



## Abstract

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1: Determining the harvest location of timber is crucial to enforcing international regulations designed to tackle illegal logging and associated trade in forest products. However, complex supply chains obscure harvest sources, which often leaves paper-based traceability systems as the sole tool for demonstrating provenance, despite its vulnerability to fraud. Stable Isotope Ratio Analysis (SIRA) can be used to verify claims of timber harvest location by matching levels of naturally occurring stable isotopes within wood tissue, to location-specific SIR predicted from reference data ('isoscapescapes'). The primary challenge in developing reliable isoscapescapes is the need to accurately predict stable isotopes in areas where no physical reference samples are available. Existing attempts to predict isoscapescapes from reference data have been hampered by the use of simple and ad-hoc statistical models, limiting the precision of estimated isoscapescapes and the confidence in derived estimates of geographical origin.

2: We present a new SIRA data analysis pipeline, designed to infer timber harvest location. We use Gaussian Processes to robustly estimate isoscapescapes from reference wood samples, which are then combined with species distribution range data to compute, for every pixel in the study area, the probability of it being the origin of the sample. Finally we present a methodology to determine priority locations to obtain new reference samples in future field expeditions.

3: We demonstrate our approach on a data set of  $n = 87$  wood samples from seven oak species in the USA as proof of concept. Our method is able to determine the harvest location up to 520-870 km, depending on the model parameterisation. Incorporating species distribution information improves accuracy by up to 36%. The new sampling locations proposed by our method decrease the variance of resultant isoscapescapes by up to 86% more than sampling the same number of locations at random.

4: The pipeline we present here combines the prediction of isoscapescapes with derivation of geographical origin estimates quickly and efficiently. It advances the toolset available to authorities addressing illegal trade in forest products and enforcing anti-deforestation legislation. Importantly, reference data can be added as available, allowing for the expansion of reference collections and increasing prediction accuracy.

56 **Keywords:** SIRA, origin traceability, timber provenance, illegal logging, isoscapescapes,  
57 Gaussian Processes

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## 1 Introduction

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One million species now face extinction, and the unsustainable exploitation of natural resources is the second largest driver of terrestrial biodiversity loss, next only to land use changes [18]. To prevent our societies triggering a new wave of mass extinctions [2], nearly 200 nations have recently agreed on a new set of targets and goals under the Kunming-Montreal Global Biodiversity Framework. Under this international agreement, human-driven species extinctions must halt by 2030, in

65 order to allow an appropriate level of natural recovery by 2050. In particular, Target  
66 5 of the agreement has the objective to "ensure that the use, harvesting and trade  
67 of wild species is sustainable, safe and legal, preventing overexploitation".

68 Meeting such an ambitious but necessary target will require overcoming a key  
69 element of unsustainable use of natural resources: the illegal harvest of threatened  
70 trees. To combat illegal logging and associated trade in illegally harvested tim-  
71 ber, various frameworks have been put into place, such as the Convention on the  
72 International Trade in Endangered Species (CITES) and security linked sanctions.  
73 At national or regional levels, additional legislation include the US Lacey Act, the  
74 UK Timber Regulation, the EU Deforestation Regulation and the Australian Illegal  
75 Logging Prohibition Act. Despite the comprehensive legal framework already in  
76 place, and the international commitments under current adoption, it is clear that  
77 none of these can effectively work without enforcement. This is where technological  
78 solutions and methodological developments become key.

## 79 **1.1 Stable Isotope Ratio Analysis for timber provenance**

80 Current legislation increasingly requires accurate, cost-effective, and high-throughput  
81 tools that can verify the specific species and origin of products in global trade [19].  
82 Scientific testing technologies have become relatively well established and are al-  
83 ready supporting companies and enforcement authorities to scrutinize traceability  
84 systems. These technologies measure the chemical, anatomical and genetic features  
85 of plants which, when compared against a robust physical reference collection, can  
86 be used to (in-)validate declared species and origin claims and support enforcement  
87 officials in their efforts to detect, for example, illegal or conflict timber and fraud in  
88 supply chains.

