

Conformal Prediction quantifies the uncertainty of Species Distribution Models

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Abstract: Providing accurate estimates of uncertainty is key for the analysis, adoption, and interpretation of species distribution models. In this manuscript, through the analysis of data from an emblematic North American cryptid, I illustrate how Conformal Prediction allows fast and informative uncertainty quantification. I discuss how the conformal predictions can be used to gain more knowledge about the importance of variables in driving presences and absences, and how they help assess the importance of climatic novelty when projecting the models under future climate change scenarios.

1 **INTRODUCTION**

2 The ability to predict where species may be found is a cornerstone of biogeography and macroe-
3 cology (Franklin 2023). Techniques from the field of applied machine learning (ML hereafter) are
4 now routinely used alongside ecological approaches to train generalizable species distribution
5 models (SDMs hereafter) (Beery et al. 2021). SDMs generate a binary response (corresponding to
6 the prediction that the species is likely present/absent under given environmental conditions) or
7 a quantitative score most often as a probability of presence or habitat suitability, indicating how
8 strongly we believe that the species may be present at the location.

9 Proper communication of the uncertainty associated to the prediction of a SDM is important,
10 since we usually seek to apply these models to look both forward and backwards in time. This
11 projection if the model to different times is usually called “transfer” (Zurell et al. 2012), whereby
12 a model trained under historical (baseline) conditions is applied to past/future projections of the
13 same predictors. The projection of SDMs can also happen in space (Petitpierre et al. 2016), to
14 predict where species may invade or be naturalized. Even when predictions are not projected,
15 spatial knowledge of the uncertainty is valuable information: it can be used to identify areas
16 where the model predictions are trustworthy. Current checklists on the reproducibility of SDMs
17 emphasize the consequences of data uncertainty (Feng et al. 2019). Yet, predictions also have
18 inherent uncertainty, which is usually not adequately communicated. This can be, for example,
19 because of genuine uncertainty about (or inability to capture through the model) the actual
20 response of the species to combination of predictors (Parker et al. 2024).

21 A common way to capture information about the variability of SDMs is to rely on non-parametric
22 bootstrapping (Valavi et al. 2021), wherein models trained on random subsets of the data are
23 compared to estimate the distribution of the response under incomplete sampling. This approach
24 captures more than one type of variability (Thuiller et al. 2019), and provide valuable information
25 about the range of performances that can be expected from a model. Other methods are built into
26 the predictor itself, as is the case for *e.g.* BARTs (Carlson 2020), which estimate their own uncer-
27 tainty. But either situation comes with drawbacks. Bootstrapping requires to train and evaluate
28 the model hundreds of times, and on partial datasets, which is computationally inefficient. Using
29 methods that are specific to a particular classification algorithm limits one to the classifier for
30 which these methods are available, which prevents for example the use of a new algorithm with
31 the same estimation of uncertainty.

32 In this manuscript, I illustrate how the ML technique of conformal prediction (CP) allows to
33 identify instances (combinations of environmental variables) for which a trained and calibrated
34 model cannot confidently make predictions (Gammerman et al. 1998). A brief introduction to CP
35 is provided in this manuscript, but the topic is covered in more depth by Shafer & Vovk (2007)
36 for the mathematical foundations, by Fontana et al. (2020) for a historical perspective, and by
37 Angelopoulos & Bates (2023) for concrete recommendations. By way of contrast to *e.g.* bootstrap-
38 ping, CP does not necessarily involve retraining the same model many times over, but instead
39 wraps the model into an additional prediction step, and returns estimates of credibility based on
40 the distribution of past model predictions compared to ground-truthed data. This is an important
41 difference, as conformal prediction makes no assumption about the distribution of data, but
42 rather captures the uncertainty associated to the distribution of observed model outcomes (Lei &

43 Wasserman 2013). Conformal prediction provides what is essentially (for classification problems)
44 a confidence interval around the presence or absence of a species in a given location. This is a
45 particularly important feature, in that CP achieves this in a way that creates several analogues
46 between ML prediction and fundamental concepts in frequentist statistics (Neyman 1937).

47 One of the reasons why CP is particularly promising for uncertainty quantification in SDMs
48 is that it is a distribution-free method: it requires neither assumptions about the model nor
49 prior knowledge of the outcome distribution to provide confidence intervals that are as small
50 as possible while being *guaranteed* to contain the true value under a set risk level (Vovk et al.
51 2018). This is particularly important when transferring a SDM to novel environments (Zurell
52 et al. 2012), where we expect covariate shift (the joint distributions of predictors are different
53 when training and predicting), a prediction context that CP is robust to (Fannjiang et al. 2022,
54 Tibshirani et al. 2019).

55 Using occurrence data about an emblematic North American cryptid, I provide a template for
56 the adoption of CP as a natural way to quantify uncertainty of species distribution models. In
57 particular, I show how predictions under CP (i) identify areas where the species range is uncer-
58 tain, (ii) estimate uncertainty differently from bootstrapping methods, (iii) can be explained using
59 Shapley values analysis, and (iv) quantify the accumulated uncertainty when transferring the
60 SDM to future conditions. I conclude by highlighting ways in which using CP can both simplify
61 the process of training SDMs, and provide information that make their discussion and analysis
62 more informative.

63 **METHODS**

64 **DATA**

65 *Occurrence data*

66 The occurrence data used in this article are geo-referenced observations of the Sasquatch (Lozier
67 et al. 2009). Although these observations are likely to be mis-categorized American black bears
68 (Foxon 2024), they nevertheless share many features of the data that are used to train SDMs: high
69 auto-correlation, uneven sampling effort, and clear association with several bioclimatic variables
70 that is robust enough to train a predictive model. The recorded locations, as well as a background
71 points, are presented in Figure 1.

72 This dataset lacks associated records of absence. This is a characteristic shared with most appli-
73 cations of species distribution models, and therefore a desirable property to illustrate the use of
74 conformal prediction. Through this article, I will rely on pseudo-absences (described in the next
75 section) to replace true absences. Because they are treated as absences in a machine learning
76 context (and, though never explicitly, also when using methods like MaxEnt), I will refer to
77 observations as “presences”, to pseudo-absences as “absences”, and the classifier will therefore
78 be described as making a prediction on the species “presence”.

79 *Pseudo-absences generation*

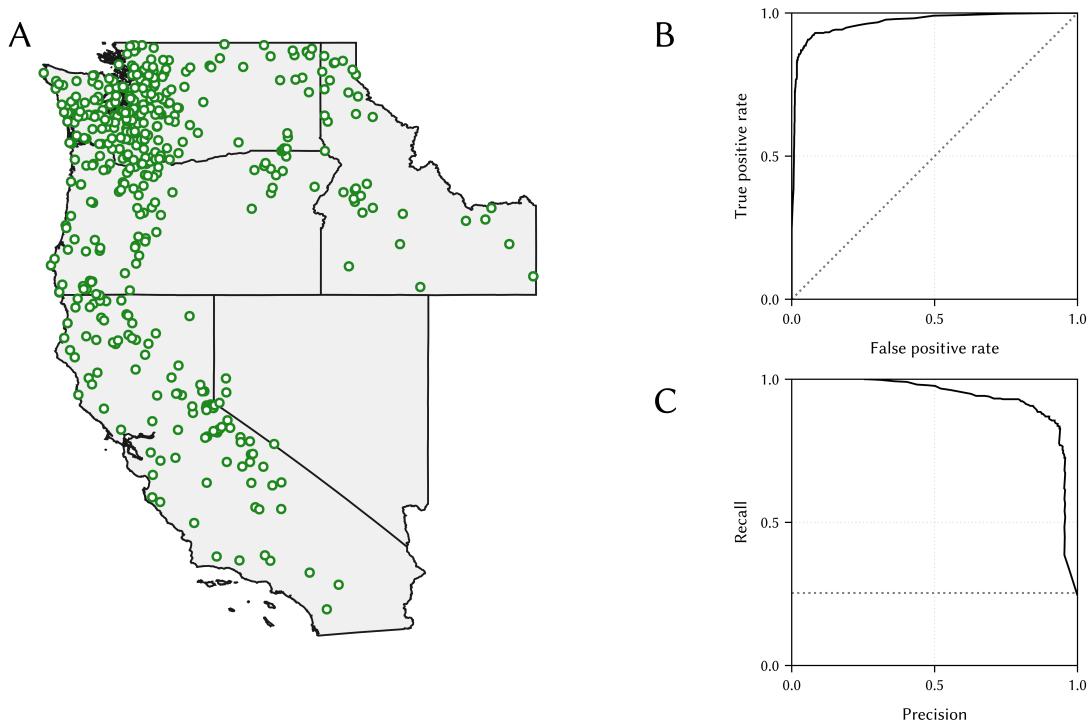
80 The dataset of observations is composed only of presences. In order to establish a baseline of
81 absences to train a binary classifier, there is a need to generate a number of pseudo-absences,
82 which simulates locations at which the species, if not absent, has not been observed. In order to
83 do so, the presence data were first spatially thinned to be limited to one for each cell, at a 5.0
84 minutes of arc resolution. Cells that had no observation were potential candidates for a pseudo-
85 absence, and were further selected by drawing a number of them, without replacement, where the
86 probability of inclusion in the sample was proportional to h_{\min}^{-1} , where h_{\min} is the Haversine (great
87 arc) distance to the nearest cell with an observation, measured in kilometers. In other words, cells
88 that were close to an observation were unlikely to be included, and cells that were further away
89 were more likely to be so. To avoid sampling pseudo-absences too close to presences, the pixels
90 less than 10 kilometers away from known observations were excluded from the background
91 data. This method of pseudo-absence selection is akin to “background thickening” (Vollering et
92 al. 2019), which avoid selecting pseudo-absences too close to known obervations, and seeks to
93 increase the importance of locations that are further from observations when picking pseudo-
94 absences.

