

1 Evolution of chemodiversity – From verbal to quantitative models

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7 **Abstract**

8 Plants harbour an astonishing amount of *chemodiversity*, i.e., diversity of specialized metabolites, at different
9 scales. For instance, individual plants can produce a large number of different specialized metabolites and
10 individuals in a population can differ in their metabolite composition. Given the ecological and economic
11 importance of plant chemodiversity, it is important to understand how it arises and is maintained over
12 evolutionary time. For other types of biodiversity, i.e., species diversity and genetic diversity, quantitative
13 models, that is, mathematical models and computer simulations, have long played an important role in
14 addressing such questions. Here we review models and hypotheses for the evolution of plant chemodiversity
15 and, in particular, explore what quantitative models have been proposed so far and what gaps there are in
16 quantitative modeling of chemodiversity. For each model or hypothesis we review its ingredients, i.e., the
17 biological processes that are assumed to shape chemodiversity, the scales at which the model explains or
18 claims to explain chemodiversity, and the extent to which the model has been formalized as a mathematical or
19 simulation model. From this review, a mixed picture emerges. We identified a small number of quantitative
20 models for the evolutionary dynamics of plant chemodiversity. In addition we found a number of models that
21 use equations to derive an optimal defense, but are not dynamic. Many influential models, however, have
22 remained verbal so far. Furthermore, we identify some quantitative models used for genetic variation that
23 have not been used for chemodiversity so far, but could be easily extended to do so. We end by outlining
24 our vision for future model building for the evolution of plant chemodiversity.

25 **Keywords:** chemodiversity, phytochemical diversity, secondary metabolites, mathematical model, simu-
26 lation

1 Introduction

Chemodiversity is the diversity of specialized metabolites (SMs) at different levels of organisation – from single tissues to entire communities (see Wetzel & Whitehead, 2020, for a review). SMs are compounds produced by an organism that are not directly involved in its most basic survival and reproduction mechanisms, but may be important for the interaction with herbivores, pollinators, and conspecifics, or protect from abiotic stresses (Wetzel & Whitehead, 2020). SMs are also often called 'secondary metabolites' (Stone & Williams, 1992; Cipollini & Levey, 1997; Hamberger & Bak, 2013; Moore *et al.*, 2014; Dyer *et al.*, 2018; Rokas *et al.*, 2020), 'phytochemicals' (in plants, Allstadt *et al.*, 2012; Richards *et al.*, 2016; Dyer *et al.*, 2018; Defosse *et al.*, 2021), or 'natural products' (Firn & Jones, 2003). Examples of SMs are alkaloid or terpenoid defense chemicals, flavonoid pigments, and numerous other compounds and compound families with known or unknown properties. The set of SMs that an organism produces is called its chemotype.

Chemodiversity can be quantified in various ways (Wetzel & Whitehead, 2020): it can focus on particular compound families or not, and it can be measured within as well as between units of scale. For example, both the number of SMs per individual and the differences in SMs between individuals in a population are aspects of chemodiversity.

The existence of widespread chemodiversity is puzzling from an evolutionary perspective. SMs are often synthesized in complex metabolic pathways that involve multiple enzymes modifying a precursor metabolite into the SM over several steps (Jones & Firn, 1991; Firn & Jones, 2003, 2009). Given the inherent costs of these pathways, one would expect evolution towards a small number of the most beneficial metabolites (Jones & Firn, 1991). Despite this, a high diversity of SMs has been found within and between plant populations and to some extent also in fungal and bacterial populations (Calf *et al.*, 2018; Rokas *et al.*, 2020; Li *et al.*, 2020; Defosse *et al.*, 2021). Evidently, there are mechanisms for the maintenance of this chemodiversity. These mechanisms are what models around chemodiversity attempt to elucidate.

For both species diversity and genetic diversity, quantitative models – both mathematical and simulation models – have long been an important part of scientific inquiry (see e.g. Wright, 1937; Kimura, 1983; Hubbell, 2001). They are for instance used as proof-of-concept models to test the validity of verbal models (Servedio *et al.*, 2014). Other functions are to generate predictions and hypotheses that can be tested empirically (Servedio *et al.*, 2014) and to estimate parameters from data. Running *in silico* experiments with quantitative models can give clues to what aspects of a system are the most relevant, and thus which measurements should be taken in an experiment. In that way they can make empirical studies more efficient, and sometimes provide a statistical model that can be fit to data. Additionally, a good quantitative model can unify several studies that differ in methodology so they form a coherent narrative (Otto & Rosales, 2019). Because of these various contributions of quantitative models to scientific inquiry, we argue that chemodiversity too should be investigated in this way.

61 A better understanding of the evolution of chemodiversity that might be conferred by such models also
62 has an applied relevance. For instance, models that predict local adaptation and geographic structure
63 of chemodiversity (Calf *et al.*, 2018; Defosse *et al.*, 2021), could be taken into account in conservation
64 planning, the way genetic diversity already is (see e.g. Frankham *et al.*, 2002, Ch. 16). Moreover, models for
65 the evolution of chemodiversity might help predict why some introductions of plants or herbivores to new
66 places succeed while others do not, which will be discussed further in Box 3.

67 In this article, we review the work that has been done so far on developing verbal and, in particular,
68 quantitative models for chemodiversity. For each model, we discuss which biological processes it focuses
69 on and at which scale it addresses chemodiversity. We are not the first to review possible explanations for
70 chemodiversity. There are for instance in the excellent reviews by Stamp (2003), Moore *et al.* (2014), Dyer
71 *et al.* (2018), and Wetzel & Whitehead (2020); therefore we do not want to dwell on reviewing the empirical
72 support for the different explanations for chemodiversity. Instead, we focus on how the verbal models
73 described in those reviews have been tested through quantitative models, and discuss how quantitative
74 models for other types of diversity could be adapted to modeling chemodiversity. We also discuss some of
75 the empirical work that has been done on them to elucidate how quantitative models can connect verbal
76 models and empirical studies. We end by outlining important avenues for future research.

2 Chemodiversity models

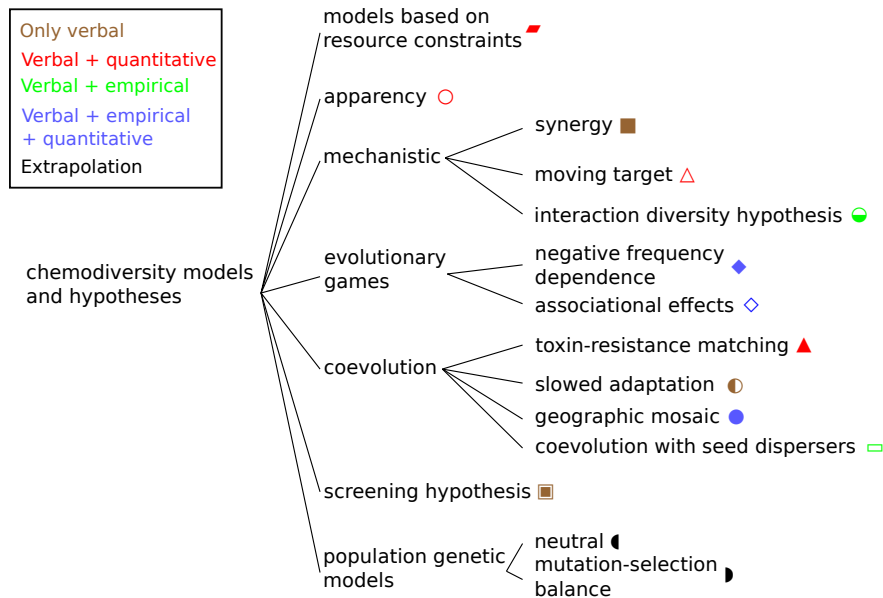


Figure 1: Tree showing a rough conceptual grouping of chemodiversity models and hypotheses. Each model or hypothesis has a unique symbol, which is then used in Fig. 2 to indicate which ingredients are part of the model and in Fig. 3 to indicate at what scales the model can explain chemodiversity. Note that similarity in shape between symbols here does not indicate any relationship between models. Models with brown symbols have only a verbal model, green symbols additionally have empirical evidence, red a quantitative model, and blue both a quantitative model and empirical evidence. Black symbols stand for models that have, to our knowledge, not been previously used in the context of chemodiversity but that we argue can be extended to modeling chemodiversity (i.e. they are our extrapolation).