89 One of the most promising and widely used scientific technologies is stable  
90 isotope ratio analysis (SIRA). The ratios of several elemental stable isotopes within  
91 natural products vary across space and can assist in verifying the geographic origin  
92 of products. Most of the abundant elements in organic compounds (Hydrogen,  
93 Oxygen, Carbon, Sulfur, Nitrogen) have naturally-occurring stable isotopes that do  
94 not undergo radioactive decay, and can be readily detected by mass spectrometry  
95 [4]. The composition of stable isotopes incorporated into the tissues of a plant is  
96 determined by the soil, climate, metabolic fractionation and other biotic and abiotic  
97 conditions characteristic of the species and its habitat [40, 8, 25]. Stable isotope  
98 ratios can be used to discriminate between geographic areas as the variation in these  
99 stable isotope ratios depend on natural variations of the underlying mechanisms (for  
100 example environmental drivers [46]). The types of products that can be analysed by  
101 SIRA to determine risk for being illegally harvested include natural resources-such as  
102 forest products [47, 4], agricultural products [9, 37], wildlife [7, 30], fish/seafood [15,  
103 41, 32], ivory [45, 53], precious metals [29] and illicit drug trafficking, including  
104 natural and synthetic opioids [33, 10], and other forensic uses to identify counterfeit  
105 and pirated goods trafficking [11].

## 106 1.2 Modelling approach

107 Current modelling practices for the use of SIRA to verify harvest location of both  
108 legally and illegally harvested products could be improved. The use of SIRA is  
109 currently limited by the relative simplicity of models used, as well as by the limited  
110 number of reference samples used as input data for such models. The practical  
111 nature of reference sampling campaigns is that they can be costly and budgetary  
112 needs are often underestimated, with sampling locations often taking into account  
113 relative ease of sampling rather than areas that yield a gain in model prediction  
114 accuracy [38]. There has been considerable development of isoscapes ("isotope  
115 landscapes"), given that stable isotopic variation is a continuous spatial variable in  
116 nature [49]. These isoscapes are geospatial maps that show the isotopic variation  
117 of the material of interest [49]. While the potential of isoscapes for determining  
118 product origins has long been recognized, few rigorous methods exist to achieve  
119 this task. The existing methods use simple prediction strategies such as linear  
120 regression [47, 48] and clustering [30], which do not fully leverage the information  
121 contained in isotope ratio data.

122 Gaussian Process (GP) regression, also known as kriging in geostatistics litera-  
123 ture, is a class of flexible regression models which use the values in sampled points  
124 to estimate the values in surrounding, unsampled points [34, 17, 50]. A key advan-  
125 tage of GP regression is that it can quantify the uncertainty of its own predictions  
126 based on the inferred spatial covariance structure of the samples. Quantifying the  
127 uncertainty of predictions is viewed as increasingly important in safety-critical [26]  
128 and forensic [43, 44] machine learning applications. GP regression also facilitates  
129 co-kriging: inferring the values of a sparsely sampled variable of interest through  
130 variables that are highly correlated with it but more densely sampled [1, 28]. In the  
131 context of plant origin estimation, co-kriging translates to inferring stable isotope  
132 ratios from atmospheric drivers (such as precipitation, temperature and water vapor  
133 pressure) known to influence the stable isotope signal in wood [25, 40]. This then  
134 provides a powerful tool for predicting the isotopic composition in areas that have  
135 not yet been sampled (examples are given in [22, 47]).

136 Previous work on isoscapes used GP regression primarily as a spatial interpo-  
137 lation technique without a probabilistic interpretation [22, 30, 28]. A more recent  
138 method uses GP variance estimates from precipitation isoscapes for origin deter-  
139 mination in animals [35]. In this work, we develop GP-based probabilistic machine  
140 learning models to infer the origin of timber samples by directly modelling timber  
141 isoscapes. We present a new data analysis pipeline that incorporates timber SIRA  
142 data, atmospheric predictors and species distribution data. We find that probabilis-  
143 tic modeling greatly enhances the utility of SIRA in estimating the geographical  
144 origin of timber and helps guide future sample collection by identifying sampling  
145 locations that will minimize prediction uncertainty. The presented framework can  
146 then be applied to trace back timber of endangered species, by assisting in deter-  
147 mining where to collect samples and by using SIRA datasets being collected by the  
148 World Forest ID [21] initiative.