95 The number of pseudo-absences was arbitrarily set to two times the number of presences.
96 Although Barbet-Massin et al. (2012) recommend to use the same number of presences and
97 pseudo-absences for classifiers, using an imbalanced dataset is not a problem: stratified k-
98 folds cross-validation is perfectly able to handle the moderate class imbalance we introduce
99 (Szeghalmy & Fazekas 2023), and the model performance (as will be established in a later section)

100 is sufficient. Moreover, most real-world applications of classification will have to deal with
101 problems with class imbalance (this is particularly likely to be true of SDM application from
102 sampling data, where presences may be the minority of outcomes); it is therefore important to
103 ensure that we do not establish a testing scenario that is too optimistic about the prevalence of
104 presences. In all cases, class imbalances is a feature of data that must be dealt with in order to
105 get the more predictive models (Benkendorf et al. 2023).

106 *Bioclimatic data*

107 The model was trained, validated, and applied on the 19 WorldClim2 BIOCLIM variables (Fick &
108 Hijmans 2017), at a spatial resolution of 2.5 minutes of arc. Preliminary analyses using 0.5, 2.5, 5,
109 and 10 minutes of arc show that the qualitative results presented hold (the results and conclusion



110 **Figure 1:** Overview of the occurrence data (green circles) and the pseudo-absences (grey points) for the states of,
111 clockwise from the bottom, California, Oregon, Washington, Idaho, and Nevada (A). The underlying
112 predictor data are at a resolution of 2.5 minutes of arc, and represented in the World Geodetic System 1984
113 CRS (EPSG 4326). The panels on the right column show the ROC curve (B) and PR curve (C), with the
114 random classifier indicated by a dotted line. The area under the ROC curve is $\approx 96\%$.

115 are equivalent). For the projection of the model under climate change, I only report the future
116 data under the SSP245 scenario (“middle of the road”), for six GCMS: MRI-ESM2.0 (Yukimoto et al.
117 2019), ACCESS-CM2 (Huneke et al. 2025), EC-Earth3-Veg (Döscher et al. 2022), CanESM5 (Swart
118 et al. 2019), GFDL-ESM4 (Dunne et al. 2020), and MIROC6 (Shiogama et al. 2023). The climatic
119 data were retrieved for the 2081-2100 period. The prediction for a score under future climates
120 is measured as the median of the predicted values for each of the GCMs, and the prediction of
121 whether this point lies within the future range of the species is done by applying majority voting
122 to the six predictions.

123 The climatic novelty of the baseline *v.* future data is estimated through the Euclidean distance
124 (Fitzpatrick et al. 2018), specifically by assigning as a novelty score for each pixel in the future
125 the distance to its closest baseline analogue. This novelty is measured on de-means predictors
126 with unit variance to ensure that predictors with different scales can be adequately compared.
127 The method from Williams et al. (2007) is adapted by using the *training* bioclimatic conditions
128 as the baseline, and then measuring the *novelty* in the contemporary data (spatial covariate shift
129 in historical predictions) and the projected data (spatio-temporal covariate shift in future predic-
130 tions). This variation allows to evaluate the effect of covariate shift in both the contemporary and
131 future climates, as covariate shift may lead to a lessened data exchangeability, thereby decreasing
132 the relevance of CP.

133 **SPECIES DISTRIBUTION MODEL**

134 All analyses are conducted using the `SpeciesDistributionToolkit` package (Poisot et al. 2025)
135 for *Julia* 1.11.

136 *Model structure*

137 The model used here is a logistic regression, with interactions terms up to a maximum degree
138 of two (preliminary analyses with random forests, naive Bayes classifiers, and rotation forests
139 resulted in similar predicted ranges and cross-validation performance, which suggest that the
140 problem can be handled well by multiple algorithms). When trained on a vector of features \mathbf{x}_i
141 (with null means and unit variances), the model will return a probability p_+ , which correspond to
142 the probability of these environmental conditions being associated to the presence of the species.
143 This probability is turned into a presence/absence decision by comparing it to a threshold, as
144 explained in a later section. Because this logistic regression is a deterministic classifier, the
145 prediction p_{i+} (the probability associated to the prediction of “presence” for prediction i) satisfies
146 $0 \leq p_{i+} \leq 1$, and we use $p_- = 1 - p_+$ as the probability that the species is absent from the location.

147 *Tuning*

148 We tune this model by (i) iteratively forward selecting the best set of predictor variables, and
149 (ii) optimizing the threshold τ above which a site with a probability for the positive class p_+
150 is considered to be positive (turning the prediction of presence into $p_+ \geq \tau$). In both cases, the
151 cross-validation strategy is the same: the dataset is split in 10 random folds, 9 of which are used
152 for training and one for validation. All folds are used for evaluation, providing exhaustive cross-
153 validation. The folds are stratified so that the relative number of present cases in the training
154 set is similar to that of the entire dataset. The performance on each set, for the purpose of
155 defining the set of variables to include of the threshold to use, is measured as the average of
156 the Matthews Correlation Coefficient (MCC) across each of the ten folds. The MCC is the most

157 accurate representation of a binary classifier performance (Chicco & Jurman 2023), and avoids
158 the pitfalls of several other validation measures.

159 For all steps of model training and validation, the identity of instances composing the different
160 folds remains fixed. This ensure that the changes in MCC are only due to the addition of the
161 variable, and not to the random sampling of a training/validation set with different properties.

162 Although some authors encourage the use of spatially-stratified cross-validation (Soley-Guardia
163 et al. 2024), this is not a desirable strategy for this use-case. The area in which the predictions will
164 be made is entirely delimited by the bounding box of observed presences, and there is therefore
165 no risk of covariate shift when shifting from validation to prediction (outside of the situation of
166 temporal transfer of the SDM).

167 The predictors included in the model have been decided through the use of forward selection.
168 This is an important step in order to perform dimensionality reduction (which generally increases
169 the predictive accuracy), but also to ensure that the set of retained variables is reduced enough
170 that it can be interpreted. Variables were retained as part of the final set of predictors if adding
171 them increased the MCC for the model once retrained with this new variable.

172 One of the most efficient ways to increase the performance of binary classifiers is to change the
173 decision rule leading to a positive (here, presence) prediction, so that presences are assigned when
174 $p_+ \geq \tau$ – a process known as moving threshold classification (Liu et al. 2013, 2016). The value of
175 τ is an hyper-parameter of the model, which is chosen to maximize the value of a measure of
176 model performance (here the MCC) when evaluated over many different values. In this instance,
177 we optimized the value of τ by picking the value out of 200 linearly spaced value between the

178 smallest and largest prediction made on the training set. The value of τ that maximizes the MCC
179 during cross-validation was selected as the optimal threshold for the classifier. Note that even
180 though our decision rule for the presence of the species is $p_+ \geq \tau$, we will keep the information
181 about p_- as is required for conformal prediction.