There is currently no single unifying theory that can explain chemodiversity at all scales and within as well as between units at each scale. However, there are numerous hypotheses and verbal models (Fig. 1), each of which focuses on a different set of “ingredients”, i.e., biological players and processes that can contribute to shaping chemodiversity (Fig. 2, Box 1). Moreover, the different hypotheses and models cover different scales of organization (Fig. 3, Box 2). Although there are connections and overlaps between many hypotheses and they are certainly not all mutually exclusive, to provide some structure for this review, we have placed all models into a hierarchy and grouped similar models together (Fig. 1). Note that sometimes different terms are used in different publications to describe the same or similar models (for instance in the excellent reviews of Stamp, 2003; Dyer *et al.*, 2018; Wetzel & Whitehead, 2020).

Although these various hypotheses on the development and maintenance of chemodiversity are commonly used to generate hypotheses for empirical research (Li *et al.*, 2020; Whitehead *et al.*, 2021), there are fewer

89 quantitative models of these hypotheses. Here, we will review extant quantitative models of chemodiversity,
 90 as well as research that could serve as a starting point for the creation of models to fill the numerous gaps
 91 in the state of the art.

92 **Box 1: Ingredients of chemodiversity models**

93 Here we explain the biological processes that act as key ingredients (Fig. 2) in current models and hypotheses
 94 for chemodiversity and, without aspiring to completeness, give some exemplary empirical studies supporting
 95 their role in shaping chemodiversity. How the ingredients are used in the various theoretical models is
 96 explained in section 3.

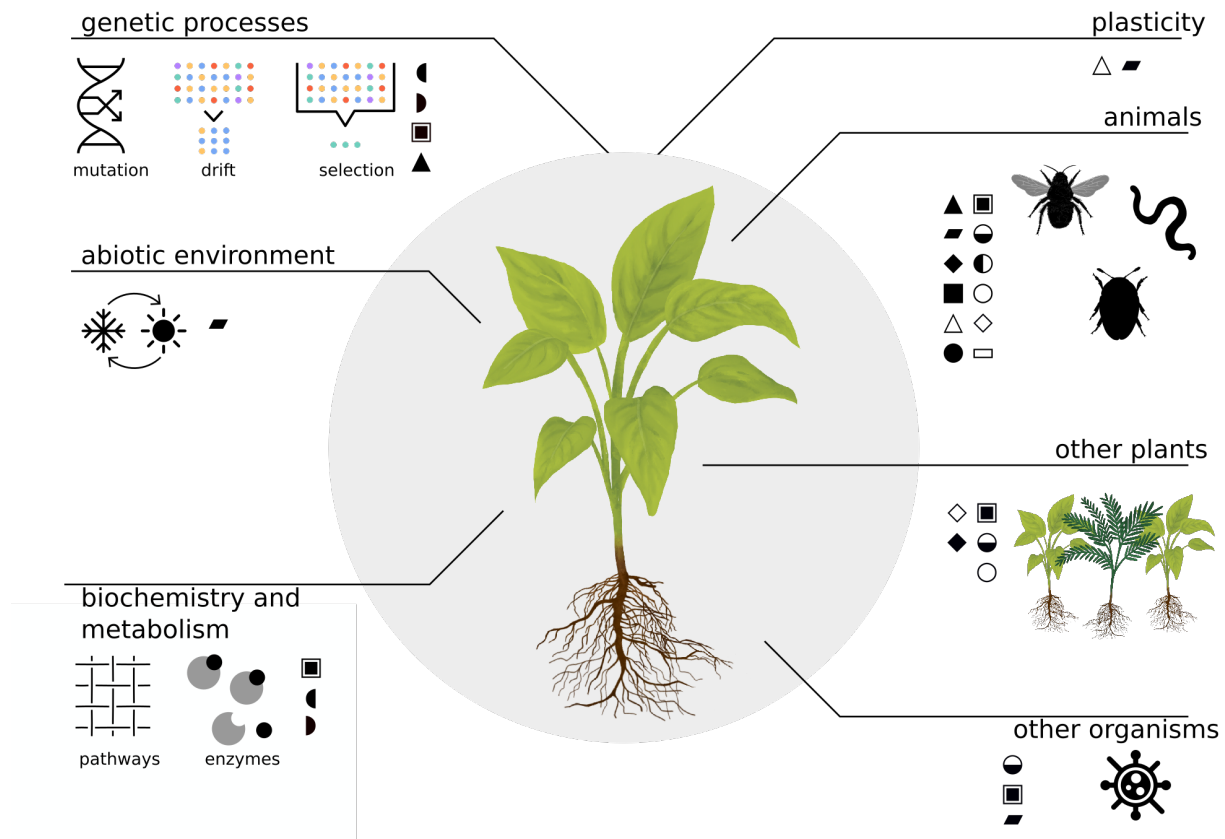


Figure 2: Overview of the key ingredients of chemodiversity models. To see which hypothesis/model a symbol stands for, please refer to Fig. 1. The ingredients listed in the figure are further explained in Box 1. Models and hypotheses are listed as connected to particular ingredients when these ingredients are key components of the model as described below in section 3.

- 97 • **Genetic processes:** SMs are produced by enzymes, which are coded for in genes, which in turn have
98 regulatory genes. Therefore, genetic processes can give rise to chemodiversity. Genes for enzymes
99 in the same metabolic pathway are often organized as operons, i.e. located closely together in the
100 genome (Hamberger & Bak, 2013). Within these operons, gene duplication and subsequent divergence
101 is common, which facilitates a flexible gene network that can undergo rapid evolution (Hamberger &
102 Bak, 2013).

103 **Mutations** may change the function of the enzyme coded for by a gene, so it has a different substrate
104 profile or catalyzes different reactions and therefore produces different SMs than its ancestor. Mean-
105 while changes in gene regulation may change the abundance of SMs, and duplication and deletion may
106 open new pathways of SM synthesis or close them off. Random mutations thus bring about genetic
107 and consequently chemical diversity.

108 **Genetic drift** is the phenomenon of changing allele frequencies through chance effects on reproduction
109 and survival rather than natural selection based on fitness. A gene might become fixed or die out
110 randomly within one population but not another, resulting in chemodiversity.

111 **Natural selection** occurs when SMs which confer fitness benefits become more common, and SMs
112 that confer fitness costs become more rare. Similarly, if there is a fitness benefit to lack of specificity
113 in the production of SMs, this trait may be selected for.

