# Eccentric trees, biased estimates: sampling design reduces error in basal area increment estimates under stem and pith eccentricity

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# Abstract

Accurate tree growth quantification is crucial in ecological research. Basal area increment (BAI) is often calculated from tree rings on increment cores, assuming the stems form perfect circles with centered piths. However, piths are often offset and rings not perfectly round, which leads to BAI estimation errors. Yet, we do not know how much estimation error results from these two types of eccentricities.

Using geometric principles that hold across all tree sizes, we quantified the effects of these eccentricities on BAI accuracy on 109 cross-sections from 25 temperate species ranging in stem out-of-roundness and pith offset. We compared BAI estimates calculated with four methods and with radii estimated from one to four cores against true BAIs taken from cross-sectional scans. We found that with one core, pith eccentricity accounts for 21% of the error in BAI estimation, and stem eccentricity 8%. Taking multiple cores significantly reduces these errors, with four cores fully accounting for both eccentricities and two cores sampled at 180° significantly reducing this error.

We recommend using multiple cores to minimize error, with two-opposite cores being the most cost-effective approach. We also provide methods for quantifying and reporting pith and stem eccentricity in the field, offering practical guidance for practitioners to calculate estimation errors based on their methods.

## Introduction

The rate of tree growth is an essential variable in plant and forest ecology, as it quantifies the performance and health of individual trees and populations (1,2). A common way to estimate tree growth is to calculate basal area increment (BAI) defined as the change in cross-sectional area of a tree between two time points (3,4). BAI can be measured either from repeated diameter at breast height (DBH) measurements or from growth rings observed on cross-sections or increment cores. Taking tree cross-sections is the most accurate method as it provides a wealth of information (age, pith location, shape of the stem and other ring irregularities) and the exposed entire growth rings allow us to accurately calculate area as well as identify incomplete rings. However, since this method kills the tree, most studies must rely on non-lethal methods. Repeated measurements of tree diameter using a diameter tape is a common alternative, but it requires multiple visits to the site, which is often not possible, and can introduce additional measurement error due to slight variations in measurement height, tape positioning, and operator technique across sampling events. Thus, taking increment cores is often the method of choice when trees cannot be killed or when multiple visits to a site are not possible.

Current BAI calculations from cores calculate basal area from the radii using the equation for the area of a circle ($A = \pi r^2$) which assumes that trees have a perfectly circular stem with centred piths (3,5 ; see Fig. 1). However, tree cross-sections often deviate from a perfect circle, and other studies have shown that this eccentricity leads to error (4,6–8). Yet, while previous studies have demonstrated that stem eccentricity can introduce bias in BAI estimates, the magnitude of this error remains poorly constrained, and it is still unclear to what extent different sampling or measurement approaches can mitigate it. Recently, increment core data has been incorporated into forest monitoring programs (9) as well as in simulation models of forest growth

(10,11), such that improving our understanding of BAI estimation error from increment cores is timely.

Numerous forms of stem irregularity can occur in trees, including stem eccentricity (out-of-roundness), pith eccentricity (off-centre pith), as well as more complex features such as concavities and stem lobing (12). These different forms of eccentricity can be quantified using a variety of metrics, reflecting differences in how stem geometry is characterized. For example, stem eccentricity is commonly described using ratios of major to minor axis diameters or related shape indices (7,13,14), whereas pith eccentricity is typically quantified as the displacement of the pith from the geometric centre of the stem (2,15).

Despite the availability of these metrics, there is no single standardized approach to characterizing eccentricity across studies, and different studies have relied on different definitions and measurement strategies, particularly given the inherent non-circularity of stem cross-sections (16). In most cases, these metrics are derived from cross-sectional measurements of stem geometry obtained from discs or increment cores (4,12).

Here, we address this gap by quantifying how much estimation error arises from two key forms of tree eccentricity: pith eccentricity (pith-off-centre, POC; defined as the offset of the pith from the stem centre) and stem eccentricity (defined as out-of-roundness, OOR; Fig. 1). We further evaluate whether this error can be decreased through alternative area calculation methods or by increasing sampling effort through multiple cores per individual.

We assume that the relationships between eccentricity and BAI estimation error are governed by geometric properties and are therefore largely independent of species and tree size, provided that the sampled range of eccentricities reflects those observed in natural populations. These questions are specifically relevant to sampling approaches that intercept the pith (called "inside-out" methods; 17), where radii can be measured directly, but not to approaches where the pith is missing and diameters must instead be used ("outside-in" methods).

