Unifying occupancy-detection and local frequency scaling (Frescalo) models

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Abstract

Frescalo's "local frequency scaling" and classical occupancy-detection models both seek to recover true species-occurrence signals from imperfect data. In this paper, we show that the two approaches rest on the same underlying detection mathematics. Occupancy models treat each site's repeat visits as independent detection trials and separately estimate occupancy probability and per-visit detectability. Frescalo, by contrast, pools data across ecologically defined neighbourhoods and infers a unified Poisson-process discovery rate and a temporal "time-factor" to capture trends. We demonstrate that the Bernoulli-trial formulation of occupancy-detection can be linked to Frescalo's Poisson framework, with occupancy and detectability represented by a single rate parameter (which approximates the product ψp when overall sampling intensity is low). This connection clarifies how Frescalo's neighbourhood-scale and time corrections function as a coarser-scale analogue of repeat-visit models. By casting Frescalo in occupancy modelling terms, we hope to promote further investigation into the adoption of occupancy model diagnostics, extensions and covariate tests within Frescalo analyses, improving transparency and rigour when working with less-structured biodiversity data.

Keywords: occupancy models, sampling effort, effort correction, citizen science, unstructured data, Hill numbers

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1. Introduction

Occupancy-detection models [9] and the Frescalo "local frequency scaling" method [6] both aim to correct raw biological records (i.e. species occurrence) data for imperfect sampling. Classical occupancy models do this at the scale of repeated visits to individual sites, explicitly estimating true presence probabilities (ψ) and detectability (p) via a hierarchical likelihood. Frescalo was designed to work at larger spatio-temporal scales, exploiting emergent patterns of relative frequency in "neighbourhoods" to derive Poisson-process-based scaling factors (α) and species' relative "time factors" indexing true fluctuations in site occupancy. Given that many datasets lack repeat-visit structure, and/or may exhibit variation in the detection process that is unmodellable due to a

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Email address: `olipes@ceh.ac.uk` (Oliver L. Pescott) lack of knowledge of its determinants [11], understanding how Frescalo recovers effort-adjusted trends from aggregated data can broaden the toolkit of ecologists.

Although the two models can appear quite different, Pescott et al. [11] informally suggested that Frescalo could be seen as a type of occupancy-detection model "where an adjustment for overlooked species is made in relation to spatial rather than temporal replication, whilst simultaneously adjusting for variable regional effort". We here show that this suggestion can be formalised due to the two model types' reliance on the same core mathematics of Bernoulli versus Poisson detections. Below we (1) recall each framework, (2) write down their key equations, and (3) algebraically map one onto the other, demonstrating that Frescalo time trends are based on an implicit occupancy-detection model whose "visits" and "occupancy" are folded into a single site/species discoverability rate parameter λ and standardised neighbourhood effort index.

2. Occupancy-detection models

2.1. Basic single-season model

Following MacKenzie et al. [9], at each site i for species j assume a latent occupancy indicator

$$z_{ij} \sim Bernoulli(\psi_{ij}).$$

Conditional on presence, v total survey visits indexed by k produce

$$y_{ij1}, ..., y_{ijv} \mid z_{ij} = 1 \sim Bernoulli(p_{ij})$$

where p is detectability. If $z_{ij} = 0$ (i.e. species absent), then all $y_{ijk} = 0$. Marginalising out z_{ij}, it is well-known that the probability of at least one detection across v visits is

$$P(\max_k y_{ijk} = 1) = \psi_{ij}[1 - (1 - p_{ij})^v].$$

Thus the model simultaneously estimates

$$\psi_{ij} = \text{Pr(occupied)}, \quad p_{ij} = \text{Pr(detect|occupied)},$$

and inference proceeds via the full likelihood over all sites and detection histories.

3. Frequency scaling using local occupancy (Frescalo)

3.1. Neighbourhood frequencies

Frescalo [6] pools presence-only data across a neighbourhood around target site i. We denote the observed proportion of neighbourhood sites in which species j was recorded by f_{ij} (in practice this frequency may relate to a weighted neighbourhood as per Hill [6], but this detail is not crucial for what follows). Under a Poisson-process model of species discovery with rate λ_{ij} and unknown total neighbourhood-level sampling effort $s_{i(N)}$, one has

$$f_{ij} = 1 - \exp(-\lambda_{ij} s_{i(N)}).$$

Subsequently, a frequency-weighted neighbourhood index

$$\phi_i = \frac{\sum_j f_{ij}^2}{\sum_j f_{ij}}$$

is then "standardised" to a target value Φ by solving for a site-specific effort multiplier α_i such that

$$\phi_i(\alpha_i) = \frac{\sum_j [1 - (1 - f_{ij})^{\alpha_i}]^2}{\sum_j [1 - (1 - f_{ij})^{\alpha_i}]} = \Phi.$$