- 114 • **The abiotic environment**, such as water and nutrient availability, temperature, or elevation, can
115 determine how much a plant can invest in growth and defense (Bryant *et al.*, 1983; Coley *et al.*,
116 1985; Herms & Mattson, 1992; Hamilton *et al.*, 2008; Smakowska *et al.*, 2016; Monson *et al.*, 2021).
117 Moreover, changes in the abiotic environment may cause stress responses that lead to changes in the
118 concentrations of certain SMs or induce biosynthesis of new SMs (Agati & Tattini, 2010).

- 119 • **Biochemistry and metabolism:** Metabolic pathways for the production of SMs can involve multiple
120 enzyme-catalyzed reactions and can be branched. Which new SMs can be produced by changes in these
121 pathways is constrained by the extant pathways. Many of the enzymes involved with SM synthesis are
122 “promiscuous”, meaning that they accept multiple substrates and thus can produce multiple different
123 SMs at low cost (Aharoni *et al.*, 2005). Thus, small modifications to the enzyme suite can have a
124 profound impact on the metabolic pathway (Moghe & Last, 2015; Shoji, 2019).

- 125 • **Phenotypic plasticity:** Phenotypic plasticity can explain chemodiversity even among genetically
126 identical individuals in a population. For example, an individual which experiences a more nutrient-
127 poor micro-environment may invest these resources differently from an individual in the same pop-
128 ulation that experiences a more nutrient-rich micro-environment, leading to a variation in chemical
129 composition (Stamp, 2003; Defosse *et al.*, 2021).

- 130 • **Interactions with animals** play a role in the majority of hypotheses and models for the evolution of
131 chemodiversity and their importance has broad empirical support (Hambäck *et al.*, 2014; Calf *et al.*,
132 2018; Li *et al.*, 2020; Whitehead *et al.*, 2021). For example, Calf *et al.* (2018) found that there were
133 differences between the composition and total amounts of glycoalkaloids produced by the bittersweet
134 nightshade *Solanum dulcamara* sampled from different locations, and that slugs *Deroceras reticulatum*
135 consistently showed preference for leaves from populations which produced fewer glycoalkaloids, which
136 corresponded to populations where few slugs were present, hinting at local adaptation of plant popula-
137 tions. Moreover, plants produce various SMs that serve as visual, olfactory and/or gustatory signals to
138 lure pollinators (Borghi *et al.*, 2021) and attract seed dispersers (Cipollini & Levey, 1997; Nevo *et al.*,
139 2018; Baldwin *et al.*, 2020). Furthermore, toxicity from SMs may influence frugivore behaviour and
140 gut retention time of seeds, and in that way influence the spread of seeds (Cipollini & Levey, 1997;
141 Baldwin *et al.*, 2020).
- 142 • **(Indirect) interactions with other plants:** Plants can interact directly with each other, for example
143 through SMs that hinder the access to resources for other plants, e.g. heterospecific competitors. For
144 example in *Brassica nigra*, the benefit of these SMs for the plants was higher when their strategy
145 was rare (Lankau & Strauss, 2007, 2008). Interactions with other plants can also be indirect, often
146 through interaction with other species in the environment (associational effects, Hambäck *et al.*, 2014).
147 In *Piper* plants, more chemodiverse communities were found to have lower plant mortality and local
148 species extinction than less chemodiverse communities (Salazar & Marquis, 2022).
- 149 • **Interactions with other organisms:** Separately from animal mutualists, a plethora of microbial
150 mutualists are guided by plant SMs to their host, and the formation of symbiotic structures is induced
151 by SMs (De la Peña & Loyola-Vargas, 2014), for example rhizobia (Cooper, 2004) and the hyphal
152 branching of arbuscular mycorrhizal fungi (AMF) (Akiyama *et al.*, 2005). Differences in root exudates
153 among populations within species have been found to alter the rhizosphere soil composition, leading to
154 distinct soil chemical communities (Mueller *et al.*, 2020). Mutualists can in turn also affect patterns of
155 chemodiversity. For example, endophytes and a symbiosis with AMF resulting in functional arbuscular
156 mycorrhiza can modify the chemical composition of different plant tissues (Schweiger & Müller, 2015;
157 Yadav *et al.*, 2022).

158 **Box 2: Scales of chemodiversity**

159 Chemodiversity can be quantified at various scales of organization (Wetzel & Whitehead, 2020). Terms
160 such as richness and evenness, alpha, beta and gamma diversity that are familiar from species diversity are
161 frequently used. However, different authors use different definitions for these terms (see e.g. discussion in

162 Kessler & Kalske, 2018; Li *et al.*, 2020), depending on the scales their work focuses on. Meanwhile Wetzel
 163 & Whitehead (2020) use alpha, beta and gamma diversity to describe general relationships between scales
 164 without either term referring to a specific scale. We argue that using the same terms to speak about different
 165 scales can be confusing, especially when comparing studies, and when there are more than two scales that
 166 may be of interest.

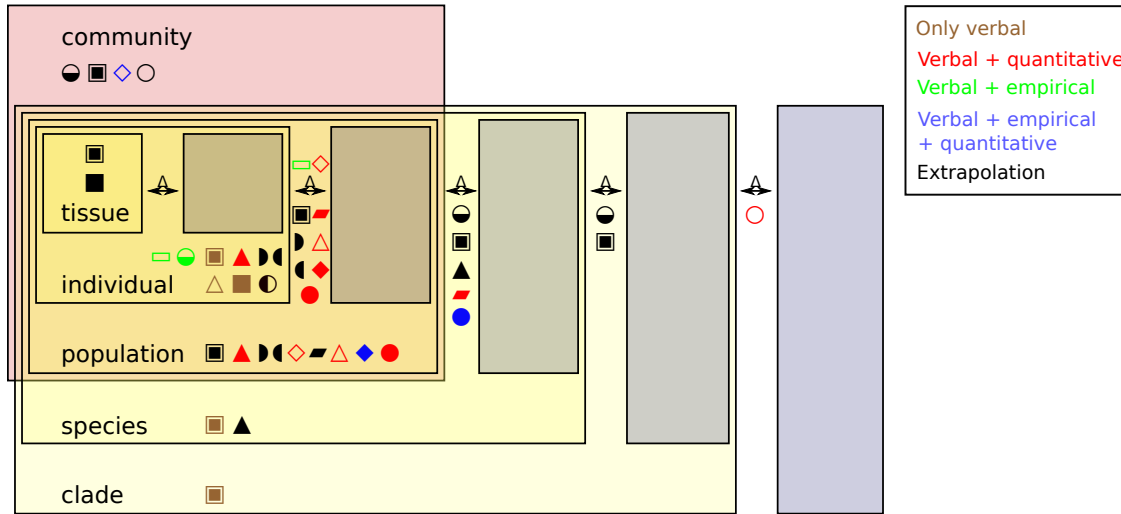


Figure 3: Schematic overview of the scales at which the different models and hypotheses (potentially) explain the evolution of chemodiversity. Please refer to Fig. 1 for the models each symbol refers to. Models with brown symbols have only a verbal model, green symbols additionally have empirical evidence, red a quantitative model, and blue both a quantitative model and empirical evidence. Black symbols stand for models that have, to our knowledge, not been previously used in the context of chemodiversity but that we argue can be extended to modeling chemodiversity (i.e. they are our extrapolation). For example, the filled red diamond next to the label “population” indicates that there is a quantitative model suggesting that negative frequency dependence can explain the maintenance of chemodiversity within a population and the same symbol between the two “individual” boxes indicates that it can maintain differences between individuals. For some symbols, we used different color in different places in the figure to indicate a different type of support for chemodiversity at the different scales. The community level is orthogonal to the species level because populations can be considered nested both in their species and in a multi-species community.