The effects of other sources of error—such as missing piths, stem lobing, and missing or false rings—are beyond the scope of this study but have been addressed elsewhere (e.g., 12,16), while recent work has focused more specifically on the role of eccentric growth and sampling effort (8).

<<Figure 1 here >>

**Figure 1. Tree eccentricity on stem cross-sections.** Tree stems can exhibit differing levels and combinations of pith and stem eccentricity, such as no eccentricity (first image on the left), only pith or only stem eccentricity (second and third images respectively), or both pith and stem eccentricity (image on right). The type and degree of the eccentricity depends on tree growth conditions, such as growing on an incline (4).

To address these gaps, we formulate three research questions focusing on the magnitude, mitigation, and methodological implications of eccentricity-related error in BAI estimation: (Q1) To what extent do pith and stem eccentricities cause BAI estimation error? (Q2) Does sampling multiple cores decrease the estimation error due to eccentricity? (Q3) Which area calculation methods perform best with eccentric cross-sections?

## Materials & Methods

### Sample Selection

Stem cross-sections were obtained from saplings of 25 different temperate hardwood species from Mont Saint-Hilaire (45°33′8″N, 73°9′3″W), a natural reserve located in Quebec, Canada. The saplings were from the subcanopy (shorter than two-thirds of the canopy height) and had basal cross sections ranging in diameter from 1.5 – 7.5 cm (Table S1 for complete species list and basal diameter ranges). Using tree cross-sections of smaller size was necessary to obtain full high-resolution scans and to measure their true BAI. Although the sampled stems are smaller than those typically studied in mature forest stands, this does not limit the relevance of our results. Our analysis focuses on geometric sources of error in BAI estimation arising from stem shape (e.g., pith offset and out-of-roundness), which are independent of tree size and are therefore expected to scale consistently across size classes and species.

From a set of 380 cross-section samples, a subset of 109 cross-sections (four to five per species) with clearly visible growth rings was selected to cover a wide range of pith and stem eccentricity. Pith eccentricity (POC) and stem eccentricity (OOR) metrics theoretically range from 0 (no eccentricity; perfectly centred pith or circular stem) to 1 (maximum eccentricity). Our samples span a broad range of eccentricities with POC values ranging from 0 to 0.6, while OOR values ranged from 0 to 0.4. Findings from this study are most relevant to populations of samples ranging in OOR from 0 to 0.4 and in POC from 0 to 0.6. Each selected cross-section was sanded with increasingly fine sandpaper, up to 600 grit (18).

### Measurements and Calculations

**Area and Radii Measurements.** Cross-sections were scanned at 1200 dpi resolution and analysed using Fiji/ImageJ software with the ObjectJ plugin (19,20). For each cross-section, the longest diameter was identified, from which 4 perpendicular radii were then drawn (Fig. 2A). A single clearly distinguishable and complete annual ring was selected on each scan as the focal ring for all measurements. The sampled rings did not exhibit stem lobing or other pronounced deviations from approximately circular growth (see 12). For each focal ring, we measured (i) true basal areas of the inner and outer rings, (ii) inner and outer ring width for radii one through four ($r_{1in}$ to $r_{4in}$ and $r_{1out}$ to $r_{4out}$), (iii) the shortest and longest radii along the longest diameter ($r_{short}$ and $r_{long}$), and (iv) the diameter of the largest circle that could be inscribed within the cross-section, used to quantify stem out-of-roundness (14 ; Fig. 2A). Basal areas of the inner and outer rings were obtained by tracing their boundaries in ImageJ to create polygons from which areas were calculated (Fig. 2B; yellow and pink polygons). Basal area increment (BAI) was then calculated as the difference between outer and inner ring areas. Radii lengths were measured from the pith to the ring boundary along the 4 lines drawn on the sample (Fig. 2B). To avoid bias when labelling cores 1 through 4 in a cross-section, core number 1 was assigned randomly when the sample had circular symmetry (henceforth 'symmetrical'). For asymmetrical cross-sections the shortest radius was assigned as radius number 1.

<<Figure 2 here>>

**Figure 2. Diagram of the measurements taken on the stem cross sections. (A)** The true BAI of the focal ring is shown as the pink polygon. The inner area is represented by the orange polygon. The dashed circle represents the largest circle that can be fully inscribed in the cross-section and is used in the OOR calculation. The black arrows show the four full radii of the

sample. Core number 1 was assigned randomly since the sample is symmetrical. $r_1$ and $r_3$ correspond to the $r_{short}$ and $r_{long}$ radii, respectively, on the longest diameter of the cross section. For legibility, “IN” and “OUT” subscripts are omitted. **(B)** Inner ($r_{in}$) and outer ($r_{out}$) radii on one of the 4 cores. For clarity, measurements along a single core (i.e. $r_{3IN}$, $r_{3OUT}$) are shown.