Mathematically, Φ is chosen so that every neighbourhood's weighted-mean frequency $\phi_i = \sum_j f_{ij}^2 / \sum_j f_{ij}$ equals Φ. Hill [6] showed that ϕ_i is equivalent to the ratio of the mean species richness to the 'effective number of common species' (often called N_2, the reciprocal of Simpson's index; Hill [5]), which means that ϕ_i isolates sampling intensity from true differences in richness and evenness. By fixing $\phi_i = \Phi$, we therefore align all neighbourhoods to the same effort scale without erasing real ecological differences.

This process yields the standardised neighbourhood frequencies

$$\tilde{f}_{ij} = 1 - (1 - f_{ij})^{\alpha_i}$$

which are independent of time (i.e. they are calculated with respect to the entire time period under consideration, rather than any subdivisions of this used for trend calculations), and serve as a proxy for the "true" discoverability- or effort-standardised neighbourhood species rank-frequency curve.

3.2. Temporal correction

Within each time period t, one chooses a set of "benchmark" species [8] and computes the proportion recorded per site and time period (Hill's s_{it}) as an index of site-level recording effort. (Note that there are potentially many ways to choose ones' neighbourhood benchmarks, but Hill [6] proposed a fixed proportion R^* of the standardised species rank-frequency curve after an additional normalisation step involving the division of species' ranks by the expected species count $\sum_j \tilde{f}_{ij}$; however, the precise method of choosing benchmarks does not affect what follows). For each species j in period t, Hill then defines a Poisson-link intensity

$$Q_{ijt} = -\ln[1 - s_{it}\tilde{f}_{ij}], \tag{1}$$

The modelled "discovery" probability is then

$$P_{ijt}(x_{jt}) = 1 - \exp(-Q_{ijt}x_{jt}).$$

Hill [6] estimates the time-factor x_{jt} by matching the total modelled to total observed presences y_{ijt}:

$$\sum_i y_{ijt} = \sum_i P_{ijt}(x_{jt}).$$

In practice one iterates x_{jt} in the exact Poisson form above until those sums coincide (e.g. see the *R* code of Pescott [10]). The difference between the (summed) observed presences y_{ijt} and the model's baseline expectation after standardising time-independent neighbourhood effort α_i and adjusting for site/time specific effort s_{it} is therefore captured by the time factor x_{jt}. Frescalo can thus deliver detection-corrected trends from unstructured data when its core assumptions are met.

4. Bridging the gap

4.1. Static occupancy and detection

We can compare the static (i.e. single season) single-species occupancy-detection model probability of at least one detection in v visits

$$\psi[1-(1-p)^v]$$

with the Poisson-process discovery rate (conditional on the all-time frequency curve) used in Frescalo

$$1-e^{-\lambda s_{i(N)}}.$$

For modest values of the product pv, one may use the standard Taylor-series fact

$$(1-p)^v \approx e^{-pv} \quad \text{for}\, pv \ll 1,$$

which turns $\psi[1-(1-p)^v]$ into $\psi[1-e^{-pv}]$. On the other hand, setting $\lambda = \psi p$ and $v = s_{i(N)}$, Frescalo's Poisson form $1-e^{-\psi pv}$ expands in exactly the same way: to first order both are $\psi(pv)$ with only $O((pv)^2)$ differences (i.e. only quadratic and higher terms differ). We therefore recover Frescalo's $1-e^{-\lambda s_{i(N)}}$ approximately whenever pv is small.

For larger pv, the neglected second-order terms no longer agree, so the approximation is lost. However, one can always recover the *exact* Poisson rate by solving:

$$1-e^{-\lambda v} = \psi[1-(1-p)^v] \implies \lambda = -\frac{1}{v}\ln[1-\psi(1-(1-p)^v)], \tag{2}$$

but that formula reduces to $\lambda = \psi p$ only in the limit $pv \to 0$.

Frescalo's Poisson rate λ is therefore exactly the function of occupancy, detectability and visit count that makes the first part of equation (2) true. Whilst in Frescalo we never observe v directly, we can infer it via the continuous neighbourhood effort index $s_{i(N)}$, which itself is aligned across all neighbourhoods by the spatial scaler α_i. Frescalo can therefore be interpreted as an occupancy-detection analogue at the neighbourhood scale: it replaces the two-parameters (ψ, p) and discrete v with a Poisson rate λ and a continuous effort-multiplier α equalising variable survey effort s_i (inferred by the neighbourhood level $s_{i(N)}$) across sites.