167 We argue that instead of using alpha, beta, and gamma diversity, it is more helpful to explicitly state
 168 the scales under consideration (tissue, individual, population, species, clade, or community) and addition-
 169 ally specify whether diversity within units at those scales or differences between units at those scales are
 170 considered (Fig. 3). For example one can quantify chemodiversity within individuals as the average number
 171 of SMs produced per individual, and between-population chemodiversity as number of SMs that are pro-

172 duced only by one of the populations. Note that between-unit chemodiversity is not the same as within-unit
173 chemodiversity at the next higher scale. For example, the average number of metabolites that are not shared
174 between randomly picked individuals in the population is not the same as the total number of metabolites in
175 the whole population. At each scale, chemodiversity could be quantified using different metrics, e.g. metrics
176 focusing on richness or evenness. In Fig. 3, we place all hypotheses at the scale or scales at which they have
177 been shown to explain, claim to explain, or can be logically inferred to explain chemodiversity.

178 3 Model descriptions

179 3.1 Models based on resource constraints

180 The SMs an organism produces are limited by the resources the organism has access to. Furthermore, the
181 organism may optimize SM production subject to constraints based on available resources or other abiotic
182 or biotic factors (Fig. 2). These ideas are the basis for a group of models that aim to explain variation in
183 quantitative defense level depending on abiotic and biotic conditions. For instance, according to the **carbon**
184 **nutrient balance hypothesis**, the relative availability of carbon and nitrogen determines which types
185 of compounds are preferentially produced (Bryant *et al.*, 1983). For instance, in low-carbon (i.e. shady)
186 habitats, likely more nitrogen-based toxins are produced. While the hypothesis at times correctly predicts
187 the chemicals which are found in empirical studies, it has come under criticism for frequently failing to do
188 so (Hamilton *et al.*, 2008).

189 Many of the hypotheses in this group are based on the idea that a plant's investment in defense competes
190 with investment into growth, reproduction, and storage. According to the **resource availability hypoth-**
191 **esis** (Coley *et al.*, 1985; Hahn & Maron, 2016), plants in environments with low resource availability are
192 selected to grow slowly and be well defended, whereas plants grown at high resource availability are selected
193 to grow fast and invest less in defense. The **growth-differentiation hypothesis** (Herms & Mattson, 1992)
194 and the **coordinated resource allocation hypothesis** (Monson *et al.*, 2021) expand on the resource
195 availability hypothesis by modeling the maximum possible growth rate in more detail and by adding reserve
196 safety margins, i.e., storage pools that allow plants to plastically respond to biotic and abiotic challenges
197 (Fig. 2). Coley *et al.* (1985) also propose an equation (which is then extended by Herms & Mattson, 1992;
198 Monson *et al.*, 2021) modeling how plant growth rate depends on investment in defense. In these models (at
199 least with the selected parameter values), plant growth rate is optimized at intermediate defense levels and
200 the optimal strategy depends on the maximum possible growth rate in the absence of herbivores, i.e. resource
201 availability. Thus, if the microenvironments of individuals in a population differ in resource availability, these
202 hypotheses thus offer a quantitative prediction for differences in defense levels between individuals within a
203 population (Fig. 3). Analogously, differences between different populations of the same species are predicted
204 if the habitats of the populations differ in resource availability. Similarly, in a verbal model ,Vannette &

205 Hunter (2011) assumes that the expression of plant defenses has a nonlinear relationship with the density of
206 arbuscular mycorrhizal fungi (AMF), such that variation in plant defense between populations in different
207 habitats could be explained by variation in AMF density.

208 Differences in defense levels between individuals or populations in response to environmental conditions
209 are often based on phenotypic plasticity. Many SMs are not produced continuously, but induced when their
210 effects are beneficial. For example, exposure to higher ultraviolet-B radiation induces the production of
211 flavonoids, which are important SMs in photoprotection (Agati & Tattini, 2010). Using a simple mathemati-
212 cal framework called **error-management theory**, Orrock *et al.* (2015) computed the optimal herbivore cue
213 strength at which a plant should express its induced defense. This threshold represents a balance between
214 the costs of a false alarm and the costs of being attacked unprepared. Since these costs but also the exposure
215 to herbivores might differ between different plant individuals and populations (Fig. 2), this model could
216 explain chemodiversity in expressed defense between individuals and between populations (Fig. 3).

217 Clearly, plasticity cannot explain all differences in defense expression though because there are often also
218 strong genetically-based differences in a controlled environment (see e.g. Garrido *et al.*, 2012). Populations
219 and species as a whole can have the SMs they produce be selected to the typical resources in their location,
220 consequently affecting their chemodiversity (Stamp, 2003; Hahn & Maron, 2016; Defosse *et al.*, 2021).
221 However, at the intraspecific level patterns appear to be more complex, in part because locations with high
222 resource availability also tend to be locations with high herbivore pressure, such that it is still unclear how
223 much of intraspecific defense variation can be explained by the resource-availability hypothesis and other
224 models based on constraints (Hahn & Maron, 2016).

225 3.2 Apparency

226 Feeny (1976) developed the **plant apparency hypothesis** as a verbal model where plants in a community
227 produce different SMs depending on how easy it is for herbivores to find them relative to other plants
228 in the community (Fig. 2): highly apparent plants (e.g. oak trees) produce high concentration, so called
229 quantitative compounds that affect most herbivores; less apparent species (e.g. small annual plants) produce
230 low concentration, highly toxic, so called qualitative compounds that some herbivores are immune to. The
231 apparency hypothesis addresses chemodiversity at the community level as well as between clades (see Fig.
232 3).

233 Partial support that the verbal apparency model can work in principle, comes from a quantitative model
234 by Yamamura & Tsuji (1995). They used an optimal control theory approach to find the optimal investment
235 in growth vs. defense over the life time of the plant. With apparency understood as herbivory pressure,
236 their model results supported the apparency hypothesis. They found that more apparent plants produced
237 quantitative defenses whereas less apparent ones did not, while for those that did, the investment in de-

238 fense increased with apparency. With apparency understood as length of the growth period, only the first
239 prediction was true and only under some parameter settings.

240 The plant apparency hypothesis has been criticized for being difficult to test in practice. SMs are not
241 easily divided into quantitative and qualitative compounds, but often have properties of both. This makes
242 it different to correlate apparency to the presence of quantitative and qualitative compounds (Stamp, 2003).
243 However, which SMs are beneficial to particular plants in a population still might differ depending on their
244 apparency.

245 3.3 Mechanistic

246 Hypotheses in this group focus on why having multiple defense SMs is beneficial to plants, but they do not
247 explicitly consider evolution on the herbivore side (as opposed to the hypotheses in the next section).

