 413 fine-grained description across groups, e.g. biomass pyramids. Instead of community stability, we can study 414 response of one group to the perturbation of another, e.g. trophic cascades. And instead of species diversity 415 versus some control factor like resources or area, we can look at the covariation between groups (e.g. predator 416 diversity versus prey diversity) across the range of the control factor. And, when the structure is not made 417 of groups but of continuous variation along a trait axis or trade-off surface, we can instead ask about the 418 slope of change of these properties across the axis (e.g. the slope of a continuous size spectrum). 419

⁴²⁰ Unfortunately, the number of predictable patterns generally grows faster than the number of parameters ⁴²¹ which might be of increasing interest, but the patterns also become more and more fragile (dependent on ⁴²² precise assumptions) as we increase the level of detail of the structure. In the limit, we are back to 'irreducible ⁴²³ complexity' where the number of parameters is at least the square of the number of species. This argument ⁴²⁴ thus mirrors how complexity affects predictability and understanding when starting from the very simplest ⁴²⁵ models.

Partially ordered models thus represent an important extension of disordered systems models because they address how potential deviations from fully disordered models may affect results. Seen in this light, fully disordered models can be thought of as 'null' hypotheses with which to compare any given partially ordered model. If no difference in predictions is observed between the two, then the structure may be irrelevant to the ⁴³⁰ prediction and we may conclude that the result is especially robust to the structure we investigated (Barbier ⁴³¹ et al., 2018). Similarly, partially ordered models can be compared with simple models based on 'modules' ⁴³² where similar species are lumped together. If no differences in predictions are observed, then it is only the ⁴³³ structure that is relevant and we may conclude that the complexity of species interactions is unimportant, ⁴³⁴ at least for the predictions tested.

⁴³⁵ More generally, we might expect that partially structured models would often reveal important differences ⁴³⁶ in predictions from fully disordered ones and thus allow us to reject the 'null hypothesis' and potentially ⁴³⁷ replace it with the hypothesis implied by the partial structure we imposed (keeping in mind that there may ⁴³⁸ be alternate structures that may do as well or even better). While the work involved in empirically testing ⁴³⁹ (either via quantification of natural patterns or by experiments) is not trivial, one could imagine building a ⁴⁴⁰ research program that uses partially structured models to identify which aspects of 'irreducible complexity' ⁴⁴¹ as we described in our introduction are most important by producing predictions that are most robust to the ⁴⁴² possible full complexity of the system.

$_{443}$ 3 Discussion

444 3.1 Metacommunities

Our original equation of interest (Equation 1) specifies local dynamics using the RLV formulation, but also 445 includes terms for dispersal with the term D_i as a species-specific 'dispersal' parameter. In equation 1 dis-446 persal is modeled as a diffusion process where the net movement between two sites is proportional to the 447 difference in abundance of species I in the two sites. This term converts our model into a 'metacommunity 448 model' in which local communities are embedded in a set of analogous sites that comprise the 'metacom-449 munity'. The biota that exists in the metacommunity can itself constrained by the cumulative dynamics of 450 all the individual sites. This is in contrast with the more abstract approach we have taken so far consisting 451 of an externally defined fixed 'regional biota' that simply and independently provides colonists to any given 452 local community with no feedback from the dynamics of local communities. 453

Thinking about the dynamics of metacommunities can be traced back to early work at least in the late 1960s (Levins and Culver, 1971) but hasn't really flourished until the early part of the 2000s (Hubbell, 2001; Leibold et al., 2004). This work has greatly expanded our understanding of a wide array of ecological dynamics (reviewed by Leibold and Chase (2018)). Needless to say, these dynamics are more complex and complicated than those we can study with the simpler externally defined regional biota.

459 However, it is hard to argue that it isn't substantially more realistic. And work done to date in meta-

community ecology has certainly shown numerous ways by which dispersal can substantially modify our expectations about the distribution of species across sites (and often, through time as well). Developing our understanding about metacommunities based on basic principles of local selection (including both 'environmental' and species interactions), dispersal, and stochasticity (as well as speciation that we ignore here) also makes for a much more satisfyingly inclusive approach than the one we have studied so far.