**Area Calculation Methods.** To approximate BAI estimates derived from increment cores, basal area increment was calculated from radial measurements. To evaluate how different approaches account for stem eccentricity, we compared four area calculation methods. Three BAI estimation methods use the area of a circle but calculate the mean radius differently: using the arithmetic (Eqn. 1), geometric (Eqn. 2), and quadratic means (Eqn. 3). The fourth estimation method uses the area of an ellipse, which requires two or more perpendicular radii. Eqn. 4a below gives the equation for the case with 4 radii and 4b for the case with 2 perpendicular radii. In the case of one radius, the equation becomes the same as Eqn. 1. In equations 1-4, *n* is the number of radii.

$$A_{arith} = \pi\bar{r}^2, where\ \bar{r} = \frac{r_1+r_2+\cdots+r_n}{n} \qquad \textit{Eqn. 1}$$

$$A_{geom} = \pi\bar{r}^2, where\ \bar{r} = \sqrt[n]{r_1 \times r_2 \times \ldots \times r_n} \qquad \textit{Eqn. 2}$$

$$A_{quad} = \pi\bar{r}^2, where\ \bar{r} = \sqrt{\frac{{r_1}^2+{r_2}^2+\cdots+{r_n}^2}{n}} \qquad \textit{Eqn. 3}$$

$$A_{ellipse,\ 4\,radii} = \frac{r_1\times r_2\times\pi}{4} + \frac{r_2\times r_3\times\pi}{4} + \frac{r_3\times r_4\times\pi}{4} + \frac{r_4\times r_1\times\pi}{4} \qquad \textit{Eqn.4a}$$

$$A_{ellipse,\ 2\,perpendicular\,radii} = r_1 \times r_2 \times \pi \qquad \textit{Eqn.4b}$$

**Core sampling design.** To assess how the number of sampled cores and their spatial arrangement on the stem (hereafter, core sampling design) affect BAI estimation accuracy, we estimated BAI for each area calculation method using one to four radii, corresponding to increasing sampling effort. Each radius was treated as a potential increment core direction. Because the ellipse-based method requires pairs of perpendicular radii, BAI for this method was estimated using either two perpendicular radii or all four radii, whereas circular methods could be applied using any number of radii.

For asymmetrical cross-sections, the identity of the radii used in the calculations influences estimated BAI when fewer than four radii are used (n = 1–3). To standardize core selection under these conditions, we assumed that eccentricity resulted from slope-induced growth, such that the shortest radius ($r_{short}$) corresponds to the uphill direction and the longest radius ($r_{long}$) to the downhill direction. Core positions were selected to reflect common field sampling practices. When a single core is taken, it is often sampled from the uphill side to facilitate coring (21), corresponding here to $r_{short}$ ($r_1$). However, we recognize that sampling position varies among studies depending on objectives; for example, coring may be performed downslope to access older rings or perpendicular to slope to avoid reaction wood (1,12,18).

For BAI estimates based on a single radius (n = 1), the uphill radius (r1) was used for asymmetrical cross-sections and selected at random for symmetrical cross-sections (Fig. 2). For two radii (n = 2), we evaluated two configurations: (i) opposite radii (uphill $r_1$ and downhill $r_3$), and (ii) perpendicular radii (r1 and either r2 or r4, selected at random). These configurations reflect common field sampling strategies, where two cores are often collected either from opposite sides of the stem (180° apart) or from perpendicular directions. For three radii (n = 3), the uphill ($r_1$) and downhill ($r_3$) radii were always included, along with one perpendicular radius selected at random. For visually symmetrical cross-sections, r1 was assigned randomly, and the remaining radii ($r_2$–$r_4$) were defined sequentially in a clockwise direction, without reference to $r_{short}$ or $r_{long}$.

**Pith and Stem Eccentricity.** Stem eccentricity was quantified using the out-of-roundness (OOR) index described by Koch (14), defined as the ratio of the minor diameter (i.e., the diameter of the largest circle that can be inscribed within the cross-section; e.g. dashed circle in Fig. 2A) to the major diameter (maximum diameter on the cross section). In this formulation, values range from 0 to 1, with 1 representing a perfect circle. To facilitate interpretation, we transformed this index as OOR = 1 - Koch's OOR, such that values of 0 correspond to a perfectly circular stem and increasing values reflect increasing eccentricity. In our dataset, OOR values ranged from 0 to 0.3 (median = 0.11; see third and fourth images of Fig. 1 for examples with OOR values of 0.3 and 0.2).