## 149 2 Materials and Methods

### 150 2.1 Data sets

151 We use data from 87 trees of the genus *Quercus*, sampled across the contiguous  
152 United States, as described in Watkinson et al. (2020) [47]. Stable isotope ratio  
153 measurements were done following the protocol described in Boner et al. (2007) [4]  
154 and Watkinson et al. (2020) [47]. Each entry contained stable isotope ratio mea-  
155 surements of oxygen  $\delta^{18}\text{O}$  (ratio between  $^{18}\text{O}$  and  $^{16}\text{O}$ ), hydrogen  $\delta^2\text{H}$  (ratio be-  
156 tween  $^2\text{H}$  and  $^1\text{H}$ ), carbon  $\delta^{13}\text{C}$  (ratio between  $^{13}\text{C}$  and  $^{12}\text{C}$ ) and sulfur  $\delta^{34}\text{S}$  (ratio  
157 between  $^{34}\text{S}$  and  $^{32}\text{S}$ ) as well as the GPS coordinates of the sampled tree. Stable  
158 isotope ratios are largely driven by environmental conditions such as precipitation,  
159 temperature, humidity and so on. Thus, it is natural to use publicly available data  
160 on those factors to aid the inference of isoscapes. We used the following atmo-  
161 spheric data:  $\delta^2\text{H}$  and  $\delta^{18}\text{O}$  isotopic composition of precipitation [6], water vapour  
162 [5] (known to affect  $\delta^{13}\text{C}$ ), surface downward shortwave radiation and precipitation  
163 (multi-satellite), both of which affect  $\delta^{34}\text{S}$  [39].

164 To inform the priors (probability distributions representing the prior belief on  
165 possible tree locations) of the models we develop, we used species inventory data  
166 across the natural range of each species within the United States [51], downloaded  
167 from: <https://www.fs.usda.gov/rds/archive/Catalog/RDS-2013-0013> on  
168 09/12/2022. This data is available as species-specific raster layers of tree abundance  
169 at 250m resolution. We then used the function `project()` of the R package *terra* [24]  
170 to bilinearly aggregate abundance data so that it matched the spatial resolution of  
171 other spatial data in the pipeline.

### 172 2.2 Model architecture

173 Figure 1 presents the pipeline overview of the data sets, components comprising our  
174 model and output. We use a rectangular grid to represent the study area. Grid points  
175 are placed every 0.125 degree latitude ( $\approx 14$  km) and every 0.06 degree longitude  
176 ( $\approx 4.3$ -6.0 km), which allows us to approximate spatial probability distributions  
177 with high accuracy. For every isotope ratio (IR), we fit a GP regression model to  
178 the training data to obtain the posterior mean and variance of the isotope ratio for  
179 every point of the grid - see Sections 2.3 and 2.4 for more details. The input to  
180 the GP consists of the coordinates and/or the climate variable values at the grid  
181 point. For a combination of stable isotope ratios  $(y_O, y_H, y_C, y_S)$  of observed stable  
182 isotope ratios, we compute the likelihood of observing it at every point in the grid  
183 using the four GP regression models estimated in the previous step. This likelihood  
184 is the product of likelihoods for each isotope ratio as we assume independence  
185 between isotopes. Given the prior and the likelihood, we perform Bayesian inference  
186 by computing the posterior probability of each grid point being the origin of the  
187 sample by applying Bayes' Theorem. For ease of interpretation, the output is a map  
188 with highest-posterior density (HPD) regions indicated for several probability levels  
189 (15%, 30%, 50%, 75%, 90%, 95%).