⁴⁶⁵ Nevertheless, work to date indicates that there are many complex consequences of taking a metacom⁴⁶⁶ munity approach. Could an approach that builds on disordered systems modeling help identify robust and
⁴⁶⁷ testable aspects of this complexity?

Doing so will take some thinking. If we stick to RLV models, we want to find some formulation of equations 468 like the general equation we started with (Equation 1), either with our without the noise term $\xi_i(t)$. To start 469 with, it immediately implies that we need some new aggregate parameters including at the very least: the 470 variance of dispersal rates among species (D_i) , and the spatial variance and auto correlations of growth rates 471 or carrying capacities $(r_i, K_i \text{ and/or } A_{ii})$, which represent the spatial features of the environment. It also 472 seems likely we would want to include correlations of these traits with the other traits we have in the basic 473 model. We also need to figure out how to structure the model to minimize these variances and correlations 474 to optimize the utility of the disordered systems approach (as discussed in section 'Application of disorder' 475 above). Possible general consequences of taking this approach are, as of now, very poorly studied, and at 476 this point our explanations outlined below should be seen as highly speculative even if we can point to some 477 existing results. 478

However, it is likely that significant new insights can arise. For example, Gravel et al. (2016) studied 479 how dispersal among local communities could alter how diversity and complexity are constrained by random 480 interactions among the species. They start with the well-known disordered Jacobian model of May (1972) 481 that predicts that $S < \langle J_{ij}^2 \rangle C$ in isolated communities. They then ask how making multiple copies of this 482 model (one for each of the different sites) and connecting them with dispersal (d, which they hold fixed and483 equal among species) affects this inequality. They also ignore possible effects due to varying growth rates 484 and carrying capacities (except in how they alter the values of J_{ij} via the transition to the relative yield 485 formulation of the RLV. They find two analytical solutions: one that a) holds at very low dispersal levels, 486 resulting in a modified inequality $\sqrt{SC\langle J_{ij}^2\rangle} < -\langle J_{ii}\rangle + d$ showing that dispersal can have a small stabilizing 487 effect and thus allows for higher S than predicted by May; and b) one that applies at very high dispersal rates 488 that results in a modified inequality $\sqrt{SC\langle J_{ij}^2\rangle}/E < -\langle J_{ii}\rangle$ in which E is the effective number of statistically 489 independent matrices in the set of local matrices in the metacommunity (if all the local communities have 490 the same matrix, E = 1 and we recover May's result; if each site is statistically independent of all the others, 491 then we divide by the number of sites; for cases in between, we have to calculate this effective value for E). 492

They did not find any analytical solution for intermediate values of d, but numerical studies showed that 493 dispersal was even more stabilizing than either of these limiting conditions. This study, while incomplete 494 in addressing metacommunity dynamics in disordered systems models, nevertheless shows that they can 495 dramatically change the predictions we would have made from the simpler non-metacommunity approach. 496 They also studied additional aspects of metacommunity dynamics and studied how these might even vary 497 with the type of dispersal (local vs global). These results show that multiple aspects of metacommunity 498 dynamics respond in a structured way to variation in dispersal. They also show that these responses are 499 relatively robust to the dispersal structure (at least in comparing local vs global dispersal). 500

Unfortunately, the fact that we focused our approach to be Jacobian matrix near a given equilibrium 501 prevents other possible predictions (biomasses, turnover, etc) from being studied. We could nevertheless 502 study this with the original spatial RLV model to quantify these effects, which is the focus of ongoing 503 work (Roy et al., 2020; O'Sullivan et al., 2021; Garcia Lorenzana et al., 2024). Metacommunities are also 504 more complex than single communities, and they can be characterized by numerous other metrics than single 505 communities, especially those that involve cross-locality components such as spatial beta-diversity (mean and 506 variance), gradient responses to productivity, etc. At this point, work is only beginning, and we anticipate 507 exciting developments to come as the field better explores these consequences. 508