Pith eccentricity (pith offset) was quantified using the pith-off-centre (POC) index (15), defined as the ratio of the difference between the shortest and average radii along the longest diameter, over the average of those two radii (Eqn. 5). This metric ranges from 0 (perfectly centred pith) to 1 (maximum offset), with values increasing as the pith approaches the stem edge. In our dataset, POC values ranged from 0 to 0.6 (median = 0.162; see second and fourth images of Fig. 1 for examples with POC values of 0.6 and 0.3).

$$POC = \frac{r_{avg} - r_{short}}{r_{avg}}, where\ r_{avg} = \frac{r_{short} + r_{long}}{2} \qquad \text{Eqn. 5}$$

**Response Variables.** BAI estimation accuracy was assessed as both percent error (%Error) and its absolute value (|%Error|). Percent error was calculated as the estimated BAI (calculated from radii) minus the true BAI (measured from polygons), divided by the true BAI and multiplied by 100. %Error captures both the magnitude and direction of estimation bias, while |%Error| reflects only the magnitude of error. We evaluated the relationship between these response variables and four predictors: pith eccentricity (POC), stem eccentricity (OOR), core sampling design, and area calculation method. In the main text, we only report results for |%Error| unless both results differ, since patterns were largely consistent between the two metrics and |%Error| avoids cancellation of positive and negative errors when averaged across samples. Results for %Error are presented in the Supplementary Materials.

## Statistical Analyses

All analyses were conducted in R version 2022.07.1 Build 554 (22). To evaluate how area calculation method, pith eccentricity (POC), stem eccentricity (OOR), and core sampling design influence BAI estimation accuracy, we fitted linear mixed-effects models predicting both |%Error| and %Error. Fixed effects included all four predictors and their two- and three-way interactions, and sample identity was included as a random effect. Models were fitted using the `lmer()` function from the **lme4**{} package (Tables S2, 23).

This full model was simplified by backwards model selection with car{} (Table S3, 24) by sequentially removing non-significant interaction terms, beginning with higher-order interactions. To avoid collinearity, we also verified that all remaining variables had a variance inflation factor (VIF) smaller than five. Model assumptions were evaluated using diagnostic plots following Zuur and Ieno (25).

To improve homoscedasticity, |%Error| was $\log_{10}$ transformed, following recommendations from Zuur *et al.* (26). For %Error, heteroscedasticity was addressed by adding a squared predictor term ($OOR^2$ and $POC^2$) to the model, as prescribed by Zuur *et al.* (26). In addition to the lme model we conducted targeted analyses addressing each research question. In all of these analyses sample identity was used as a random effect. Prior to these analyses, we identified the area calculation method that minimized overall estimation error using a one-way ANOVA with Tukey post-hoc tests based on BAI estimates derived from four radii, and used this method in subsequent analyses.

To assess the effect of eccentricity on BAI estimation error (Q1), we regressed error metrics against POC and OOR under minimal (one radius) and maximal (four radii) sampling effort. To evaluate the effect of core sampling design on BAI estimation error arising from pith and stem eccentricity (POC and OOR) (Q2), we performed ANCOVAs of the error metrics against each core sampling design. Finally to compare area calculation method (Q3) we performed ANCOVAs of error metrics against estimation method. This analysis was conducted using BAI estimated from two perpendicular cores, as a single core yields identical area estimates across methods, precluding meaningful comparison, and two cores sampled at 180° produce identical estimates for some methods (e.g., ellipse-based approaches). The perpendicular two-core design therefore represents a low-information sampling design that is likely to produce BAI estimation error, a scenario that allows differences among area calculation methods to be detected.

# Results

## Linear Mixed Effect Model

The simplified linear mixed-effects model retained pith eccentricity (POC), stem eccentricity (OOR), area calculation method, and core sampling design as predictors of estimation error, along with several two-way interaction terms (Table S3). A significant negative interaction between POC and OOR was observed, indicating that the effect of POC on estimation error decreases as OOR increases, and that the effect of OOR decreases as POC increases. The marginal $R^2$ of the model was 0.34, indicating that fixed effects explained 34% of the variance in estimation error. Since the results of the linear mixed-effects model were consistent with those of the targeted analyses, below we present the latter as they provide the simplest answers to the specific research questions.