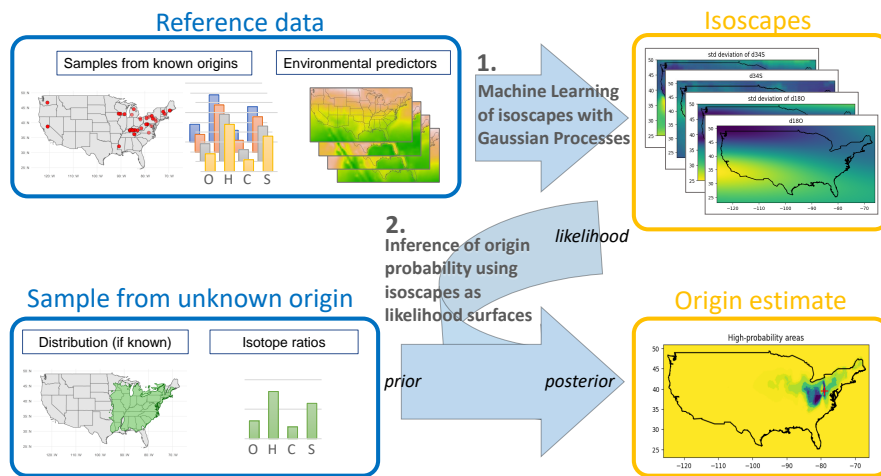


Figure 1: Model workflow. We use a training set of isotope ratios from trees collected at known locations and atmospheric data layers ("Reference data"). We fit a Gaussian process regression model to infer isoscapes and associated variance estimates, and compute the likelihood of observing the IR value for each element across the study area. To estimate the source of material with uncertain provenance ("Samples from unknown origin"), the isoscapes are then combined with prior information on the geographical distribution of the species, to yield a probability distribution of origin for the sample. We visualize predicted probability maps by plotting highest-posterior density regions for several probability levels (15%, 30%, 50%, 75%, 90% and 95%, dark blue to light green).

190 **2.3 Gaussian Process regression**

191 In the following, we give a brief overview of GPs. For a more thorough explanation,  
 192 see [50].

193 GPs provide a flexible framework for regression, which enables the modeller to  
 194 quantify the uncertainty of specific inferences. A GP is a random process such that  
 195 all of its marginals are jointly normally distributed (Gaussian). Let  $\mathbf{x} = [x_{lon}, x_{lat}]$   
 196 be the GPS coordinates of a sample. For any set of positions  $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ ,  
 197 the responses  $y_1, y_2 \dots, y_n$  at those positions are assumed to be jointly normally  
 198 distributed

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \sim \mathcal{N} \left( \begin{bmatrix} m(\mathbf{x}_1) \\ m(\mathbf{x}_2) \\ \vdots \\ m(\mathbf{x}_n) \end{bmatrix}, \begin{bmatrix} k(\mathbf{x}_1, \mathbf{x}_1) & k(\mathbf{x}_1, \mathbf{x}_2) & \dots & k(\mathbf{x}_1, \mathbf{x}_n) \\ k(\mathbf{x}_2, \mathbf{x}_1) & k(\mathbf{x}_2, \mathbf{x}_2) & \dots & k(\mathbf{x}_2, \mathbf{x}_n) \\ \vdots & \vdots & \ddots & \vdots \\ k(\mathbf{x}_n, \mathbf{x}_1) & k(\mathbf{x}_n, \mathbf{x}_2) & \dots & k(\mathbf{x}_n, \mathbf{x}_n) \end{bmatrix} + \sigma^2 \mathbf{I} \right) \quad (1)$$

199 where:

- 200 1.  $m(\mathbf{x})$  is the *mean function* describing the *a priori* expected value of the re-  
 201 sponse  $y$  at point  $\mathbf{x}$ . Usually, this is a parameterized function whose pa-  
 202 rameters are estimated by fitting the model to the training data. The most  
 203 common choice of mean function is the constant mean  $m_c(\mathbf{x}) = \theta_m$  for all  
 204  $\mathbf{x}$ , which we also use in this work.
- 205 2.  $k(\mathbf{x}_1, \mathbf{x}_2)$  is the *covariance function* describing the *a priori* covariance be-  
 206 tween responses at points  $\mathbf{x}_1$  and  $\mathbf{x}_2$ . This is also a parameterized func-  
 207 tion. Popular choices of  $k$  are the *squared exponential*  $k_{se}(\mathbf{x}_1, \mathbf{x}_2) =$   
 208  $A \exp(-|\mathbf{x}_1 - \mathbf{x}_2|^2 / \rho^2)$ , or the Matern function [50], which both reflect the  
 209 common assumption that similar predictor values will lead to similar response  
 210 values. In this work, we use the Matern function with separate scaling pa-  
 211 rameters for latitude and longitude to model spatial covariance.
- 212 3.  $\sigma^2$  is the intrinsic noise parameter
- 213 4.  $\mathbf{I}$  is the  $n \times n$  identity matrix.