509 3.2 Conclusions

The assumption of disorder or emergent simplicity is that many model details matter only inasmuch as 510 they contribute to aggregate processes at the community scale. Thus, using disordered systems models, and 511 even more so partially structured models, can be a powerful and general way to extend insights that are 512 based on basic mechanistic processes (e.g. dispersal, environment, interactions) to models that involve large 513 numbers of species. Such models can therefore be critical tools that can allow the empirical study of natural 514 communities (that virtually always contain dozens to thousands of species) while retaining insights that have 515 come from typically small scale and few-species studies (both experimental and theoretical). To date, work 516 using this approach have been largely restricted to idealized theoretical models. Our goal in this paper is to 517 explain the approach in the hope that empirical scientists will be motivated to apply it to a wide array of 518 basic and applied questions involving community and metacommunity ecology. 519

Here we have focused on applying the disordered (or 'partially structured') systems approach to Lotka-Volterra models because they represent the most obvious, and historically established, basis to approach (meta)community ecology. They can be used as a baseline to study a broad class of models that share the premise that population dynamics is the central driver of the patterns that we are interested in, and all other processes (e.g. species interactions, spatial fluxes) are modelled phenomenologically through their contributions to population growth or decline. However, we note that the approach could also be extended to other formulations of ecological systems (e.g. ecosystem models that are based on nutrient fluxes and stocks, which would more likely display linear than exponential dynamics or colonization-extinction models that ignore abundance patterns). Consequently, the idea of disordered systems is likely to be useful for a number of other ecologically relevant issues even if the Lotka-Volterra structure does not apply.

The idea of approximating species interactions as random is, of course, not new – it was famously em-530 ployed by May (1972) to discuss the relationship between ecological complexity and stability. Yodzis (1981) 531 challenged May's conclusions by pointing out that food web structure might be important in a way that 532 closely reflects our thoughts about using partially structured models. Nevertheless, until recently, theoretical 533 predictions based on (partially) random interactions had largely focused on a limited set of (often hard to 534 measure) stability properties such as asymptotic resilience (Gravel et al., 2016; Allesina et al., 2015). The 535 technical approach used by May and its successors makes less definite assumptions about ecological dynamics, 536 but is restricted to these stability properties, whereas the Random Lotka-Volterra model can make predic-537 tions on many other observables such as abundance distributions across species and across space and time. 538 complex dynamical regimes, etc. It potentially encompasses, qualitatively or even quantitatively, the predic-539 tions of a number of other randomness-based approaches that did not focus on species interactions: models 540 of independent species such as the Stochastic Logistic Equation (Grilli, 2020) and stochastic geometry ap-541 proaches (McGill, 2010), but also neutral theory (Hubbell, 2011) (except for speciation and phylogeny-related 542 questions). 543

We argue that there are important advantages in using this method. Under full disorder, theory predicts (and simulations confirm) that the many parameters required to describe a community matter only in aggregate, through simple statistics such as mean and variance (Galla, 2006; Bunin, 2017). Partially structured models admit more parameters, but still far fewer than a full description of the community. Thus, it is possible to make predictions about (meta-)communities without having to estimate all their intricate details. Furthermore, using disordered models helps identify empirically robust patterns that are not likely to be fragile to the everchanging details of community processes.

As is true of any approach however, there also important caveats and remaining challenges:

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• How to structure interactions in order to apply partially structured models?

- How to infer process from patterns (reverse modeling)?
- Generalizing beyond Lotka-Volterra (e.g. including material cycling or other ecological features that could significantly change the types of dynamics observed here)

556 557 • Long-term dynamics (all the regimes seen here, whether equilibria or chaos or noise-driven, are some sort of stationarity, i.e. long-term dynamics whose properties do not change anymore) vs transients

Nevertheless, it is worth keeping in mind that this overall approach has only just started and that 558 improvements are likely in near term. One of the more important ongoing directions is the idea of partially 559 structured models. An initial quantitative, albeit limited, exploration of this was proposed by Barbier et al. 560 (2018) and here we point to other possible extensions. Work in this direction is particularly important in 561 better addressing the various form of structure that empirical scientists have identified and because they can 562 provide important testable hypotheses to determine if such structure actually matters for explaining natural 563 systems. What is increasingly important is for empirical scientists to capitalize on these insight and develop 564 methods to evaluate their utility in natural communities. It is our intent in this review to facilitate this effort 565 by providing what we hope is a useful introduction to doing so. 566