Underpinning all of this is the assumption that, within any neighbourhood, the sequence of species/site detections behaves like a multivariate (i.e. multi-species) Poisson point-process. It is the assumption which justifies the log-link given in equation (1) above, and the Poisson moment relations that let us quantify sampling effort via ϕ_i. When sampling intensifies (so that pv is no longer small), higher-order Poisson moments become more important and the simple $\psi p \approx \lambda$ approximation breaks down.

A key step in recognising the equivalent elements of these models is to appreciate that Frescalo applies its discoverability standardisation at a large scale: not only is the adjustment done with respect to the multi-site neighbourhood and across all species, but it is also calculated across all time periods in the analysis. The standardised neighbourhood frequencies $\tilde{f}_{ij}$ and the species rank-frequency curve they form is estimated once, independently of time, before temporal change is examined.

4.2. Time trend interpretation

A time trend in occupancy derived from a classical occupancy-detection model is modelled simply by letting ψ_{ij} vary linearly or non-linearly over time, conditional on both model-specific [16, 17] and other standard survey sampling assumptions [3] being reasonable. Frescalo, by contrast, posits a single time-independent set of discoverability-adjusted baseline frequencies $\tilde{f}_{ij}$, and then uses benchmarks and the site/period effort index s_{it} to compute expected frequencies under an assumption of stasis, subsequently letting the time factors x_{jt} absorb any residual differences as true ecological change.

This underscores a key difference in how effort-adjustment processes function in each model type. Occupancy-detection models assume that true site occupancies, and so trends in these, are directly recoverable from visit-level information. Frescalo assumes that fine-scale visit data is generally unavailable and/or uninformative for all or part of the time series of interest, and so models species' discoverability at a much larger scale. The main aim of this adjustment is to ensure a common scale across which neighbourhoods, and therefore sites, can be compared: without the harmonisation of effort across neighbourhoods, the time factors estimated for each site for a species would not be comparable, making average time factors and trends in these meaningless.

Another fundamental difference is the meaning of the site occupancy values produced. As noted, ψ_i has the simple meaning of predicted site occupancy under the classical model (notwithstanding debates around usage versus occupancy when these types of models are applied at different scales; [14]). The Frescalo time factor x_{jt} is, however, defined by the benchmark average, and values >1 or <1 indicate that a species is at a higher or lower average frequency relative to the common species where it occurs, rather than in absolute occupancy probability. This may be an important limitation to inferring effort via observable recording outcomes, rather than having knowledge of those factors that directly map onto effort, such as the actual number of visits and covariates that are known to explain an important portion of observed variance in species' visit-level detectability [6, 15, 7].

One way around this issue is the observation of Bijlsma [1] that site occupancy probabilities can actually be back-calculated from Frescalo via the combination of the standardised species' frequencies $\tilde{f_{ij}}$, the species' time-factors x_{jt}, and by setting $s_{it} = 1$ across all sites and time periods (i.e. constant effort), and this has been exploited in at least one published analysis [4]. However, this requires a note of caution: whilst sensitivity analyses published in Hill [6] suggested that the trends in time-factors estimated by Frescalo can be relatively insensitive to the choice of R^*, the benchmark threshold (variation in this parameter changing the intercept of estimated trends but not their slope), the same is not true of back-calculated site occupancy probabilities. Because the relationship between time-factors and species frequencies is non-linear, the shifts in time trend intercept seen using different values of R^* will not translate into the same proportional changes in predicted site occupancies over time (Pescott, pers. obs.) This may be particularly important when these trends are used to classify species' into risk categories, as for example happens in Red Listing exercises [e.g. 12].

5. Conclusions

Unstructured species occurrence data are too valuable to ignore, especially for historical periods where no information about the visit-level data collection process survives [11]. Hill's "frequency scaling using local occupancy" or Frescalo method allows the careful analyst to infer a large-scale detectability metric that can subsequently be used to place neighbourhoods on a common footing for the estimation of distribution trends. The large-scale formulation of this approach not only allows for the potential inclusion of more data sources (e.g. records extracted from Atlases or museums), but may also act to reduce the actual error in species' trends intrinsically [2, 13].

By demonstrating how Frescalo represents the classical occupancy-detection model's ψ and p with λ, and how it infers visit-related effort via an emergent community-level mean rate, the approach performs an occupancy-detection-type correction even when explicit or informative temporal repeat-visit data are lacking. By highlighting this link, we hope to promote the development of additional diagnostics, extensions and more rigorous uncertainty quantification for the frequency scaling using local occupancy method.

6. Acknowledgements

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