182 *Bootstrap variability*

183 Bagging (bootstrap aggregating) is often used as a measure of uncertainty to the underlying data
184 when training SDMs (Beale & Lennon 2012). When performing bagging, the model is trained
185 on samples drawn with replacement from the training set (which leaves out approx. 37% of the
186 dataset). Models are then evaluated on samples that were not used as part of their training, usually
187 using cross-validation (Bylander 2002) or measures of the out-of-bag error (Janitz & Hornung
188 2018). Although ensemble models *can* result in a better predictive performance compared to
189 single models (Drake 2014), this is not a guarantee (and depends on the structure of the bias/
190 variance trade-off for the specific model and its training set). The many models trained on the
191 bagging dataset form an homogeneous ensemble, which is to say a set of models that share the
192 same algorithm and hyper-parameters, and only make different predictions as the result of having
193 been trained on different subsets of the full training set.

194 Measures of whether the different models composing the homogeneous ensemble agree can
195 provide a measure of the effect of data and parameter uncertainty (Petropoulos et al. 2018),
196 or what Davies et al. (2023) termed the “SDM uncertainty”. The best model identified after
197 thresholding was evaluated on a hundred bootstrap samples, yielding an homogeneous ensemble
198 model from which we estimate bootstrap variability (Chen et al. 2019). Because the model is kept

199 constant in this analysis, the measure of variability we will derive from the ensemble model is
200 an estimate of how sensitive the estimation of the model parameters is to small perturbations
201 (specifically: spatially homogeneous under-sampling) to the training data.

202 **AN INTRODUCTION TO CONFORMAL PREDICTION**

203 Conformal prediction differs from regular prediction in that, rather than a single point prediction,
204 it returns sets corresponding to the ensemble of *credible* outcomes given an input x representing
205 environmental conditions at which we seek to make the prediction. Given the observed quantiles
206 of the model output on validation data, these sets are obtained through a simple calibration step.
207 Therefore, CP requires an already trained model, and is agnostic to the process through which
208 this model is trained. In this section, I highlight two important features of CP: the notion of
209 *prediction sets* (and how they are obtained), and the notion of *coverage*, which is a measure of
210 tolerance to error.

211 **UNDERSTANDING CONFORMAL PREDICTIONS**

212 By contrast to the non-conformal SDM, the conformal classifier returns, for an input of environ-
213 mental predictors x , a set C containing the “credible outcomes” for this prediction. This set is
214 termed the *prediction set*, and under a binary classification task (the species is either present or
215 absent), there are four possible combinations for the content of prediction sets: $C = \{+\}$, $C = \{-\}$,
216 $C = \{+,-\}$, and $C = \emptyset$.

217 The first two cases are simple: if the prediction set contains a single output, the model can
218 confidently make a prediction that excludes the other class. In the case of $C = \{+\}$, for example,

219 the point prediction for the presence score p_+ is high enough that the outcome of absence can
220 be ruled out given the known predictions on training examples. In some cases, the prediction
221 set may contain both classes, as in $C = \{+, -\}$. Although they may not be *equally likely* (there is
222 no guarantee that $p_+ \approx p_-$), the scores are close enough to not confidently exclude one of the
223 outcomes from the model prediction. In the specific cases of SDMs, these correspond to areas
224 of true uncertainty, where the known training examples credibly support both the presence or
225 absence of the species. The final situation, $C = \emptyset$, corresponds to pathological cases where *neither*
226 outcome can be credibly supported. Given the training data (and the distribution of presences
227 and absences), the model is not able to make a prediction for this input. The increased frequency
228 of such predictions is most likely a strong sign that the risk level is too high (equivalent to a too
229 broad confidence interval) for the training data given to the conformal model.

230 These situations correspond to four different outcomes in terms of the SDM certainty about the
231 distribution of the species. The most intuitive situation is $C = \{+\}$ or $C = \{-\}$, in which case the
232 conformal model predicts that the absence (resp. presence) of the species is *not* a credible outcome
233 for the environmental conditions given as an input. Throghout this manuscript, I will refer to
234 these predictions as “sure presences” and “sure absences”, as they convey the information that
235 there is no reason to expect that the prediction is uncertain. The second situation, $C = \{+, -\}$,
236 corresponds to inputs for which the presence and the absence of the species are credible, and I
237 will refer to them as “unsure”. The rare cases where $C = \emptyset$ will be “undetermined” predictions.

239 There are several ways to decide whether a point prediction from the model results in which
 240 prediction set. A core assumption of CP is that the data used for training should be exchangeable,
 241 or in other words, their joint probability distribution should be (close to) invariant under finite
 242 permutations (Aldous 1985). This will almost never be the case for data with a spatial structure;
 243 nevertheless, this does not rule out the use of CP for species distribution modeling, as Oliveira
 244 et al. (2024) show that CP is acceptably robust to lack of exchangeability. The purpose of this
 245 section is to establish a general overview of how conformal predictions are obtained, and some
 246 of the multiple variations that exist will be introduced throughout the text.

247 The central idea of CP is to associate a conformal score to a point prediction. This can be achieved
 248 by applying the softmax function to the values for p_+ and p_- (note that the values of p are
 249 bounded, and proportional to the true event probability), giving

$$s_+ = \frac{\exp p_+}{\exp p_+ + \exp(1 - p_+)}, s_- = \frac{\exp(1 - p_+)}{\exp p_+ + \exp(1 - p_+)} \quad (1)$$

250 The conformal score associated to a prediction is $1 - s_\cdot$, where \cdot is the prediction (+ or -) made
 251 by the model. We call the distribution of conformal scores \mathcal{S} . Note that this can be done without
 252 using the softmax function (*i.e.* $s_+ = p_+$, $s_- = 1 - p_+$), but it is used here as it is best practice for
 253 classification (Dey et al. 2023). The use of the softmax function is appropriate here because not all
 254 algorithms for species distribution models will return well-calibrated, probabilities, even though
 255 $0 \leq p_+ \leq 1$ and $p_- = 1 - p_+$.

256 The next step is to identify a critical value \hat{q} above which a conformal score indicates that the
257 prediction it describes is credible. This critical value is picked by examining the empirical quantile
258 distribution of the conformal scores in the distribution \mathcal{S} calculated over n training examples,
259 and an acceptable level of risk α (explained in depth in the next sub-section). Specifically, this is
260 done by identifying the q_i -th quantile of the distribution of model scores, where

$$q_i = \frac{[(n+1)(1-\alpha)]}{n} \quad (2)$$

261 The corresponding value of S below which a proportion q_i of values lies is \hat{q} . In other, more
262 intuitive words, the value q_i indicates what proportion of wrong classification events we must
263 accept before we have accumulated enough evidence to be confident about a prediction. When
264 performing the prediction, we calculate the score of a new prediction according to Equation 1.
265 For every possible class x , if $s_x \geq (1 - \hat{q})$, this class is retained as part of the prediction set. Note
266 that some approaches to conformal prediction, some of which will be discussed in the following
267 sections, keep the distribution of scores separate for each class, *i.e.* \mathcal{S}_+ and \mathcal{S}_- , in which case
268 the quantiles are also class-specific rather than global.

269 The value of \hat{q} can be obtained either through using a holdout set for training (Split Conformal
270 Prediction), using adaptive prediction sets (Angelopoulos & Bates 2023), by retraining the model
271 in a way akin to Leave-One-Out cross-validation (Full Conformal Prediction), through the use
272 of quantile regression (Romano et al. 2019), or through taking the median of several estimates of
273 \hat{q} after cross-validation (Vovk et al. 2018).

274 To summarize, the output of the conformal classifier is, in a sense, a point estimate of the credible
275 outcomes of a model, using the value estimated for p_+ as well as knowledge about which of these
276 were associated to the correct label in the training data. A location is defined as included in the
277 range if the positive outcome is included within the prediction set returned by the conformal
278 classifier, and as excluded from the range when it is not. Because the conformal classifier can
279 identify that both outcomes are credible based on the training data (while giving them different
280 weights), predictions in which both the positive and negative outcomes are included in the
281 prediction set can be seen as “uncertain” at this given risk level.

282 How frequently a specific prediction is uncertain is termed the inefficiency of the classifier,
283 which is defined as the average cardinality of all prediction sets. The inefficiency is bounded
284 upwards by the number of classes (two for binary classification); when the inefficiency is ≈ 1 , the
285 conformal classifier behaves (essentially) like deterministic classifier, by returning a single class
286 for each instance. An inefficiency close to unity is not desirable: smaller sets can hide our actual
287 uncertainty (Sadinle et al. 2018). Because the conformal models wraps the logistic regression
288 model, we can further divide the “unsure” predictions as a function of whether they would be
289 within the range as predicted by the SDM, which I will call “unsure presences”; the other unsure
290 predictions are referred to as “unsure absences”.