248 The **synergy hypothesis** proposes that the effect of a mixture of SMs is more than the sum of the effects
249 of each SM in isolation. These nonlinear effects of mixtures of SMs then explain why organisms produce
250 multiple SMs. On the molecular level, synergistic effects are explained by molecular interactions between
251 SMs, for example by one SM facilitating the movement of another SM across cell membranes (Richards *et al.*,
252 2016). The synergy hypothesis addresses chemodiversity within tissues and within individuals (see Fig. 3,
253 and Wetzel & Whitehead, 2020). In empirical studies, it was found that mixtures of SMs affect generalist
254 herbivores more than specialists (Richards *et al.*, 2016). Synergistic effects are difficult to study in the lab
255 in part because it is difficult to isolate large enough amounts of the SMs involved (Dyer *et al.*, 2018). Here,
256 quantitative modeling can be a useful tool to formulate hypotheses and guide experiments. So far, to our
257 knowledge, such models do not exist.

258 The **moving target hypothesis** is a term proposed by Adler & Karban (1994) and used by Li *et al.*
259 (2020) to refer to an inducible defense where a plant in a population randomly changes its phenotype in
260 response to an attack by herbivores (Fig. 2). This is closely related to the 'novel weapons hypothesis' that
261 is proposed in biological invasions (Box 3). Herbivores then cannot adjust effectively to this defense as it is
262 unpredictable. Wetzel & Whitehead (2020) uses the term more broadly to cover hypotheses that describe
263 changes in SMs within individuals over time, functioning like a moving target that herbivores have a hard
264 time adjusting to, even if it does not follow the precise mechanisms described in Adler & Karban (1994).
265 Relatedly, variation between plant individuals in a population can suppress herbivores because they need
266 to adjust their detoxification apparatus when moving between plants with different defense metabolites,
267 which costs energy and is not instantaneous (Pearse *et al.*, 2018). However, the conditions under which this
268 phenomenon would give an advantage to individuals with rare metabolites and promote the evolution of
269 chemodiversity between individuals are not well understood yet and would probably depend on the details
270 of herbivore behavior.

271 Adler & Karban (1994) analysed a quantitative, stochastic, general plant-herbivore model. In this model,
272 plants could have three defense strategies: a strategy that does not adapt to herbivore damage (constitutive
273 defense), one where plants adapt to herbivore damage by randomly changing their defense (moving target),
274 or one where all plants switch to whichever defense results in the most optimal growth rate in the current
275 situation (optimal inducible defense). They found that in an environment with a fluctuating number of
276 herbivores, the optimal inducible defense is superior when there is just one herbivore species. When there
277 are at least two herbivores with different resistances to defense strategies, the moving target strategy is
278 superior as long as the cost of defense is not too high. When the cost of defense is high, constitutive low
279 levels of defense are superior.

280 Eagle-eyed readers will have noticed that this model only included chemodiversity on the within popu-
281 lation and between-individual level, not on the within-individual level, although this is part of the verbal
282 model (Fig. 3). It is arguable that the defense strategy is analogous to a chemotype. To make this an
283 explicit chemodiversity model capable of investigating chemodiversity on the individual level, a chemical
284 model would need to be added that simulates the actual metabolites.

285 The **interaction diversity hypothesis** (Kessler & Kalske, 2018; Whitehead *et al.*, 2021) suggests that
286 plants possess a variety of SMs because they interact with a large number of other organisms (both antagonists
287 and mutualists, as well as interspecific or intraspecific competitors, see Fig. 2) and different compounds are
288 active in interaction with the different organisms. Independent selection on each compound, potentially in
289 coevolution with the respective interacting species, then leads to the emergence of chemodiversity. Although
290 the name is rather new, the idea is relatively old and has also been called the “common-sense scenario”
291 (Berenbaum & Zangerl, 1996). Although the interaction diversity hypothesis mostly seeks to explain the
292 diversity of SMs within an individual (Fig. 3, with empirical support by Whitehead *et al.*, 2021), the reasoning
293 can be extended to explain chemodiversity at other scales. For example, if two populations or species differ
294 in the set of interacting animals, we would expect differences in their metabolite composition. Although the
295 ideas of multiple interaction partners driving chemodiversity is very prominent and well-supported, there do
296 not currently appear to be any quantitative models based on this idea.

297 **3.4 Evolutionary games**

298 Models and hypotheses explaining plant chemodiversity or variation in defenses via evolutionary games focus
299 on the competition of plants “playing” different defense strategies. That is, they have interactions with other
300 plants as important ingredient (Fig. 2).

301 One way in which variation in defense traits can be maintained between individuals in a population and
302 within populations (see Fig. 3) is **negative frequency dependence** where the respective rare phenotype
303 or genotype has an advantage. Sato *et al.* (2017) and Augner *et al.* (1991) created quantitative models of

304 evolutionary change in plant defense in populations over time. Sato *et al.* (2017) found a pattern of negative
305 frequency-dependent selection that could lead to stable coexistence of undefended plants and defended plants
306 which paid a cost for their defense. The underlying mechanism is that when defended plants become too
307 common, optimally foraging herbivores start to forage also on defended plants despite lower profitability
308 because it is too costly to search for the few undefended plants. This matched up well with their data on a
309 field site of *Arabidopsis halleri* that comprised hairy and glabrous morphs. Similarly, Augner *et al.* (1991),
310 showed that, in a game-theoretical model of a grazed population with two morphs, both could coexist if
311 the fitness benefit a defended player has if an undefended opponent is grazed is higher than the profit a
312 non-defended player makes if another non-defended opponent is grazed. These models show that diversity,
313 including chemodiversity, can exist when there is a benefit to being the rare morph in a mixed population.

314 A coexistence mechanism closely related to negative frequency dependence is a rock-paper-scissors game.
315 This appears to explain the coexistence of high-sinigrin *Brassica nigra* genotypes, low-sinigrin *B. nigra*
316 genotypes, and other competing species (Lankau & Strauss, 2007). Here the high-sinigrin type has an
317 advantage in the competition with other plant species and thus has an advantage when *B. nigra* is rare.
318 As *B. nigra* is then becoming more common and intraspecific competition becomes stronger, it no longer
319 pays to produce so much sinigrin which is not effective against conspecifics, and the low-sinigrin type has an
320 advantage. These dynamics were also captured in an individual-based simulation model by Lankau (2009).

321 **Associational effects** are the effects that plants have on each other without direct interaction with each
322 other but through interaction with the other species, in particular herbivores (Hambäck *et al.*, 2014) (Fig.
323 2). These exist under many names. One kind of associational effect is social heterosis, which can explain
324 chemodiversity at the within-population scale. Social heterosis refers to a scenario wherein individuals have
325 different traits that are beneficial both to themselves and others in their neighbourhood but cannot all exist
326 in one organism at the same time. In this case, individuals in a diverse group have higher fitness than those
327 in a monoculture, as demonstrated in a quantitative model by Nonacs & Kapheim (2007). In chemodiversity,
328 this could take the form of a population of plants where different plants have different chemical profiles which
329 repel different insect herbivores, both from the individual plant itself and its neighbours. If the production
330 of SMS is costly, it may be detrimental to an individual to produce too many SMS, but beneficial to be in
331 an environment where other individuals produce different SMS from oneself. In this way, social heterosis
332 generates negative frequency-dependent selection on a single or community scale (Fig. 3) as rare types are
333 more likely to find themselves in mixed neighbourhoods with other types. This type of associational effect
334 is also called associational resistance (increased resistance through associational effects).