## Effect of area calculation method on BAI estimation error

On average, the geometric mean method produced significantly higher |%Error| than the other methods, irrespective of eccentricity (5.37% vs. 3.39%; ANOVA, $p = 1.28 \times 10^{-7}$; Tables S4, S5; Fig. S1). In contrast, the arithmetic, ellipse, and quadratic methods did not differ significantly in |%Error| (Tukey post-hoc tests, $p > 0.05$; Table S5). To standardize subsequent analyses, we selected the quadratic method. Although its mean error did not differ significantly from the arithmetic and ellipse methods, it showed minimal bias (distribution centered near zero; Fig. S2), and can be applied across all core sampling designs.

## To what extent do pith and stem eccentricities cause BAI estimation error?

Because the effects of pith eccentricity (POC) and stem eccentricity (OOR) on |%Error| varied with core sampling design, we present results for the lowest and highest sampling effort (one and four cores). When using four cores, POC did not significantly affect |%Error|, which remained low across the full range of POC values (adj. $R^2 = 0.006$, $p = 0.21$; Fig. 3A; Table S6). The mean |%Error| under this sampling design was approximately 6%. In contrast, when using a single core, POC had a strong effect on |%Error|. At POC = 0, mean |%Error| was approximately

13%, whereas at POC = 0.6, mean |%Error| increased to 88% (adj. $R^2 = 0.21$, $p = 5.24 \times 10^{-7}$; Fig. 3B; Table S7).

Similarly, when using four cores, OOR had a weak effect on |%Error|. |%Error| increased slightly with OOR, and this effect was marginally significant (adj. $R^2 = 0.018$, $p = 0.089$; Fig. 3C; Table S8). In contrast, when using a single core, OOR had a stronger effect on |%Error|. At OOR = 0, mean |%Error| was approximately 13%, whereas at OOR = 0.4, mean |%Error| increased to 79% (adj. $R^2 = 0.084$, $p = 0.0014$; Fig. 3D; Table S9).

<<Figure 3 here>>

**Figure 3. Effect of pith-off-centre (POC) and stem out-of-roundness (OOR) on log(|%Error|) using the quadratic method, with 1 and 4 cores.** For ease of interpretation, |%Error| is also shown on the right as a second y axis. Panel A. With four cores, the effect of POC on log(|%Error) is not significant ($p = 0.208$; Table S6). Panel B. With one core, the effect of POC on log(|%Error|) is significant ($p < 0.05$, Adj $R^2 = 0.21$; Table S7). Panel C. With four cores, the effect OOR on log(|%Error|) is marginally significant ($p = 0.089$, Adj $R^2 = 0.018$; Table S8). Panel D. With one core, the effect of OOR on log(|%Error|) is significant ($p < 0.05$, Adj $R^2 = 0.084$; Table S9). Dotted lines show non-significant slopes and solid lines show significant and marginally significant slopes.

### Does core sampling design reduce the error due to eccentricity?

For both types of eccentricities, increasing the number of cores sampled consistently reduced BAI estimation error, as reflected by decreasing intercepts in the regression models (POC: Table S10, Fig. 4A; OOR: Table S11, Fig. 4B). Core sampling design also influenced how eccentricity affected error. For samples with low POC or OOR eccentricity, two-opposite and two-perpendicular core configurations produced similar errors. However, for highly eccentric samples, two-opposite cores resulted in lower error than two-perpendicular cores (Fig. 4). This difference arises because the slopes of the regressions were steeper for the two-perpendicular configuration, indicating a stronger increase in error with increasing eccentricity.

For pith eccentricity (POC), the slopes were shallow when two-opposite, three, or four cores were used, indicating that error increased little with eccentricity (Fig. 4A; Table S10). In contrast, using one core or two-perpendicular cores resulted in slopes that were significantly steeper than the four-core reference, indicating that error increased with eccentricity. As a result, the single-core approach produced the highest errors, reaching up to 102% at POC = 0.6 (Fig. 4A; Table S10).

For stem eccentricity (OOR), the effect on error was generally weak across most sampling designs, with similar slopes observed for one, two-opposite, three, and four cores (Fig. 4B; Table S11). However, under the two-perpendicular configuration, estimation error increased significantly more quickly with OOR. Consequently, for highly eccentric samples, the two-perpendicular design produced errors comparable to those obtained with a single core. For example, at OOR = 0.4, mean |%Error| was 76% for one core and 71% for two-perpendicular cores, compared to 25% for two-opposite cores and 18% for three cores (Fig. 4B; Table S11).