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₆₇₃ Box: The interpretation of disorder

Taking the goal of understanding aggregate properties, rather than detailed aspects at the species level, 674 means that we might be able to parameterize our model with less detail than would be needed to make 675 the more complex and precise predictions. Fully disordered systems models take an approach that is based 676 on statistical moments of sets of parameters. We start by discussing 'fully disordered' models where the 677 theory is simpler but we subsequently expand our approach to develop 'partially structured systems' mod-678 els. Figure 2 contrasts the classical approach to complexity that starts with a single species and builds in 679 increasingly complex dynamics via modules, networks, etc., with the disordered systems approach that starts 680 with completely disordered RLV models, then modifies these with partially structured components as they 681 may reveal themselves to be important (e.g. trophic levels or spatial structure). The two can even be seen 682 as complementary methods that can cross-check each other. 683

⁶⁸⁴ Three non-exclusive justifications for the disordered systems approach are worth considering:

685 Objective motivation: a confluence of many factors

When there are many independent sources of variation, their combined effects often behave like random 686 variables (see e.g. discussion in Barbier et al. (2021)). The canonical example is a dice throw where many 687 deterministic microscopic factors (spin, speed, orientation, angle to substrate, etc.) actually determine the 688 outcome but interfere so much with each other that none of them dominates, and the result is effectively 689 random. Similarly, we may treat pair-wise species interactions as random if the impact of one species on 690 another is not predictable from the interactions of these two species with other ones (i.e. pair-wise interactions 691 are not inter-correlated or are affected by other factors that interfere with each other so as to make them so). 692 This implies that sufficiently complex interactions appear as 'random', in the sense that they cannot 693 be resolved in detail, even though this aspect of the dynamics is actually deterministic (determined by the 694 complexity of interactions that interfere with each other) and therefore may be hard to distinguish from 695 other forms of stochasticity ('noise'). Such randomness can be seen as a conceptual "attractor" that can 696

describe our expectations for the absence of any additional structuring features of the interactions and can consequently generate 'null hypotheses' by which to compare other possible effects (see the section below on Partially structured models). These null expectations thus sum both 'noise' on the one hand and 'unresolved determinism' on the other.

701 Pragmatic motivation: baseline models for robust phenomena

Even if we do not accept randomness as being "objectively" a good representation of the dynamics we are 702 modelling, we can still adopt it on pragmatic grounds. We admit that we are dealing with intricate complex 703 systems, possibly devoid of any true randomness, but we only focus on behaviors and patterns that are robust 704 to these details because we believe these predictions would also hold even if we were to make changes in the 705 details of the studied system (e.g. shuffle carving capacities, growth rates, or interaction coefficients). It is 706 then plausible that the same phenomena or patterns will be very frequently be observed if these details are 707 drawn at random and deviations would require very specific changes that would be very unlikely (Figure 3). 708 Choosing a random model is useful simply because it is easier to manipulate than most non-random ones 709 with the same behaviors, and it thus serves as a "baseline" model for this aspect of the dynamics. 710

⁷¹¹ Subjective motivation: aggregation and statistical equivalence

A third option is a subjective Bayesian viewpoint: 'Randomness' simply reflects the degree to which we are 712 uncertain or uncaring about certain details. If so, aggregating implies that the variables we are aggregating 713 over (e.g. the abundances of various species) are, in some sense, equivalent or exchangeable – that we do not 714 mind adding apples and oranges if what we care about is total fruit biomass. This does not mean that these 715 variables are identical in every way, but only that none is "special" in how it contributes to the aggregate. 716 This clearly depends on which aggregate pattern is our focus, it may seem strange to add together the biomass 717 of predators and prey to predict total biomass given that they have very different ecological consequences, 718 but some predictable patterns assume an equivalence between them even if others many not. 719

Random models appear like a natural choice when we believe that we are indeed justified in treating species as statistically equivalent for a given pattern. In a random interaction network, species are not identical, but no species occupies a very unique role – even a well-connected "hub" species may not be a remarkable outlier because it occurs within the expected distribution of means and variances, but can be modeled as a representative sample of the overall community's distribution of connectedness. We come back to this notion of statistical equivalence below in Sec. 1.3.