335 Another example of associational effects of chemodiversity is the model by Hambäck *et al.* (2014), which
336 modeled associational effects in a community of plants with different traits involved with both visual and
337 olfactory detection by herbivores. Volatile SMS form odor plumes that can be detected by herbivores. The

338 model included different plant trait values for the rate of detection of these plumes from a distance, the
339 relative herbivore attraction to different types of plants within the patch when SMs act as attractors, and
340 the rate of movement away from a plant to a different plant in the patch, and the rate of movement out of
341 the patch of the herbivores when SMs act as repellants. Their model found both scenarios where mixed-trait
342 plots had associational resistance to herbivory, and scenarios where mixed-trait plots had associational sus-
343 ceptibility, and scenarios where one trait displayed associational susceptibility and the other associational
344 resistance, depending on the nature of the trait of the plant and whether mixed-trait plots had the same
345 number of plants as the monocultures they were compared to. Hambäck *et al.* (2014) did not model evo-
346 lutionary changes, nor were SMs explicitly modeled in the model. However, their model could be modified
347 in the future to include these aspects, and in a multi-chemotype system these kinds of associational effects
348 should certainly be taken into account.

349 3.5 Coevolution

350 Coevolution is a phenomenon where the evolution of two or more interacting species is influenced by the
351 evolution of the respective other species in the interaction (Ehrlich & Raven, 1964). In the case of the
352 evolution of chemodiversity, chemodiversity can be conceptualized in different ways in the co-evolutionary
353 framework. In some studies, the richness and/or evenness of SMs is used as a trait of individuals (e.g.
354 Calf *et al.*, 2018), in others, the individual SMs are treated as separate traits (Speed *et al.*, 2015). Most
355 coevolutionary models and hypotheses for chemodiversity focus on coevolution with herbivores and envision
356 plants and herbivores to be in an "arms race". There is evidence from field, lab and phylogenetic studies
357 for the coevolutionary arms race hypothesis of chemodiversity on a population, species and lineage level
358 (Thorsteinson, 1953; Becerra *et al.*, 2009; Jander, 2014; Richards *et al.*, 2016).

359 Many quantitative models of **coevolution with herbivores** can be found in the literature (e.g. Gilman
360 *et al.*, 2012; Speed *et al.*, 2015; Ashby & Boots, 2017; Sandoval-Castellanos & Núñez-Farfán, 2023). However,
361 the one that most explicitly addresses chemodiversity is by Speed *et al.* (2015). This model deals with
362 chemodiversity within individual plants and within populations (see Fig. 3). In this individual-based model,
363 the traits that make plants produce toxins have direct resistance counterparts in insect herbivores, so that
364 an insect that feeds on a plant with trait A has to have the corresponding resistance A to the same degree
365 as the plant to be able to reproduce. We have dubbed this style of model a **toxin-resistance matching**
366 **model**. When plants and insects were allowed to evolve toxins and resistances through random mutation
367 and selection (Fig. 2), an arms race developed. Selection favoured plants which produced whatever toxin the
368 insects were least resistant to at a time, while other toxins became less prevalent until the insect resistances
369 evolved in turn and a different toxin would be the toxin they were least resistant to. This caused cycles in
370 which multiple toxins were present in the plants at any time in different concentrations, giving one possible

371 explanation of chemodiversity within individuals, within populations, and in populations through time (Fig.
372 3). Which chemicals dominate in the population at any given time is independent of the chemicals which
373 dominate in separate populations which are going through their own coevolutionary processes, thus causing
374 chemodiversity between populations and within the species.

375 Other quantitative coevolutionary models do not directly address chemodiversity, but contain elements
376 which could be used in chemodiversity models. Gilman *et al.* (2012) developed a model similar to the
377 one by Speed *et al.* (2015) and also show that coevolving with multiple traits can give the 'victim' species
378 an edge, although their model is not explicitly about chemodiversity. Ashby & Boots (2017) developed
379 a host-parasite model which uses a toxin-resistance matching model to model host defenses and parasite
380 resistance traits. This model displayed dynamics of both stable equilibrium where either all traits or none
381 were present, and infinite cyclical evolution. In the case of infinite cycles, there were two types of cycle
382 occurring in the same simulation: fast-cycling of the possible resistances at the same number of resistances
383 in a plant and slow cycling of number of resistances that existed in a plant. This was possible through a
384 dynamic where immediate benefits or detriments to the exact resistances an individual possessed caused
385 rapid changes in the resistances present, while subtle fitness costs of maintaining resistances created a slow
386 dwindling of the number of resistances until a threshold was reached where having many was beneficial
387 again. Both types of cycles displayed negative-frequency-dependent-selection and are thus linked to the
388 game theory models discussed above. These dual cycles are a feature not found in the model of Speed *et al.*
389 (2015) or in the model by Gilman *et al.* (2012) which is very similar to the model by Ashby & Boots (2017)
390 but only found stable equilibria. While the models of Ashby & Boots (2017) and Gilman *et al.* (2012) are not
391 specific to chemodiversity, these and other matched-trait models of coevolution could easily be translated to
392 a chemodiversity context by making the resistance traits correspond to metabolites.

393 Some authors have proposed that chemodiversity can come about when individual plants produce multiple
394 toxins to make it more difficult for herbivores to adapt to counter every single toxin of the set a plant pro-
395 duces (Speed *et al.*, 2012; Wetzel & Whitehead, 2020). This **slowed adaptation hypothesis** is essentially
396 one verbal description of an evolutionary process for which toxin-resistance matching is the mathematical
397 description. The slowed adaptation hypothesis may play a role in biological invasions (Box 3). However, this
398 is not the only possible verbal explanation for a mechanism of toxin-resistance matching. In the model by
399 Speed *et al.* (2015), the observed chemodiversity is the result of constant, moving-target like, innovation and
400 the remains of innovations past, rather than of slowed adaptation of herbivores to a standing set of toxins.
401 Additionally, while all models described in this review that have an explicit or implicit description of indi-
402 vidual metabolites use a toxin-resistance model, this is not the only possible way to model interactions with
403 and between metabolites. Quantitative models which explore synergy between metabolites, for example,
404 would require the effects of metabolites to differ depending on which other metabolites are present.

405 Another coevolutionary hypothesis which is based on spatial structure is the **geographic mosaic of co-**
406 **evolution hypothesis**. In separated populations, different concentrations in SMs may derive from opposite
407 selection pressures of specialists, which use certain SMs for host plant finding, versus generalists that are
408 repelled or deterred by the same SMs (van der Meijden, 1996; Enge *et al.*, 2012). Geographic differences in
409 herbivore abundance may thus result in a selection pressure mosaic (Thompson, 1999; Zangerl & Berenbaum,
410 2003). In a general mathematical model for the geographic mosaic of coevolution that could also be applied
411 to toxin-resistance matching, Gomulkiewicz *et al.* (2000) showed that a spatial setup with coevolutionary
412 hot spots (mutual selection) and cold spots (only one species exerts selection on the other, but not vice
413 versa) could under some conditions allow the maintenance of polymorphism in both species and differences
414 in allele frequency between populations. For a plant toxin coevolving with a herbivore, this would mean
415 that the model could explain coexistence of toxic and nontoxic plant individuals in the same patch as well
416 as differences in the frequency of toxic plants between patches (see Fig. 3).