214 We will write  $\mathbf{y}$ ,  $\mathbf{m}$  and  $K$  to denote the responses, means and the covariance matrix  
 215 of the training data, respectively, so that we can write Eq. 1 as  $\mathbf{y} \sim \mathcal{N}(\mathbf{m}, K + \sigma^2 \mathbf{I})$ .  
 216 The choice of mean and covariance functions reflects prior knowledge and modelling  
 217 assumptions about the regression problem. The function parameters as well as  
 218 the noise parameter  $\sigma$  are estimated by maximizing the likelihood of the training  
 219 data. We use GPyTorch [20] to efficiently find the maximum likelihood parameter  
 220 estimates.

221 After parameter estimation, the GP regression model can be used to predict  
 222 responses at previously unseen data points. Let  $S$  be the locations and responses  
 223 comprising the training data set. Since the responses at training and test points  
 224 are assumed to be jointly Gaussian, the conditional distribution of the response at  
 225 a test point  $\mathbf{x}^*$  given the training data is also Gaussian with mean

$$\mu(\mathbf{x}^*|S) = m(\mathbf{x}^*) + \mathbf{k}^*(K + \sigma^2\mathbf{I})^{-1}(\mathbf{y} - \mathbf{m}) \quad (2)$$

226 where  $\mathbf{k}^* = [k(\mathbf{x}^*, \mathbf{x}_1), k(\mathbf{x}^*, \mathbf{x}_2), \dots, k(\mathbf{x}^*, \mathbf{x}_n)]$  is the vector of a priori covariances  
 227 between responses at  $\mathbf{x}^*$  and training data points. The posterior variance of  $y^*$  is  
 228 given by

$$\sigma^2(\mathbf{x}^*|S) = k(\mathbf{x}^*, \mathbf{x}^*) + \sigma^2 - \mathbf{k}^*(K + \sigma^2\mathbf{I})^{-1}\mathbf{k}^{*\top} \quad (3)$$

229 - see [50] for a derivation.

230 For a specific response value  $y^+$ , its likelihood at  $\mathbf{x}^*$  is just the Gaussian prob-  
 231 ability density with mean  $\mu$  and variance  $\sigma^2$  found by applying Equations 2 and  
 232 3

$$p(y^* = y^+|\mathbf{x}^*, S) = \frac{1}{\sqrt{2\pi\sigma^2(\mathbf{x}^*|S)}} \exp\left(\frac{-(y^+ - \mu(\mathbf{x}^*|S))^2}{2\sigma^2(\mathbf{x}^*|S)}\right) \quad (4)$$

## 233 2.4 Incorporating atmospheric data

234 For any  $\mathbf{x}$ , let  $\mathbf{u}_i(\mathbf{x})$  denote the 12-entry vector of monthly values of atmospheric  
 235 variable  $i$  at location  $\mathbf{x}$ . We use a linear covariance function to model the covariance  
 236 component corresponding to the variation in atmospheric variable  $i$

$$k_i(\mathbf{x}_1, \mathbf{x}_2) = \theta_i[\mathbf{u}_i(\mathbf{x}_1)]^\top \mathbf{u}_i(\mathbf{x}_2) \quad (5)$$

237 where  $\theta_i$  is a parameter to be estimated during training. The linear covariance  
 238 function models a linear relationship between the atmospheric variable and the  
 239 response and is mathematically equivalent to Bayesian linear regression [50].

240 The overall covariance function is the sum of the spatial term and the linear  
 241 terms

$$k(\mathbf{x}_1, \mathbf{x}_2) = k_{spatial}(\mathbf{x}_1, \mathbf{x}_2) + \sum_{i \in V} \theta_i[\mathbf{u}_i(\mathbf{x}_1)]^\top \mathbf{u}_i(\mathbf{x}_2) \quad (6)$$

242 where  $V$  is the set of atmospheric variables impacting the considered isotope ratio.