291 THE COVERAGE LEVEL

292 CP allows users to set a desired error rate, α , which appeared in Equation 2. Intuitively, what CP
293 does, is inform the user on whether the prediction set contains the true value with probability
294 $1 - \alpha$, which allows to directly interpret this value as a true confidence interval. This error rate

295 is usually referred to as the *marginal coverage*, in that it captures the probability of success
296 marginalized over the known validation points. Because the estimate of uncertainty involves the
297 original model, it is important to apply CP on a model with adequate performance.

298 coverage is a well-defined, classical property of confidence intervals in statistics

299 Chaning the risk level α leads to different estimates of how commonly multiple classes will
300 be accepted as a credible outcome. Using a low level of risk ($\alpha \approx 0$) yields usually leads to all
301 outcomes being credible ($\hat{q} \approx 1$), at the cost of a very high uncertainty. When values of α get
302 too large ($\hat{q} \approx 0$), no class can be confidently predicted, and the model will eventually always
303 return $C = \emptyset$. Although this later situation is more difficult to make sense of intuitively, a value
304 of inefficiency that gets smaller than unity should be interpreted as a model that accumulates
305 more uncertainty (at a given risk level) than the data can support (Romano et al. 2020). Conformal
306 prediction can therefore inform us on the acceptable risk levels we can operate under given a
307 trained predictive model.

308 In the rest of this analysis, I will set $\alpha = 0.05$. As noted by Angelopoulos & Bates (2023), this
309 corresponds to estimating whether a specific prediction falls within, or outside of, the 95%
310 confidence interval across all predictions, which is a convenient callback to frequentist statistics'
311 usual risk tolerance. Recall that the CP prediction sets are estimated based on the model output,
312 and therefore even when aiming for full coverage, there may be non-ambiguous combinations
313 of environmental predictors.

314 **IMPORTANT VARIANTS ON CP THAT ARE RELEVANT FOR SDMs**

315 As mentioned previously, conformal prediction is a general framework, which has been imple-
316 mented in a variety of ways. Some of these are more immediately relevant to SDMs, and in this
317 short section I will introduce two: Mondrian-CP, and risk-aware CP.

318 A core feature of occurrence data (whether based on documented or simulated absence data) is
319 that they suffer from class imbalance, wherein the proportion of presences tends to be lower
320 than the proportion of absences. As this imbalance gets extreme, having a single threshold for
321 the inclusion of a class in the prediction set ceases to be equitable. A way to handle this issue
322 is suggested by Mondrian-CP (Boström et al. 2021), where the scores are accumulated to class-
323 specific distributions, here \mathcal{S}_+ and \mathcal{S}_- , and the number of calibration instances in these two
324 classes are used to estimate a class-specific threshold (quantiles are, in other words, estimated for
325 each separate distribution). Importantly, this approach has been shown to respect the coverage
326 guarantees for each class. (Sun et al. 2017) have established that Mondrian-CP can be used in a
327 cross-conformal context; therefore, in this manuscript, I will rely on cross-validated Mondrian-
328 CP cutoffs for the inclusion of either the positive or the negative cases in the credible set.

329 Depending on the purpose for which the SDM is produced, the uncertain areas can be treated
330 differently. As Prescott et al. (2025) argue, when dealing with invasive species, it may be more
331 reasonable to interpret SDMs by erring on the side of caution, which here would mean consid-
332 ering that unsure presence area (outside the range of the non-conformal prediction, but where
333 the positive case is part of the credible set) should be considered part of the species's range. On
334 the other hand, when SDMs are meant to guide conservation actions that are costly or should

335 be focused on areas of high certainty of suitability for the target species (Pěknicová & Berchová-
336 Bímová 2016), it may make sense to ignore the unsure presences. Note that recent developments
337 in CP, such as conformal risk control, allow to penalize the loss function used to build the credible
338 set to reflect the consequences of different types of mispredictions (Angelopoulos et al. 2025).

339 **RESULTS**

340 **PERFORMANCE OF THE BASELINE MODEL**

341 In panels B and C of Figure 1, we report the ROC and PR curves for the model. As evidenced
342 by both these diagnostic tools, the model achieves a very high predictive accuracy. In Table 1,
343 we report additional measures of performance for the training and validation set of the model
344 (so as to ensure that the model is not performing better on training data), as well as a measure
345 of the performance of the ensemble, to show that it can make valid predictions in addition to

346 **Table 1:** Overview of measures of model performance for the validation and training sets of the SDM, as well as
347 the same measures for the ensemble model (measured on the out-of-bag models only). The values of κ and
348 the true-skill statistic are generally comparable to the MCC, but are included as they are commonly
349 reported in the SDM litterature (Allouche et al. 2006). The high values of the negative and positive predictive
350 values indicate that the model is suitable to detect both presences and absences. NPV and PPV are,
351 respectively, the negative and positive predictive values, which indicate the ability of the classifier to make
352 reliable predictions for the negative and positive outcomes.

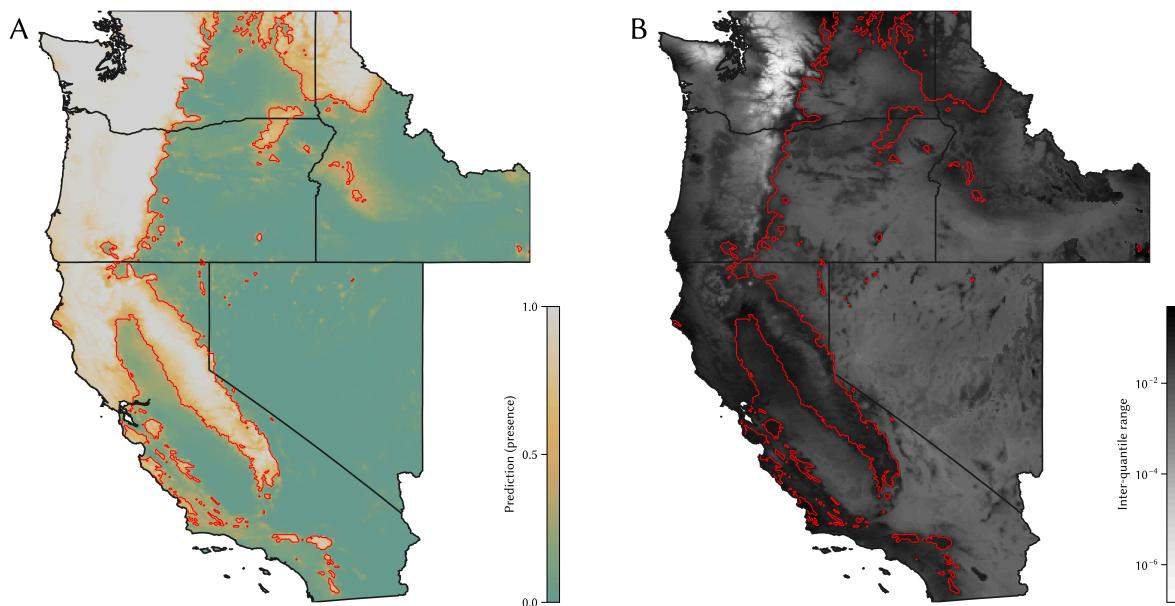
Measure	Validation	Training	Ensemble
MCC	0.75	0.76	0.76
NPV	0.93	0.93	0.94
PPV	0.82	0.83	0.82
κ	0.75	0.76	0.76
TSS	0.74	0.75	0.76
Accuracy	0.91	0.91	0.91

360 quantifying variability. These results confirm that the model is able to identify areas that are
361 suitable to the species, and can be used for CP.

362 Before applying CP, it is useful to examine the output of the SDM in space. The predictions of
363 the model for the entire region are given in Figure 2, alongside information about the model
364 variability. Areas of lowest variability (according to the IQR based on non-parametric bootstrap
365 results from the ensemble) seem to be associated with the absence of the species, with the
366 variability mostly increasing within the predicted range. Note that this bagging model is used
367 only to estimate variability due to lack of observation data, and not to estimate the species range.