417 A recent coevolutionary model that does not fit clearly into any of the previously discussed categories
418 investigates the circumstances under which non-linearity of costs and benefits of herbivore resistance can
419 lead to a mix of herbivory-resistant and herbivory-tolerating plants in a population (Sandoval-Castellanos
420 & Núñez-Farfán, 2023). The model is an individual-based model that compares additive and multiplicative
421 versions of a fitness function that includes the fitness costs and benefits of resistance and tolerance as well as
422 the fitness cost of inbreeding depression. It concludes that the nonlinear fitness function needs to be concave,
423 the allocation of costs and benefits multiplicative, selfing non-heritable, and tolerance costly to promote a
424 mix of strategies. The 'resistance' in this model does not necessarily refer to SMs, but but the model could
425 be easily modified to include different SMs.

426 To really make the models specific to chemodiversity, introducing ingredients like the branching pathways
427 through which chemicals are created would be useful. Additionally, while toxin-resistance matching has been
428 used in most quantitative coevolutionary models, empirical studies frequently test linear additive or nonlinear
429 synergistic effects of SMs (Richards *et al.*, 2016; Dyer *et al.*, 2018). Therefore, developing models that use
430 these additive or synergistic effects could really bring the empirical and quantitative modeling research in
431 this field together.

432 **Coevolution with seed dispersers** is another way in which coevolution can potentially promote
433 chemodiversity. For example, in the genus *Piper*, fruit toxicity affects dispersal by coevolved frugivores.
434 This was statistically modeled by Baldwin *et al.* (2020) based on empirical data collected in a system where
435 *Carollia* fruit bats consume *Piper* fruits. They found that higher levels of defensive amides correlate with
436 shorter gut retention times and lower dispersal distances, while lower concentrations of amides correlate with
437 higher dispersal distances as well as optimal ripeness of the fruit and maximum attractiveness to the fruit
438 bats. In this way, the ripe *Piper* seeds are dispersed as far as possible, while unripe fruits are less likely to

439 be eaten in the first place. Models like this can be used to make quantitative predictions for future empirical
440 studies. In its current form, such a model could explain differences in amide concentration between fruits
441 within an individual or between individuals when fruits differ in ripeness (Fig. 3). For purposes of studying
442 chemodiversity, it would be interesting to not only include total amide concentrations as a predicting variable,
443 but also diversity of amides. It may be possible to shine a light on how coevolution of seed dispersal influences
444 chemodiversity on the within-plant and between-population scale. Moreover, gut retention time in frugivores
445 is not the only way chemodiversity may play a role in the coevolution of fruit-bearing plants and their seed
446 dispersers. For example, SMs also may act as olfactory cues to frugivores, inhibit seed germination, and act
447 as toxins Cipollini & Levey (1997). So far, we did not find any quantitative models addressing the potentially
448 important consequences of coevolution with seed dispersers for the evolution of plant chemodiversity.

449 **3.6 Screening hypothesis**

450 The models reviewed in the previous sections generally assume that every metabolite has a specific biological
451 activity in the interaction with one or more species. Metabolites without a specific strongly advantageous
452 activity would be expected to be selected away because of their inherent costs. The screening hypothesis first
453 proposed by Jones & Firn (1991) (see also Firn & Jones, 2003; Firn, 2009) challenges this view. Based on the
454 observation that many metabolites produced by plants and fungi have no known function, they argue that
455 strong specific biological activity is in fact a rare phenomenon. Thus, in order to defend themselves against
456 a variety of herbivores, plants need to “screen” a large number of “candidate metabolites”. They keep a
457 diversity of metabolites, even many without a function, because this diversity and the underlying multitude
458 of enzymes allows them via mutation to rapidly generate new metabolites and thus increase their chances
459 to find at least some that have strong activity. Since maintaining a diversity of enzymes and metabolites
460 still has costs, the screening hypothesis predicts that the metabolic pathways would be selected such that
461 they can produce a high number of metabolites with as little enzymatic machinery as possible, mostly via
462 promiscuous enzymes that can take multiple substrates, leading to grid-like metabolic pathways. Keeping a
463 diversity of metabolites around can then also allow plants to rapidly respond to new herbivores or herbivores
464 that have evolved a counter-defense (Jones & Firn, 1991).

465 In summary, the screening hypothesis is based on the ingredients of random mutation and natural selec-
466 tion, metabolic pathways, and interactions with other organisms (Fig. 2). Though the focus in the original
467 formulation is on herbivores, by the same logic, a diversity of metabolites can help plants to develop metabo-
468 lites that are beneficial in the interaction with other plants or with mutualists and soil communities. The
469 screening hypothesis has been formulated as a verbal model and to our knowledge, it has not been formulated
470 as a quantitative model. Nor are the ideas of grid-like metabolic pathways, promiscuous enzymes, and rare
471 metabolic activity incorporated into other quantitative models.

472 The screening hypothesis mostly addresses chemodiversity within units of scale, i.e. it attempts to explain
473 why a plant individual, or a plant population, species or clade produces so many different metabolites (Fig.
474 3). Although this has been less discussed by the authors of the screening hypothesis, if a complex metabolic
475 network allows plants to rapidly generate new metabolites, this could also help explain differences between
476 individuals, populations, and species. This appears to be supported by the observation that many enzymes
477 involved in generating chemodiversity are less conserved than those that are involved in more primary
478 metabolism (Weng *et al.*, 2012).

479 The screening hypothesis has been criticized because evolutionary foresight appears to be required for
480 evolution to create a complex network of pathways just based on the chance that some of its products might
481 eventually prove useful (Pichersky *et al.*, 2006). Proponents of the hypothesis counter that no evolutionary
482 foresight is required if the rapid adaptation made possible through the network is sufficiently beneficial to be
483 selected for (Firn & Jones, 2006). A formal mathematical description or computer model could demonstrate
484 to what extent these criticisms are warranted. Such models have, to our knowledge, not yet been developed.

485 3.7 Population genetics models

486 Chemodiversity is, at least in part, underpinned by genetic variation. Thus, it would seem natural to extend
487 classical population genetic models to study chemodiversity.

488 In biodiversity research (Hubbell, 2001) and in population genetics (Kimura, 1983), **neutral models** play
489 an important role as null models for more interesting ecological and evolutionary processes though there is also
490 much debate over how useful they are (see e.g. Clark, 2009; Kern & Hahn, 2018). While neutral biodiversity
491 models assume that species within a guild are equivalent in terms of their birth, death, and migration rates,
492 neutral models in population genetics assume that allele frequencies change just through mutation and drift
493 without selection. We argue that similar neutral models should also be created for chemodiversity. That
494 is, diversity in such models would come about and be maintained through random mutation and genetic
495 drift without selection (Box 1). A neutral model for chemodiversity should additionally include pathways
496 and enzymes that are encoded by the modeled genes and that thus also evolve neutrally (Fig. 2). A
497 neutral model could in principle be formulated at any scale of organization, though a neutral model derived
498 from neutral models in population genetics would most straightforwardly address chemodiversity within
499 populations, within and between individuals (Fig. 3). This model would serve as a null model for the origin
500 and maintenance of chemodiversity. Any explanations of the origin and maintenance of chemodiversity which
501 propose that there is a selective advantage to producing a diversity of SMs can then be compared to it. We
502 have not found any sources where a model of chemodiversity is compared to such a null model, but we think
503 that such an approach would be extremely useful.