<<Figure 4 here>>

**Figure 4. Effect of eccentricity on BAI estimation error, as a function of the core sampling design.** For ease of interpretation, |%Error| is also shown on the right as a second y axis. In both panels, the four-core reference slope is shown as a dotted line because it does not differ from 0.

For other sampling designs, dotted lines indicate slopes not different from this reference, whereas solid lines indicate slopes that differ from it. Panel A. Effect of pith-off-centre (POC) on log(|%Error|). Increasing POC increases estimation error but increasing the number of cores sampled can correct for this. Taking 2-opposite cores is better than 2-perpendicular. See regression equations for each sampling design in Table S10. Panel B. Effect of stem out-of-roundness (OOR) on log(|%Error|). Increasing OOR increases estimation error but increasing the number of cores sampled can correct for this. Taking 2-opposite cores is better than 2-perpendicular. See regression equations for each sampling design in Table S11.

## Which area calculation methods perform best with eccentric cross-sections?

The effect of area calculation method on BAI estimation error differed for pith eccentricity (POC), but not for stem eccentricity (OOR) (Fig. 5; Tables S12, S13). For POC, methods differed in both intercept and slope (Table S12; Fig. 5A). The ellipse and geometric methods showed higher intercepts, but lower slopes compared to the arithmetic and quadratic methods. As a result, at low POC values, the ellipse and geometric methods produced marginally higher errors (6.6 v. 8.3% error, $p = 0.02$). However, as POC increased, their lower slopes resulted in lower errors compared to the arithmetic and quadratic methods (slope estimates = 0.71 vs. 1.35, $p = 0.001$). For example, at POC = 0.6, mean |%Error| was approximately 22% for the ellipse and geometric methods, compared to 43% for the arithmetic and quadratic methods (Fig. 5A).

<<Figure 5 here>>

**Figure 5. Effect of eccentricity on BAI estimation error, as a function of the method.** For ease of interpretation, |%Error| is also shown on the right as a second y axis. For both panels, the Arithmetic reference slope is shown as a solid line because it significantly differs from 0. For other area calculation methods, dotted lines indicate slopes not different from this reference, whereas solid lines indicate slopes that differ from it. Panel A. Effect of pith-off-centre (POC) on log(|%Error|). None of the explored methods can fully account for the estimation error arising from POC. The Ellipse and Geometric methods perform significantly better than the other two methods at high POC values ($p = 0.02$). See regression equations for each calculation method in Table S12. Ellipse and geometric overlap each other and thus only Ellipse is visible. Panel B. Effect of out-of-roundness (OOR) on log(|%Error|). None of the four methods explored can correct for the increase in estimation error due to OOR. See regression equations for each calculation methods in Table S13.

## Discussion

Overall, the results show that basal area increment (BAI) estimation error is strongly influenced by pith and stem eccentricity when sampling effort is low, but can be substantially reduced through appropriate core sampling design. When only a single core is used, both pith eccentricity (POC) and stem eccentricity (OOR) can generate large errors, with POC generally having a stronger effect than OOR. Increasing the number of cores markedly reduces this error; when two cores are used, their spatial arrangement becomes important, with cores taken 180° apart reducing error more effectively than perpendicular sampling, particularly in highly eccentric stems. In contrast, when four cores are used, the effect of eccentricity becomes negligible, indicating that increased sampling effort can effectively compensates for irregular stem geometry. Together, these results demonstrate that BAI estimation error is not only a function of stem geometry but can be effectively mitigated through sampling choices, with direct implications for ecological studies relying on increment cores to estimate tree growth.

Our findings are consistent with previous studies showing that both OOR (4,7,8) and POC (2,7,8) influence BAI estimation. They also support recommendations to increase sampling effort, such as those of Visser et al. (8), who, based on simulations of tree growth, suggested that four cores provide reliable BAI estimates, and Buras and Wilmking (12), who, in empirical studies on shrubby growth forms, showed that multiple radial measurements improve representation of stem growth. Together, these results reinforce a growing consensus across a range of study systems and methodological approaches that estimation error arising from eccentricity decreases with increasing sampling effort. Accordingly, when sampling one to three cores, researchers should assess and report the potential influence of pith and stem eccentricity on BAI estimates.