## 243 2.5 Bayesian inference of sample origin

244 Given a prior distribution  $p(\mathbf{x})$  over possible sample origins and a GP regression  
 245 model for every isotope, we perform Bayesian inference of sample origin. For a  
 246 sample  $\bar{y} = (y_O, y_H, y_C, y_S)$  of observed values, the Bayes' theorem gives the  
 247 posterior distribution of possible origins:

$$p(\mathbf{x}|\bar{y}, S) = \frac{p(\mathbf{x}) \prod_{i \in \{O, H, C, S\}} p_i(y_i|\mathbf{x}, S)}{\int_{\mathbf{x} \in A} p(\mathbf{x}) \prod_{i \in \{O, H, C, S\}} p_i(y_i|\mathbf{x}, S) d\mathbf{x}} \quad (7)$$

248 where the probabilities  $p_i$  are computed from the GP models for the respective  
 249 isotopes using Equation 4 and  $A$  is the study area. The integral in the denominator  
 250 is computed by averaging the probabilities over the spatial grid.



## 251 **2.6 Incorporating species distributions**

252 We use the spatial density maps developed by Wilson *et al.* [51] to design two  
253 prior distributions for sample origin that account for the spatial distribution of oak  
254 species. The first, which we call the *density prior*, holds that the probability of a  
255 sample originating at a grid cell is proportional to the basal area (average amount  
256 of area occupied by tree stems per unit of space) recorded at the grid cell. The  
257 second, which we call the *range prior*, assigns equal probability to every grid cell  
258 where above-zero basal area has been recorded. In addition, both priors allow for a  
259 small probability that a sample might occur outside its observed range - we set that  
260 probability to 0.01 and diffuse it uniformly over all grid points within the contiguous  
261 United States where the species does not occur according to Wilson *et al.*.

## 262 **2.7 Performance evaluation**

263 We perform 5-fold cross-validation on the data set and report the average values of  
264 all performance metrics over all data points. Samples with incomplete or ambigu-  
265 ous species information and samples collected in botanical gardens outside of their  
266 species' native range are excluded from the test sets, but not from the training sets,  
267 resulting in a total of  $n=74$  test samples across the 5 folds.

268 Rigorously evaluating the performance of our model is a non-trivial task as it  
269 produces a distribution over possible locations, rather than a single location. For  
270 this reason, we have defined several metrics to investigate different aspects of our  
271 predictions.

### 272 **2.7.1 Predictive log-likelihood and log-posterior**

273 We report the log-likelihood (Eq. 4) and the log-posterior (Eq. 7) of observing the  
274 sample  $\bar{y}$  at its true origin  $\mathbf{x}_t$ . Both of those measure how well the model fits the  
275 test data.

### 276 **2.7.2 Mean posterior distance to true location (MPD)**

To investigate how distant our predicted locations are from the truth, we report the  
expected distance between the true location and a location sampled randomly from  
the posterior distribution returned by our model

$$MPD = \int_{\mathbf{x} \in A} d(\mathbf{x}_t, \mathbf{x}) p(\mathbf{x} | \bar{y}, S) d\mathbf{x}$$

277 where  $d()$  is the great circle distance between the two points. A perfect prediction  
278 would have the distance of 0. In practice, isoscapes often predict similar isotope  
279 ratio values at distant locations, so even a statistically efficient method might yield  
280 a high MPD value. This metric will favour predictions concentrated around the  
281 true location over equally dispersed predictions concentrated elsewhere. It will also  
282 favour less dispersed predictions generally.

283 **2.7.3 Area scored higher than the true location (ASH)**

The behaviour of MPD is influenced by the shape of the posterior distribution, which favours unimodal over multimodal shapes. We report the total surface area corresponding to the points that the model considers more plausible than the true origin of the sample.

$$ASH = \int_{\mathbf{x} \in A} I [p(\mathbf{x}|\bar{y}, S) > p(\mathbf{x}_t|\bar{y}, S)] d\mathbf{x}$$

284 where  $I(\cdot)$  is the indicator function that yields 1 when the statement is true and 0  
 285 otherwise. In contrast to MPD, this metric is likely to give a low value to a posterior  
 286 distribution that is concentrated in several small areas as long as one of those areas  
 287 contains the true location.