504 A slightly more complex explanation for chemodiversity is **mutation-selection-drift balance** (see e.g.

505 Wright, 1937). Mutation-selection-drift models in population genetics generally assume that mutations are
506 either unconditionally beneficial or deleterious. There are no complex selection patterns like balancing
507 selection or negative frequency dependent selection. Thus variation under this model is observed due to
508 mutations that are transiently segregating in the population before they are getting either lost or fixed. In a
509 chemodiversity extension of such models, new SMs would come about through random mutation in existing
510 metabolic pathways. Here it is assumed that at least some SMs improve the organisms' fitness and are under
511 positive selection, while others may be detrimental or are not worth the cost of their production and are
512 thus under negative selection. The probability that a mutation is fixed or lost and the expected time to do
513 so then depends on the strength of selection relative to genetic drift. During their time to loss or fixation,
514 mutations and their effects on the chemical phenotype would then transiently contribute to chemodiversity.
515 Like neutral models, mutation-selection-drift models could explain chemodiversity within populations, and
516 within and between individuals (Fig. 3), but potentially also at all other scales. For example if different SMs
517 randomly become fixed in different populations and clades this could explain differences between these units.
518 So far, there do not seem to be any mutation-selection-drift models of chemodiversity in the literature.

519 **Box 3: Biological invasions as testing ground for chemodiversity** 520 **models**

521 Connecting models for the evolution of chemodiversity to empirical data can be challenging because in natural
522 population we can usually only observe a snapshot of the evolutionary process at the current time, but the
523 long evolutionary history remains hidden. Biological invasions of plants or interacting animals offer exciting
524 opportunities for testing these theories because here the interaction partners often experience sudden large
525 changes in selection regime, sometimes resulting in rapid evolution on ecological time scales (Cox, 2004).

526 Invasive species may also benefit from high intraspecific variation in chemical composition, i.e. chemodi-
527 versity on a population level. For example, individuals of the plant *Tanacetum vulgare* (Asteraceae) are highly
528 diverse in their terpenoid profiles, with different terpenoids occurring in different concentrations (Wolf *et al.*,
529 2011). Terpenoids have various ecological functions (Cheng *et al.*, 2007), and effects on antagonists are highly
530 species-specific. Therefore, a high chemodiversity within invasive populations may impede the adaptation
531 of herbivores and microorganisms native to the habitat (Wolf *et al.*, 2011; Tewes *et al.*, 2018), as predicted
532 by the slowed adaptation hypothesis. Furthermore, the simple unfamiliarity of the novel SMs is thought to
533 provide a competitive advantage. This is called the 'novel weapons hypothesis' (Callaway & Ridenour, 2004)
534 in the context of biological invasions and closely related to the moving target hypotheses described above.

535 Furthermore, multiple introductions from different source populations to North America may have led to
536 hybridisation of individuals of different chemotypes, producing novel mixtures of terpenoid profiles and thus

537 novel chemotypes. Indeed, few chemotypes were only found in individuals in North America, where *T. vulgare*
538 is invasive (Wolf *et al.*, 2011). Moreover, potential inbreeding in invasive populations in interaction with the
539 environment may lead to rapid evolutionary changes (Schrieber & Lachmuth, 2016), potentially also resulting
540 in novel chemotypic variation. Overall, changes in chemodiversity and the functions in species-interactions in
541 non-native populations have been largely neglected (Tewes *et al.*, 2018), although chemical composition can
542 highly vary between native and non-native populations (Tewes *et al.*, 2018; Pankoke *et al.*, 2019) and due to
543 hybridization (Piola *et al.*, 2013), offering great opportunities to develop theoretical models and test them
544 with empirical studies. The novel-weapons hypothesis has already been mathematically modeled (Allstadt
545 *et al.*, 2012), albeit not as a chemodiversity model, and is ripe for expanding upon.

546 4 Vision

547 It would be easy to fall into the trap of assuming that the different frameworks and hypotheses described
548 here must be in competition, that through modeling and empirical work, we can find one correct and all
549 others incorrect. Instead, we agree with Dyer *et al.* (2018) that the hypotheses do not have to contradict
550 each other. In fact, many of the hypotheses and selection mechanisms described here may be at play at once
551 within the same plant species. While the members of particular set of SMs cannot all have both synergistic
552 and toxin-resistance matching fitness effects at the same time, this set of SMs might have synergistic effects
553 on herbivory at the plant level, and also be under negative frequency-dependent selection on the population
554 level. This set can then simultaneously be in a co-evolutionary arms race with herbivores on the lineage level,
555 where new SMs come about within a pathway through the process well-described by the screening hypothesis.
556 Another set of SMs in the same plant may only exist transiently as random mutations in a pathway under
557 mutation-selection-drift balance. For this reason, it is useful to think of the different hypotheses and selection
558 methods as different possible factors in the evolution and maintenance of chemodiversity, and not as mutually
559 exclusive. At the same time, some of these hypotheses may well be invalid, or some might contribute less
560 to explaining chemodiversity than others. Quantitative models can serve to investigate the validity and
561 explanatory power of these hypotheses. Combining multiple hypotheses in a single quantitative model would
562 enable us to compare the relative explanatory power of different models.

563 This does not mean that every quantitative model of chemodiversity has to be able to test all possible
564 mechanisms involved with the evolution and maintenance of chemodiversity. A model should include no
565 more complexity than it needs to (Servedio *et al.*, 2014), and there is also the risk of overfitting in overly
566 complex models. Nevertheless, one should be mindful of alternative explanations when making a model
567 that investigates a particular set of hypotheses. A model can show a verbal hypothesis to be flawed if the
568 stated assumptions cannot be made to produce the suggested result, but it cannot prove a hypothesis to be
569 definitely correct (Otto & Rosales, 2019).

570 When we make a model to test a verbal hypothesis, we need to incorporate enough detail to describe
571 what that hypothesis is about. That means that a test of the coevolutionary arms race needs to incorporate
572 a focal species and an interacting co-evolving species. For the screening hypothesis it is necessary that a
573 molecular model be included that works as described by Jones & Firn (1991); Firn & Jones (2003), but it is
574 not fundamentally necessary that any other species is involved. When modeling a very specific system, it is
575 useful to ensure that all aspects of that system are carefully considered, and all relevant details are included,
576 and all assumptions are justified (Servedio *et al.*, 2014). For example, if one were to model a biological
577 invasion as described in Box 3, it would be prudent to include elements of the slowed adaptation and novel
578 weapons/moving target hypotheses.

579 We have shown here that there is a great body of work on the description of current chemodiversity and
580 its evolutionary history, and there are good evolutionary frameworks for the evolution of chemodiversity that
581 are compatible with various mechanisms for the maintenance of chemodiversity. For many of the hypotheses,
582 we also found a small number of quantitative modeling studies. However, quantitative models that explicitly
583 address chemodiversity where individuals produce a diverse set of metabolites are few and far between. Such
584 models will often be sufficiently complicated that it might not be possible to capture them in a system of
585 equations that can then be solved analytically. Thus, numerical solutions and individual-based models as in
586 Speed *et al.* (2015) appear to be the most productive ways to model the evolution of chemodiversity in these
587 cases.

588 **5 Conclusion**

589 In conclusion, while there has been much verbal theory-crafting on chemodiversity and there is a small
590 number of interesting quantitative models for many of the verbal hypotheses, there is much space for the
591 development of mathematical models. This can be done by expanding existing models of chemodiversity,
592 adapting existing models of diversity, sometimes by combining different submodels to create coherent models
593 of chemodiversity. In this way, the existing theory can be tested for validity, the explanatory power of different
594 hypotheses can be explored, and the results can be used to interpret empirical data to better understand
595 the dynamics underlying extant chemodiversity.

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