⁷²⁶ 4 Box: Diagram of dynamical regimes



727

We show here dynamical regimes for the Random Lotka Volterra model without noise or space. Existing theory on these regimes crucially depends on the overall strength of interactions. We can define three main situations leading to distinct predictions: i) each species has a number of interactions α_{ij} that are each individually important (not negligible compared to $\alpha_{ii} = 1$ or stronger), ii) each pairwise interaction has limited impact on a species, but the total impact of all its interaction partners is important, and iii) important interactions are sparse and the others are either small or absent.

In the first case, each species can have a significant impact on many others' abundances, and negative interactions can easily lead to extinction. In speciose communities including such strong negative interactions, we expect that few species will coexist (Mallmin et al., 2024). In the second case, each species can be understood as interacting with the community as a whole, and a simple community-level description emerges readily. We can define aggregate parameters μ and σ^2 as the mean and variance of the sum of all interactions experienced by a species in the community

$$\mu \equiv \left\langle \sum_{j}^{S} \alpha_{ij} \right\rangle = S \left\langle \alpha_{ij} \right\rangle, \qquad \sigma^{2} \equiv \operatorname{var}\left(\sum_{j}^{S} \alpha_{ij}\right) = S \operatorname{var}(\alpha_{ij}) \tag{4}$$

These parameters then turn out to be the main ones controlling all the outcomes (Bunin, 2017). Many other possible details such as which distributions interactions are drawn from (normal vs uniform vs other) do not affect the conclusions we make about their behavior. Finally, we may have very sparse interactions, where a substantial majority of pairwise coefficients A_{ij} are negligible, while some are strong. This regime is increasingly studied theoretically (Fried et al., 2016; Marcus et al., 2022). We note that interactions that can most plausibly be treated as fully disordered involve competitive or multifactorial interactions within a guild of similar species, e.g. competition between grasses in a field, or a mixture of positive and negative effects between soil bacteria. Contrary to e.g. food webs or plant pollinator networks, it is not obvious why interactions within a guild should be sparse, and thus we do not discuss this situation in depth here.

⁷⁴⁹ Most existing theoretical studies based on disordered systems are limited to the diffuse interaction regime ⁷⁵⁰ shown in inset, where species interactions are important in aggregate but very weak individually (i.e. μ and σ ⁷⁵¹ remain moderate even for very large species number S), as this is the regime where theoretical tools are most ⁷⁵² applicable and the most robust predictions can be made. The strong interaction regime is less understood, ⁷⁵³ mainly through simulations, except in some particular limits. The grey region indicates where abundances ⁷⁵⁴ become infinite due to an excess of mutualistic interactions, i.e. a breakdown of the LV model, which can be ⁷⁵⁵ solved in various ways.

One might expect that many dynamical behavior types could be possible in the Random Lotka-Volterra model. However, and perhaps surprisingly, the RLV only has a limited set of behaviors of interest, which occur predictably whereas all the other possibilities become increasingly unlikely when species numbers are 'large' (in practice greater than 15-20, or more, depending on the behavior). Previous studies, e.g. Hu et al. (2022), have found suggestive empirical evidence for the first three phases.

⁷⁶¹ *Global coexistence:* First, it is possible that all species in the biota can coexist and reach a stable point ⁷⁶² equilibrium together (Bizeul and Najim, 2021).

Unique fixed point with lower diversity: Here, a fraction of species go extinct, but the remainder coexist
 stably in an equilibrium that cannot be invaded by any of the extinct species (if we reintroduce them, they
 go extinct again).

Chaotic turnover: Here, the dynamics tend to approach an equilibrium where a significant fraction of species thrive at significant abundances, but all such equilibria can be invaded by other species that were previously going extinct. There is consequently a constant turnover of species through a kind of "pinball" dynamics (Roy et al., 2020; O'Sullivan et al., 2021). We note that when S is small, we can observe limit cycles instead of chaos in that region.