288 **2.8 Guiding future sampling efforts**

289 Field sample collections are time-consuming and expensive. By having an idea  
 290 which sample points need to be collected for increased origin estimation accuracy,  
 291 we can optimize future field collections. The isoscape variance estimates provided  
 292 by GPs can be used to guide future sampling efforts, which in turn will maximize  
 293 the performance of the model. This approach is known as *active learning* in the  
 294 machine learning literature. Here, we propose a strategy to minimize the error of  
 295 our isoscape estimates by carefully choosing future sampling locations.

296 Early attempts at efficient active learning in GPs involved collecting samples  
 297 at points with highest response variance or, equivalently, picking a set of points  
 298 that maximizes the entropy of responses [14]. Unfortunately, this approach tends  
 299 to recommend collecting samples on the boundaries of the study area, which is  
 300 wasteful as the newly collected samples improve isoscapes in a smaller fraction of  
 301 the study area than if they were placed away from the boundary. This motivated  
 302 researchers to propose several criteria for optimizing sampling [23, 36]. Here, we  
 303 adopt an approach similar to that of Guestrin *et al.* [23] with a few modifications  
 304 designed to address the large size of our spatial grid, which renders their original  
 305 method computationally intractable for our data set.

306 We seek to maximize the *average* reduction in predictive variance across our  
 307 study area that can be achieved by adding a sample to the training set. Let  $S$   
 308 be the set of sampled locations and  $G$  be the set of grid points. We define the  
 309 information gain (IG) associated with adding a new point  $\mathbf{x}^*$  to the training data  
 310 set as

$$IG(\mathbf{x}^*) = \sum_{\mathbf{x} \in G} [(\log(\sigma^2(\mathbf{x}|S)) - \log(\sigma^2(\mathbf{x}|S \cup \{\mathbf{x}^*\})))] \quad (8)$$

311 where the predictive variances are computed using Equation 3. The algorithm then  
 312 picks the point in the grid that yields the highest IG. Importantly, the predictive  
 313 variances depend only on the sampling locations and not on the sampled values,  
 314 so it is possible to sequentially propose multiple sampling points before collecting  
 315 the samples. Our method sequentially proposes sample collection points until a  
 316 user-specified number of samples is reached. We assume that samples can only

317 be collected in locations where at least one of the species is present. Thus, grid  
318 points that lie outside every species range are excluded from the procedure. Our  
319 active learning strategy requires repeatedly computing a large number of predic-  
320 tive variances for varying training sets. To reduce computation time, we randomly  
321 downsample our grid to 15000 points before running the analysis. In addition, we  
322 assume that the reduction in variance is negligible for grid points situated more than  
323 15 degrees away from  $x^*$  in longitude or more than 7.5 degrees in latitude.

## 324 **3 Results**

### 325 **3.1 Model accuracy and comparison**

326 Table 1 shows performance metrics for all the models on the test data set. In  
327 all settings, the plausible origin areas identified by our models consisted of points  
328 within an average distance of 520-870 kilometers from the true locations. Even  
329 with a relatively small training data set of 69 training samples, our model is able  
330 to exclude the vast majority of the study area as a possible source of the sample  
331 under consideration.

332 Incorporating species distribution information improves prediction performance  
333 for every model and every metric examined except the log-likelihood, which does  
334 not depend on the prior. Informative priors improve MPD by 16% to 35% and ASH  
335 by 15% to 57% with most improvement for the pure spatial model and least for  
336 the spatial+atmospheric model. The more informative density prior gives better  
337 accuracy than the range prior according to all the metrics. Predicted probability  
338 maps for a few test points are shown in Fig. 2 and 3.

339 The spatial-only GP model gives the closest predictions to the true location,  
340 except when a flat prior is used. In general, the spatial-only and the combined  
341 spatial+atmospheric model give similar results on all metrics and they both outper-  
342 form the atmospheric-only model in almost all settings. Somewhat surprisingly, the  
343 combined model does not outperform the spatial-only model. This might be due to  
344 the relatively small data set size.

345 The predictions of atmospheric GP models appear qualitatively different from  
346 those from the purely spatial GP. Atmospheric model predictions often emphasize  
347 geographical areas with distinct climate patterns, such as the Appalachia or the Gulf  
348 Coast. Unsurprisingly, the purely spatial GP identifies areas that are more spatially  
349 cohesive but do not share any obvious physical features.