368 **CONFORMAL PREDICTION OF THE SPECIES RANGE**

369 Before discussing the spatial output of running the conformal model, it is worth considering
370 why the thresholding step as visualized in Figure 2 is not really providing us with a set of

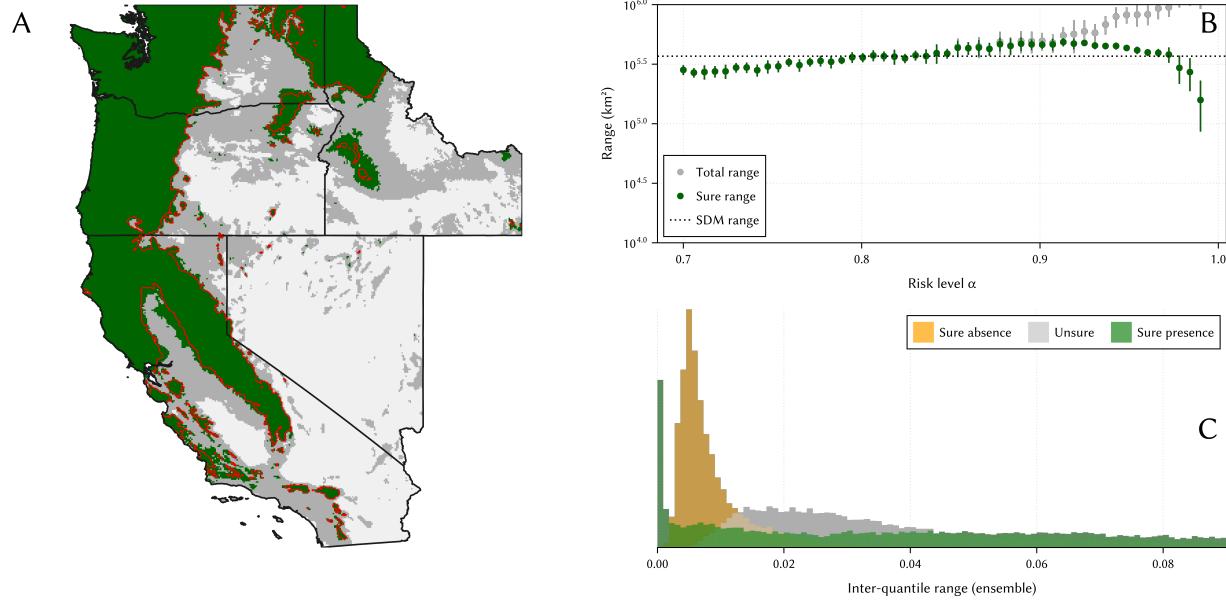


371 **Figure 2:** Overview of the probability p_+ returned by the model (A), and the inter-quantile range of the non-
372 parametric bootstrap model predictions (B). The range, *i.e.* the limit of cells for which $p_+ \geq \tau$, is indicated
373 by a solid red line; I maintain this convention for all subsequent figures. Note that the scale of the variability
374 is logarithmic, as the model shows good performance and therefore has low variability overall.

375 certain presences and absences. When optimizing the threshold τ above which a prediction p_+
376 from the non-conformal model is determined to be a presence, we inherently establish a sort of
377 certain presences and certain absences, specifically by ignoring the possibility that there can be
378 uncertain predictions. Indeed, the space covered by positive predictions is usually interpreted as
379 the (potential) distribution of the species. But this prediction conveys a false sense of certainty,
380 that has to do with the very nature of the threshold we optimize. By definition, the threshold is the
381 value that finds the best balance between the false/true positive/negative cases on the validation
382 data for a given measure of model optimality this is in fact why the optimal threshold is the point
383 closest to the corners of the ROC and PR curves indicating a perfect classifier (Balayla 2020).

384 When a prediction p_+ gets closer to the threshold, a small perturbation to the environmental
385 conditions locally could bring it on the other side of the threshold, and therefore flip the predicted
386 class using the non-conformal classifier. Around the threshold is where we expect uncertainty
387 to be the greatest.

388 To bring these considerations into a spatial context: we expect the areas where the score for the
389 present class are closer to the threshold (the limits of the predicted range of the species) to be
390 the most uncertain. Importantly, this is true *both* for areas that are inside the range (for which
391 p_+ is just above the threshold) and for areas that are outside of it (for which p_+ is just below
392 the threshold). CP is perfectly suited to solving this issue, by identifying the areas where one
393 class is predicted, but the other class is also credible. In this section, we will project the areas
394 with uncertain predictions, and compare the uncertainty quantified by the conformal model to
395 the uncertainty derived from the ensemble model.



396 **Figure 3:** Overview of areas where the presence of the species is certain according to the CP model under a risk
 397 level $\alpha = 0.05$ (A). The certain areas are in dark green, and the uncertain areas, wherein both presence and
 398 absence are credible, are in dark grey. (B) Surface covered by the sure absence and total range (including the
 399 superficy of the unsure area) for different risk levels (expressed as the desired confidence, $1 - \alpha$). Note that
 400 for $\alpha \approx 0.1$, the total predicted range starts being lower than the range predicted by the SDM, and the
 401 uncertain range collapses. (C) Distribution of variability from Figure 2B by type of CP model outcome under
 402 $\alpha = 0.05$.

403 In Figure 3, we show that this prediction indeed stands: the range as predicted by the SDM
 404 (fig. 3A) falls within the range of unsure predictions. We also see that lowering the risk level
 405 α leads to a contraction of the area (in km²) considered to be credibly associated to only
 406 the presence of the species ($C = \{+\}$), while the range that is ambiguous ($C = \{+, -\}$) increases
 407 (Figure 3B). As far as ecologists are concerned, the areas in which the prediction set only has a
 408 score for the absence of the species are the easiest to make sense of: they correspond to regions
 409 where the model is certain (under the specified risk level) that the species is absent. All other areas
 410 (assuming that there are no predictions for which the prediction set is empty, which I discuss in
 411 the next section) are *potentially* part of the range of the species: some certainly (would have been
 412 included in the non-conformal prediction), some uncertainly (would not have been included in
 413 the non-conformal prediction).

415 Note that the relationship between the certainty associated to CP, and the variability under the
416 ensemble model presented in Figure 2B is nuanced: in fig. 3C, it appears that although areas
417 identified as unsure using CP tend to have higher variability, there is considerable overlap
418 between the categories. Intriguingly, the overlap between areas that are uncertain according to
419 the conformal classifier, and areas that are uncertain according to the bootstrap model, is imper-
420 fect. There are a number of points classified as sure presences for which the IQR is very high, *i.e.*
421 points whose certainty is not affected by undersampling the training data. Notably, the results
422 in fig. 3C show that it is not possible to find a cutoff in the measure of bootstrap variability that
423 would identify areas of model uncertainty. This suggests that the classification of predictions as
424 certain/uncertain according to the conformal prediction is in part reflecting genuine uncertainty
425 in the underlying data, but also contributing novel information about the fact that some instances
426 are more difficult to call.

427 These results can be better understood by contrasting what “uncertain” means in the context of
428 CP, and how it differs from the uncertainty in the ensemble model. The uncertainty derived from
429 the ensemble model represents whether many models trained on small perturbations of the full
430 training dataset would agree on a specific prediction task, represented by an array of environ-
431 mental predictors. Therefore, the uncertainty from the ensemble originates in the estimation of
432 the parameters, and its sensitivity to being able to access the full information within the training
433 data. Uncertainty in the conformal classifier is coming from comparing a specific model predic-
434 tion for a known input to all other predictions in the training (calibration) data, thereby allowing

435 to estimate the model prediction scores leading to possibly the prediction of both the presence
436 (or absence) outcome. Therefore, the uncertainty from the conformal predictors accounts for all
437 the predictions the model can make, and accounts for the variability *across* predictions within a
438 fully known dataset.