The range of pith and stem eccentricities observed in this study is consistent with values reported in previous work. The observed range of pith eccentricity (POC; 0–0.6) aligns with values reported across species and environmental conditions (e.g., 2,7,15). Similarly, stem out-of-roundness (OOR) values in our dataset (0–0.3) fall within the range typically reported for temperate tree species, where stems vary from nearly circular to moderately elliptical (e.g., 7,8,13,27). This correspondence indicates that our dataset captures realistic variation in stem geometry and supports the broader applicability of our findings. However, while the range of eccentricities is comparable to that reported for mature trees, the frequency and underlying drivers of eccentric growth may differ across size classes and environmental contexts, and this should be considered when extrapolating these results.

While previous studies have emphasized the importance of increasing sampling effort (e.g., 8,12), they have not explicitly evaluated how the spatial arrangement of cores influences estimation accuracy. Our results extend this body of work by showing that core orientation plays a critical role: when stems are eccentric, two cores taken 180° apart consistently produce lower error than perpendicular sampling, particularly at high levels of eccentricity (Fig. 4). This likely reflects the fact that opposite cores better capture variation along the major and minor axes of the stem, whereas perpendicular sampling can fail to account for asymmetric growth patterns. In addition, this sampling configuration allows for estimation of pith eccentricity (POC).

Pith and stem eccentricities are not the only sources of error when estimating BAI from increment cores. In our analysis, the linear mixed-effects model explained 34% of the variance in estimation error, indicating that two thirds of the source of error is not captured by the factors considered here (the two types of eccentricities, the area calculation method and the core sampling design). This suggests that additional sources of error, including more complex geometric irregularities such as stem lobing, concavities, or localized growth asymmetries, may contribute to inaccuracies in BAI estimation but are not captured by the POC and OOR metrics (see 8).

In addition, other sources of error that were not present in our samples but are common in increment cores may interact with eccentricity-related errors. These include missing piths and the presence of missing, partially missing, or false rings (8,12). For example, when the pith is absent, basal area is often estimated from diameter measurements rather than radii, implicitly assuming a centred pith and potentially introducing additional bias when this assumption is violated. To our knowledge, the interaction between eccentricity and these additional sources of error has not been systematically evaluated. Future research addressing how these sources of uncertainty combine would therefore improve BAI estimation. In practice, when multiple sources of error are present, a conservative approach is to treat them as additive (28), although this likely overestimates total uncertainty.

## Practical recommendations

**Area calculation method.** Our results show that the choice of calculation method can influence BAI estimation accuracy. We recommend avoiding the geometric mean method, as it consistently produced higher error than the other approaches. The arithmetic, quadratic, and ellipse methods performed similarly overall; however, the quadratic method did not show systematic over- or underestimation of BAI. When radii are available (i.e., in "inside-out" approaches where the pith position is known), estimating basal area using a circular formula with a quadratic mean radius provides a robust approach. Our findings differ from those of Visser et al. (8), who found that the ellipse method minimized error. This discrepancy likely reflects differences in methodology, as their study used an "outside-in" approach based on diameter measurements, whereas our results are based on radii measurements from known pith positions.

**Core sampling design:** Although increasing sampling effort to three or four cores minimizes estimation error, this approach is often impractical in field settings, where obtaining even a single core that reaches the pith can require multiple coring attempts. In many cases, the additional information gained from sampling multiple cores on a single tree may be outweighed by the benefits of sampling a larger number of individuals instead, particularly in studies focused on capturing population-level variability. When accurate measurements are needed at the individual level, our results show that taking two cores 180° apart provides a substantial improvement over single-core sampling and represents a practical compromise between accuracy and feasibility. We therefore recommend sampling two opposite cores to reduce error associated with both pith and stem eccentricity (POC and OOR).

In sloped terrain, pith eccentricity is often related to mechanical responses to gravity, with the pith displaced uphill in gymnosperms (compression wood formation) and downhill in angiosperms (tension wood formation). Sampling cores on the uphill and downhill sides of the stem is therefore likely to capture the longest and shortest radii. Because these measurements are also required to estimate pith eccentricity (POC), this sampling strategy provides the information to quantify potential bias associated with this type of eccentric growth and assess uncertainty in BAI estimates. However, this recommendation may conflict with current best practices in quantitative wood anatomy or dendroclimatology, where cores are typically taken perpendicular to slope to avoid compression or tension wood where rings are more difficult to identify (18). As a result, it may be difficult to simultaneously optimize sampling for both wood anatomical measurements and accurate BAI estimation in eccentric trees. Pith eccentricity may also arise from asymmetric competition, such as when a tree grows adjacent to a large neighbor. In such cases, identifying and sampling along the longest and shortest radii may be more challenging due to limited access around the stem.