Multiple attractors: When interactions are individually strong, the dynamics may reach multiple stable and uninvadable states, the simplest case being bistability between two competitors that can exclude each other. In the case of sparse interactions, this may lead to dynamics that are *directional*, i.e. progressing over long times toward more "mature" states (Bunin, 2021), contrary to chaotic turnover which is typically *adirectional*.

⁷⁷⁶ Box: Step-by-step guide to applying disordered systems theory

⁷⁷⁷ This box serves as a quick summary of the points made throughout the manuscript.

A step-by-step description of the process could be provided.

Find a parameterization of the system that tries to make the species as equivalent and their interactions
 as uncorrelated as possible (Section 1.3), e.g. expressing them in a common currency like total biomass
 or total resource consumption.

- 2. Introduce all elements in the model that are needed to resolve measurable behaviors and patterns
 (Section 2.1)
- Try to fit the disordered model to data using the most discriminating patterns, bearing in mind that
 some parameters are hard to resolve depending on which patterns are available to measure (e.g. it is hard
 to distinguish temporal fluctuations due to environmental perturbations versus species interactions).
- 4. If disordered predictions are unsuccessful, introduce the minimal amount of structure needed to reach
 satisfactory predictions (Section 2.2).

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Figure 1: a) Our overall goals is to relate basic ecological processes to community level patterns. b) To do so, we break the process down into four parts: the model ingredients (Section ??), the different regimes, dynamics or behaviors it can display (Section 4) and the 'observables' chosen to characterize empirical patterns (Section 2.1) that we aim to predict from the model and use to validate it and infer its parameters. This tripartition is useful since different ingredients can produce the same behavior (e.g. many mechanisms can lead to chaotic fluctuations in abundances), and different behaviors can produce the same pattern (e.g. intrinsically chaotic dynamics or dynamics driven externally by environmental fluctuations can lead to similar-looking time series), see also McGill (2010).



Figure 2: DSM and PSMs. When species differ importantly in just a few ways, e.g. trophic height, we can organize them in simple modules like food chains. When they differ in a vast number of ways, their interactions become largely unpredictable and, in effect, random-like. Real communities probably lie somewhere inbetween, and can be approached from both sides, e.g. with Partially Structured Models.



Figure 3: We may use random models because they are a simple way to access 'robust' or 'generic' outcomes that we expect to arise in many nonrandom models (Barbier et al., 2018). Drawing parameters at random is thus different from 'exploring all the possibilities' – on the contrary, when the number of parameters becomes large, the model is unlikely to stumble upon any non-generic possibility.



Figure 4: Theoretically predicted coexistence, abundance and stability properties for the assembled state of the random Lotka-Volterra model, as we vary $\mu = S \langle A_{ij} \rangle$, $\sigma = \sqrt{S \operatorname{var}(A_{ij})}$. The uniform area in the left of each graph signals the parameter region where abundances diverge (breakdown of the LV model due to strong mutualism). The bottom-right graph showcases the phase parameter: the sharp line where it diverges indicates the transition from the single-equilibrium regime (below the line) where our analytical results are exact for the reference model, to the chaotic regime (above the line) where they are approximate. Reproduced from Barbier et al. (2018).



Figure 5: An archetypal representation of a metacommunity. Here we study the Jacobian Matrix foor a linearizeddynamic model, but the representation still captures the essence of the idea. Local interactions are represented by the A matrix that has intraspecific interactions (along the main diagonal that represent environmental selection), interspecific interactions (in the off-diagonal elements of the submatrices along the main diagonal), and dispersal in the off-diagonal matrices. A relevant formulation of DSM model for this representation would have each of these elements characterized by means and variances. And additional structure to generate relevant PSMs would involve modifying this general model to partition these matrices in relevant ways or by imposing correlations among the parameters. For example, the set of dispersal matrices could be constrained to allow only local dispersal (only matrices along the most proximal matrix to the main diagonal have non-zero values) vs global dispersal (there are no off diagonal matrices that are zero).