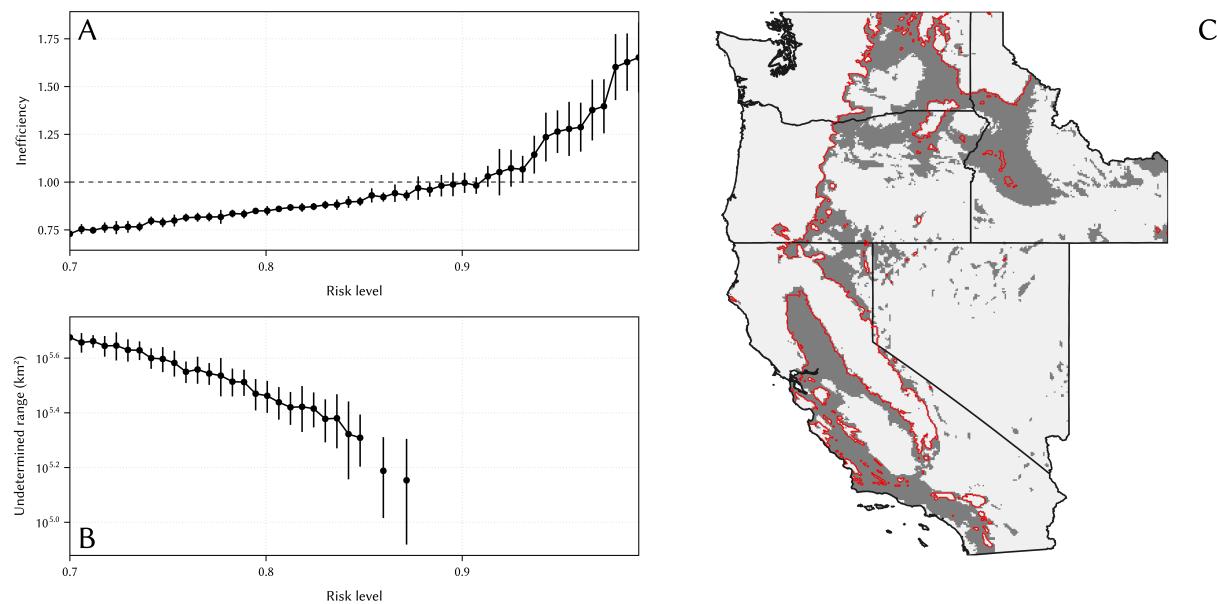
439 Despite differences in the type of uncertainty captured by bootstrap *v.* CP, it remains noteworthy
440 that there is an association between the two. Bootstrap uncertainty simulates the effect of
441 knowing a little less about the species occurrences, and therefore high bootstrap uncertainty
442 areas would be good candidates to collect additional presence (or true presence) data. By contrast,
443 CP is more likely to identify areas of model uncertainty, where the presence or absence of the
444 species is genuinely more difficult to decide, and where therefore uncertainty may be reasoned
445 about biologically. It may not be unexpected that even when the bootstrap variability decreases,
446 because we have collected enough information about the system, there would remain some CP
447 uncertainty because the presence of a species may, in some habitats or under some environmental
448 conditions, be more intrinsically uncertain.

449 **IDENTIFICATION OF UNDETERMINED AREAS**

450 In Figure 3B, we see that there is a risk level above which the total predicted range starts to get
451 lower than the range predicted by the SDM. We can explain this behavior through the lens of the
452 number of undetermined predictions, *i.e.* the number of inputs for which the CP model returns
453 $C = \emptyset$.

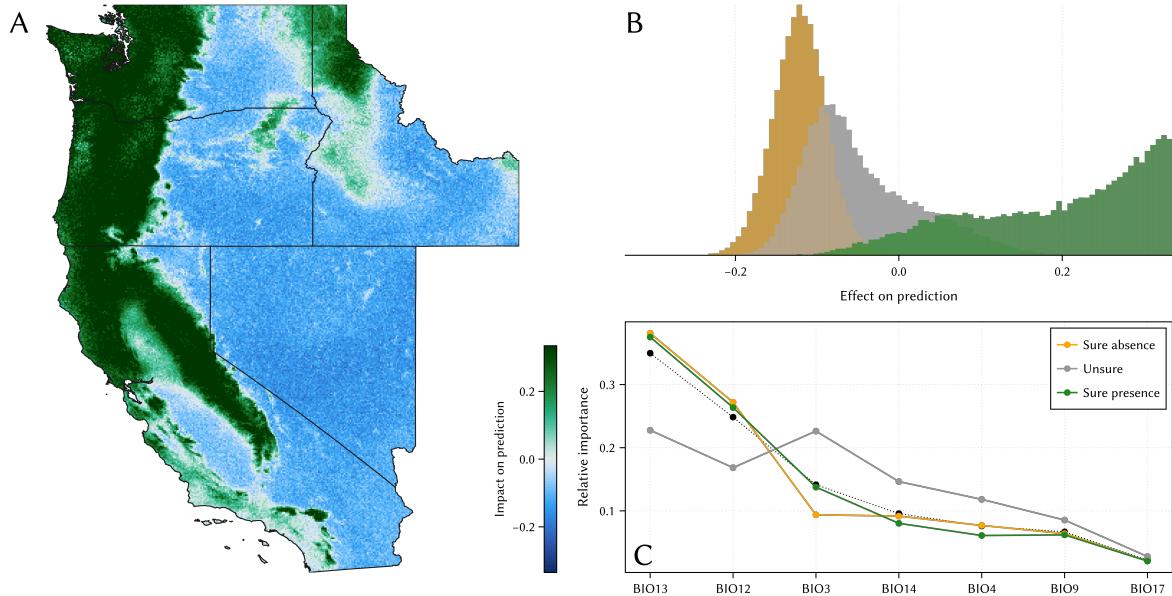
454 In fig. 4A, we see that above $\alpha \approx 0.1$, the inefficiency of the classifier starts to fall under 1 - this
455 indicates that *on average*, the model is returning fewer than one output for each prediction. In

456 a sense, this creates an upper limit to the risk we can accept: the model trained on this dataset
 457 does not support conformal prediction for larger risk levels. In fig. 4B, we see that this change
 458 of behavior in the model is indeed resulting in an increase in the range for which the model
 459 makes no prediction, which gets larger when the risk level is too high. The spatial distribution of
 460 undetermined areas is shows in fig. 4C for $\alpha = 0.2$: these areas are concentrated around the range
 461 limit as identified by the SDM. This suggests that using a risk level that it too high would result
 462 to no conformal predictions being made for the areas where our need to accurately quantify
 463 uncertainty are the most important, and calls for a cautious investigation of the appropriate
 464 risk level.



465 **Figure 4:** Inefficiency (average number of classes in the prediction set) for various levels of α (A); above $\alpha \approx 0.1$,
 466 the conformal prediction starts returning empty prediction sets. This results in an increase in the spatial area
 467 for which no prediction can be made (B). For $\alpha = 0.2$, these areas are distributed around the limit of the
 468 predicted range, showing that the areas in which uncertainty quantification are most important cannot be
 469 predicted.

471 In this section, I perform an analysis of Shapley values of the conformal predictor, in order to
472 (i) assess the importance of variables and (ii) provide explainable results about the relationships
473 between predictors and response. I rely on the common Monte-Carlo approximation of Shapley
474 values (Roth 1988, Touati et al. 2021). Monte-Carlo Shapley values represent, for each prediction,
475 how much the i th variable contributed to moving the prediction away from the average predic-
476 tion. The Shapley value associated to variable i is $\varphi_i \in [-1, 1]$, which measures how much this
477 variable modified the *average* prediction for this class. Shapley values have a number of desir-
478 able properties regarding the explanation of prediction of responses for environmental studies
479 (Wadoux et al. 2023), including their additivity: for any given prediction, $p = \hat{p} + \sum_i^{\text{variables}} \varphi_i$.
480 Because of this additive property, the importance of variables across many predictions is usually
481 measured as the average of $|\varphi|$, where both positive (the class is more likely) and negative
482 (the class is less likely) are counted. This measure of variable importance represents the relative
483 impact that each variable had on the process of moving all predictions away from the average
484 prediction and towards its actual value. Because Shapley values are both additive and indepen-
485 dent, they can be measured and aggregated for any arbitrary stratification of the data (which
486 allows reporting them conditional on the uncertainty status of the prediction).
487 As the predictions of the conformal model can be split by whether they are certain or uncertain,
488 they offer a unique opportunity to delve into the mechanisms that *generate* this uncertainty.
489 Namely, if the relative importance of variables is different across these classes of predictions, this
490 is strongly suggestive of the fact that there are certain environmental conditions (represented by



491 **Figure 5:** Overview of the effect of the most important predictor (A); areas with high values indicate that the
 492 value of BIO13 at this location make the presence of the species more likely. These values are associated to
 493 different prediction certainties (B), with predictions within the unsure range being centered around 0 (*i.e.*
 494 not moving the needle on the average prediction one way or another). Nevertheless, the contribution of the
 495 variables in different uncertainty categories are different (C), suggesting that Shapley values can help create
 496 explanations of where uncertainty originates. The proportion of certain/uncertain predictions as a response
 497 to changing values of BIO3 is presented in Figure S1.

498 combination of values for each variables) that create or reduce uncertainty. Furthermore, because
 499 we can split the certain predictions into a presence and absence class, this is a unique opportunity
 500 to investigate whether the factors leading to a species being present or absent are the same. An
 501 example of the spatial contribution of a variable is given in Figure 5A.