**Estimating eccentricity:** Because BAI estimation error arising from eccentricity can be very large when only a single core is used (in some cases exceeding a hundred percent), we strongly recommend collecting more than one core whenever possible. However, when additional coring is not feasible for all trees, estimating pith eccentricity (POC) in the field after extracting a first core can help determine whether a second core is warranted. This decision can be guided by a pre-defined threshold of acceptable error.

If a core intercepts the pith, the observed radius can be compared to the expected radius under circular geometry (i.e., half the stem diameter). This diameter can be estimated in the field using a DBH tape, which provides the average stem diameter based on circumference. Deviations from the expected radius indicate the presence of pith eccentricity, with shorter or longer radii reflecting sampling on the short or long side of the stem, respectively. To remain consistent with the definition of POC (Eqn. 5), which requires the shortest radius, the observed radius ($r_{obs}$) must first be interpreted relative to the expected radius under circular geometry (D/2). If a core is taken perpendicular to the bole and the observed radius is smaller than D/2, it can be approximated as the shortest radius. If the observed radius is larger than D/2, it is assumed to correspond to the longest radius, and the shortest radius can be approximated as the difference between the stem diameter and the observed radius (i.e., $r_{short} \approx D - r_{obs}$). The resulting estimate of $r_{short}$ can then be used to approximate POC following Eqn. 5, using D/2 as an estimate of the average radius. Although this approach provides only a rough estimate of pith eccentricity, it can be used as a practical screening tool to guide sampling decisions in the field. For example, if one aims to maintain the error in BAI estimation arising from POC under 30%, based on the equations provided in Table S10, any POC value higher than ca. 0.25 would warrant taking a second opposite core (Fig. 4A)

Stem out-of-roundness (OOR) can also be estimated in the field using two simple diameter measurements. OOR is defined as the ratio of the diameter of the largest circle that can be inscribed within the cross-section to the maximum stem diameter. On stems without pronounced concavities or lobes, the maximum and minimum diameters can be identified in the field by placing a tree caliper around the stem at breast height and rotating it until the largest and smallest value are obtained. The ratio of these two diameters provides an estimate of OOR. If one aims to maintain the error in BAI estimation arising from OOR under 30%, based on the equations provided in Table S11, any OOR value higher than 0.2 would warrant taking a second opposite core (Fig. 4B)

These measurements include bark thickness and may therefore introduce some error if bark thickness or compressibility varies around the stem. When higher precision is desired, practitioners can measure bark thickness at the points of diameter measurement using a bark gauge and subtract it to approximate xylem diameters. However, the proposed approach provides only an approximation of pith and stem eccentricity. While it does not reproduce the exact geometric measurements used in this study, it relies on simple and widely used field measurements and is sufficient to guide sampling decisions to avoid large errors in BAI estimation.

The supplementary materials provide examples of how to extract the relevant formulas from the linear mixed-effects model coefficients to estimate |%Error| as a function of POC, OOR, area calculation method, and core sampling design. We also show how these relationships can be used to estimate the expected magnitude of BAI estimation error for individual samples based on their POC and OOR values. When possible, practitioners may parameterize these equations for their specific study systems, which may exhibit different ranges of POC and OOR.

In conclusion, our results show that pith and stem eccentricity can introduce substantial error in BAI estimation when sampling effort is limited, but that this error can be effectively reduced through appropriate core sampling design. While increasing the number of cores improves accuracy, practical constraints in field settings often limit sampling intensity. Our findings demonstrate that sampling two cores 180° apart provides a robust and feasible compromise, substantially reducing error while remaining operationally realistic. By combining simple field-based estimates of eccentricity with model-based expectations of error, this study provides a practical framework for improving the reliability and transparency of BAI estimates. Last, irrespective of the number of cores collected, we recommend as a best practice that ecologists assess and report the potential uncertainty in BAI estimates arising from both pith and stem eccentricity using the approaches described above.

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## Supporting Information

**S1 File.** Supplementary material including statistical analyses of |%Error| and %Error, a demonstration of how to estimate |%Error| for specific sampling designs, and supporting tables (S1–S20) and figures (S1–S4).

## Author Contributions

CR and JM designed the study, JM collected the cookie samples, CR collected the images and processed them to obtain the measurements, CR performed the statistical analyses, CR and JM interpreted the results, CR, BGM and JM wrote the manuscript.

## Data Availability Statement

All data is available on Zenodo: https://doi.org/10.5281/zenodo.19831133.