502 We find that, for the most important variable (*i.e.* the one with the largest $\sum|\varphi|$), the contribution
 503 of this variable tracks the status of the prediction: it tends to be negative when the absence is
 504 certain, positive when the presence is certain, and around zero when the prediction is unsure
 505 (fig. 5B). This is a fairly remarkable result, in that it ties Shapley values (a tool to help with
 506 ML models interpretation) to CP (a technique to accurately convey uncertainty). In Figure 5C, I
 507 present the relative contribution of all selected variables split by the status of the prediction; this
 508 reveals that the Shapley values for sure presences and unsure areas are distributed in different

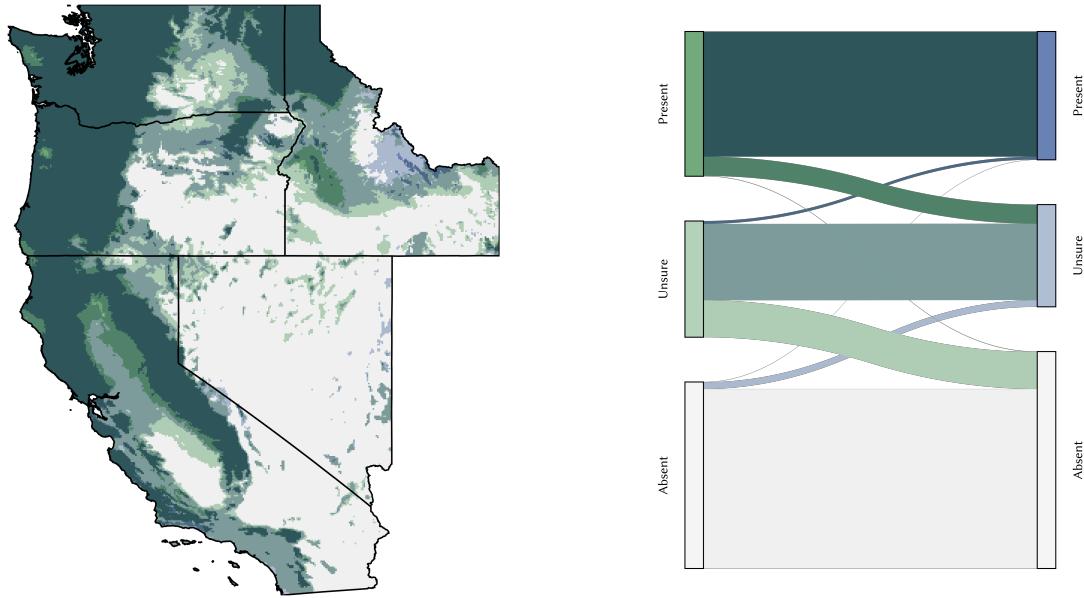
509 ways. Notably, BIO15 is far more important in areas of high model uncertainty than in areas
510 of either sure presences or absences. This suggests that the division of the prediction according
511 to CP status can provide information about which sets of environmental conditions are driving
512 the uncertainty, thereby providing useful information to guide future sampling or model inter-
513 pretation.

514 CONFORMAL PREDICTION UNDER CLIMATE PROJECTION

515 *Certain and uncertain range shifts*

516 In a recent contribution, Smith & Levine (2025) suggest that because of issues around the use
517 of thresholds, projections of SDMs under climate change scenarios may benefit from a more
518 continuous perspective. In this section, I present a comparison of the conformal prediction of the
519 range under a climate change scenario (SSP370, 2081-2100), to illustrate how the future conformal
520 range can convey information about the certainty of some types of range shift. These results are
521 presented in Figure 6.

522 Based on the comparison between the baseline (fig. 3A) and projected (fig. 6A) ranges, we can
523 establish identify areas where the species range is conserved ($\{+\} \rightarrow \{+\}$), is lost ($\{+\} \rightarrow \{-\}$),
524 becomes uncertain ($\{+\} \rightarrow \{-, +\}$, $\{-\} \rightarrow \{-, +\}$), or was uncertain but becomes certain ($\{+, -\} \rightarrow$
525 $\{-\}$, $\{-, +\} \rightarrow \{+\}$). By mapping these situations, we can identify large areas that are confidently
526 lost towards the Southern edge of the species's range, with very limited areas of either possible or
527 sure gain, strongly suggesting that this species would undergo range contraction. Note that the
528 area corresponding to ambiguous transitions is relatively large, which provides a good under-



529 **Figure 6:** Overview of the conformal prediction of the range for the future climate data, equivalent to fig. 3A
 530 (panel A). Sankey diagram for the transitions between absent, unsure, and present predictions for the
 531 current (left) and future (right) bioclimatic variables (panel B). The colors in panels A and B are the same.

532 standing of the possible spatial variation (and uncertainty) to be expected under the considered
 533 climate change models and scenario.

534 *Uncertainty and bioclimatic novelty*

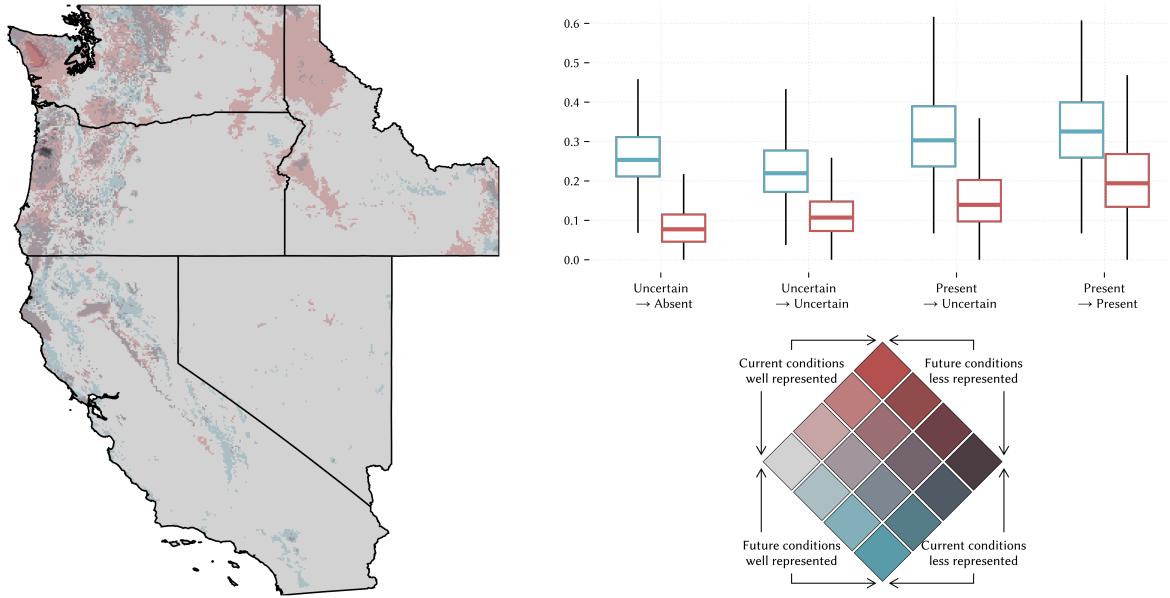
535 Zurell et al. (2012) highlight the importance of fully considering uncertainty when transferring
 536 the model to novel climate data: there is a chance that the future climate conditions will not
 537 have occurred in the training dataset, and therefore our confidence in the model outcome should
 538 be lowered. This covariate shift is well documented to decrease the performance of models
 539 (Mesgaran et al. 2014), and CP offers an opportunity to shine a different light on this phenomenon.
 540 Understanding covariate shift in the context of CP is particularly crucial given that entirely novel
 541 climatic conditions are likely to become the norm (Mahony et al. 2017), which in turn will drive
 542 the emergence of a novel biosphere globally (Kerr et al. 2025, Ordonez et al. 2024).

543 Yet although novelty is expected to emerge through climate change, it also emerges for current
544 climate data, because the training dataset is a subset of all the data on which the model is applied.
545 For this reason, Figure 7 compares how different future *and* current climates are to the bioclimatic
546 data in the training set, and describe the variation of this novelty across different types of range
547 shifts identified in Figure 6. The study area shows higher novelty in future data, but does not
548 show that the novelty is higher in areas that become uncertain in the future. This is an interesting
549 observation, as it suggests that the response of species distribution to novelty changes may be
550 more complex than “higer novelty leads to more uncertainty”. Indeed, the highest novelties were
551 observed in areas where the species was exected to conserve its range.

552 There are additional techniques to handle covariate shift in conformal prediction (Barber et al.
553 2023). In addition, Allen et al. (2025) suggest that in-sample calibration (using the training data) is
554 enough to get the coverage guarantees that are required for conformal prediction. Furthermore,
555 Balinsky & Balinsky (2025) show that the training dataset can be re-used to calibrate the model,
556 without a loss of performance of CP. These results suggest that CP may be more robust to
557 covariate shift (and therefore, appropriate to use to project under future climates) than expected.

558 CONCLUSION

559 Conformal prediction, like most SDM methods, is not quite delivering a true estimate of the
560 probability of presence (Phillips & Elith 2013). Nevertheless, it brings valuable information, in the
561 form of a quantified measure of whether a prediction comes with uncertainty (are both presence
562 and absence in the prediction set?) in a way that is directly comparable with the non-conformal



563 **Figure 7:** Climate novelty measured as Euclidean distance to the closest contemporary or future analogue (left
 564 map); note that the colors and their explanation are given in the bivariate legend. The boxplots on the right
 565 correspond to the difference (novelty value) for current conditions (red) and future conditions (blue) for the
 566 different types of distribution changes presented in Figure 6.

567 prediction. “Class overlap”, where both presences and absences are observed under the same
 568 values of the predictions, decreases the predictive performance of models (Valavi et al. 2022) –
 569 CP is naturally suited at handling this, by assigning the area where overlap occurs to uncertain
 570 predictions.

571 A useful categorization of uncertainty is to differentiate between its aleatoric and epistemic
 572 component. Mansfield & Christensen (2025), for climate prediction models, suggest that aleatoric
 573 uncertainty stems from the variability in input data, whereas epistemic uncertainty stems from
 574 an inability to identify the parameters that unambiguously map an input to a model prediction.
 575 The same idea has been suggested for ecological dynamics models (Reimer et al. 2022). Sale et al.
 576 (2025) recently suggested that CP could capture both forms of uncertainty, although primarily
 577 because the disentanglement of epistemic *v.* aleatoric uncertainties is a difficult task, especially

578 under climate change (Kujala et al. 2013). Under this perspective, CP could serve as a mapping
579 of the aggregate uncertainty for a given prediction problem.

580 **INCREASING THE RELEVANCE OF CP TO SPECIES DISTRIBUTION MODELING**

581 Davis et al. (2024) previously suggested using CP to approximate a confidence interval around
582 a probability of species presence, which considers species prediction as, fundamentally, a
583 regression problem. As SDMs are more traditionally viewed as classification problems, a proper
584 accounting of the method for CP for classification is required in order to understand what
585 future research efforts should focus on. This is particularly important as alternative frameworks
586 around CP, like Adaptive Conformal Inference (Szabadváry & Löfström 2026), are emerging: the
587 ontologic status of SDM as a machine learning practice must be clear.

588 Although the change in climatic conditions has been measured through climate velocity (Brito-
589 Morales et al. 2018), measures of climate *novelty* are likely to be more informative for the
590 interpretation of CP. Exchangeability of the data is a core assumption of CP, and although some
591 recent evidence suggests that CP is relatively robust to violations of this assumption (they have
592 been discussed in earlier sections of this manuscript), a very high novelty is likely to result in
593 locally non-exchangeable data: the model would be applied (and its uncertainty quantified) on
594 data points that are outside of the (joint) distribution of variables in the training set. Beyond
595 climatic novelty, measurement of potential covariate shift between the training dataset and *both*
596 the current and future climate conditions, may provide a clearer understanding of where and
597 when predictions are likely to be more uncertain.

599 Transparent communication of uncertainty, meaning that it is both spatially explicit, quantified,
600 and expressed under a risk set by the user, is important: we do not expect a fully trained model
601 to always be certain, as some areas are genuinely more difficult to predict. For example, small
602 organisms are more inherently stochastic (Soininen et al. 2013) any form of stochastic event will
603 drive species distribution even when there is strong environmental signal (Mohd et al. 2016) these
604 stochastic events can even manifest in areas that are close to the species' environmental optimum
605 (Dallas et al. 2020). For these reasons, CP can produce interpretable estimates of uncertainty in
606 species distribution models, and does not require the adoption of additional modeling tools or
607 paradigms as it functions on an already trained model.

608 Because this technique is computationally efficient and works on pre-trained models, it opens
609 up the opportunity for more systematic uncertainty quantification in SDMs. CP, in short, can
610 deliver the “maps of ignorance” that Rocchini et al. (2011) argued for: how difficult is it to make
611 a prediction for the range at a given risk level is, in and of itself, an important information to
612 frame the reliability of the results. Finally, CP can provide guidance on the feedback loop between
613 SDM training and field validation (Johnson et al. 2023) – areas where the range is certain are
614 a much lower priority for sampling. CP contributes to dispel what Messeri & Crockett (2024)
615 called the “illusion of understanding”, which is often associated with ML models: it generates an
616 understanding of the uncertainty from observations of a pre-trained model, and expresses this
617 uncertainty both in absolute (is the “presence” event in the prediction set?) and relative (is the
618 point estimate of the score for presence larger than for absence?) terms.

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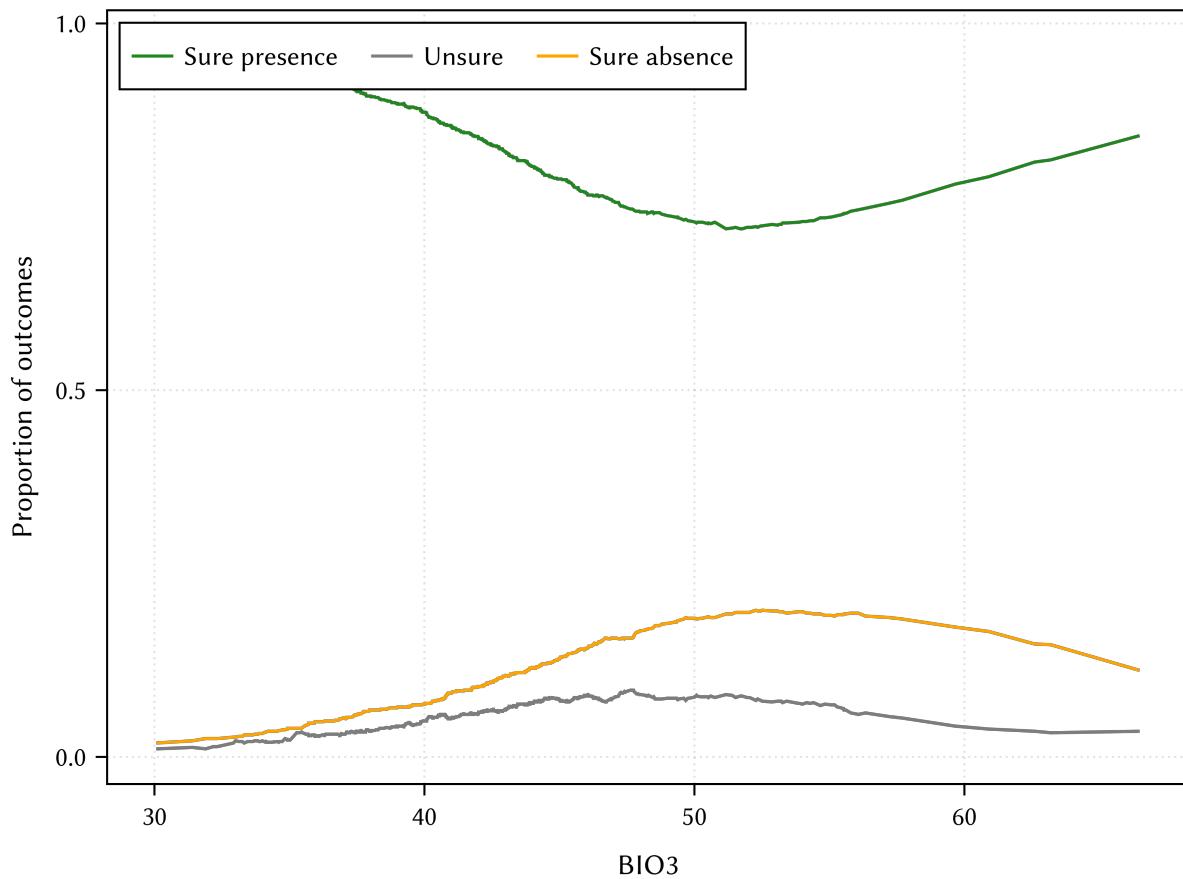
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829 **Figure S1:** Effect of changing the value of the BIO3 variable, on the prediction, as measured by inflated partial
 830 responses (Fiorentino et al. 2025). The partial responses have been measured on a random sample of a 1000
 831 draws, and for each draw, the prediction has been classified with the conformal predictor at a risk level $\alpha =$
 832 0.05. The proportion of each outcomes for the classification is presented as a function of the variable value.
 833 This analysis illustrates how conformal prediction can be used to identify range of predictor variables that
 834 are most likely to be associated to uncertain predictions.