### 350 **3.2 Guiding future sampling efforts**

351 We investigated the performance of our active learning strategy on the US oak  
352 data set. For the spatial-only model, we let our method propose  $n_s = 10$  new  
353 sampling locations to add to the training data set in the first cross-validation fold and  
354 computed the predictive variances before and after including the proposed locations.

355 The resulting isoscape standard deviation maps are shown in Figure 4. Our active  
356 learning strategy proposes sampling locations in currently undersampled regions with

model	prior	log L	MPD (km)	log-posterior	ASH (km <sup>2</sup> )
Spatial-only	flat	<b>-6.964</b>	809	-9.582	470000
Spatial-only	range	<b>-6.964</b>	600	-9.537	327000
Spatial-only	density	<b>-6.964</b>	<b>520</b>	-9.059	<b>203000</b>
Atmospheric-only	flat	-7.362	870	-9.972	576000
Atmospheric-only	range	-7.362	606	-9.797	450000
Atmospheric-only	density	-7.362	567	-9.428	311000
Atmospheric+Spatial	flat	-7.149	794	-9.518	382000
Atmospheric+Spatial	range	-7.149	627	-9.431	315000
Atmospheric+Spatial	density	-7.149	536	<b>-8.978</b>	213000

Table 1: Mean test set performance for all the models used in the study. Best values across all models are shown in bold. The Spatial-only GP combined with the density prior gives the highest predictive log-likelihood and log-posterior and the lowest MPD and ASH values for all priors used. The Spatial-only model outperforms the other models when range or density priors are used, while the Atmospheric+Spatial model performs best in terms of MPD and ASH when flat priors are used. The inclusion of species distribution information decreases MPD and ASH values for all models used.

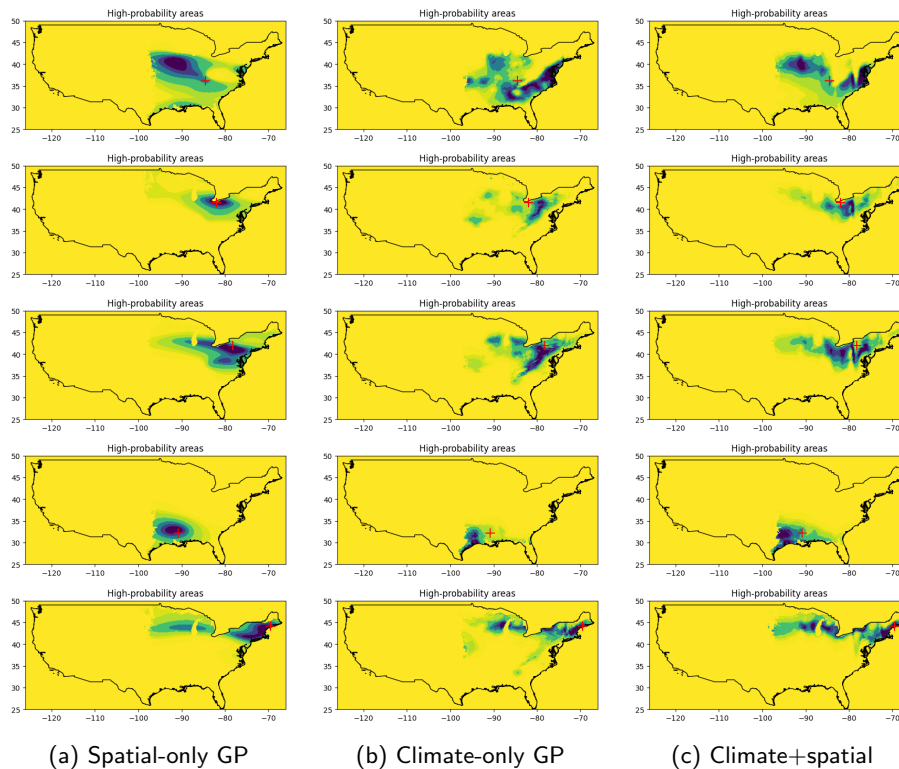


Figure 2: Spatial predictions from the three models for 5 points from the test set using the range prior. Darker shades denote areas with higher probability mass and the red cross indicates the actual location of the tree.

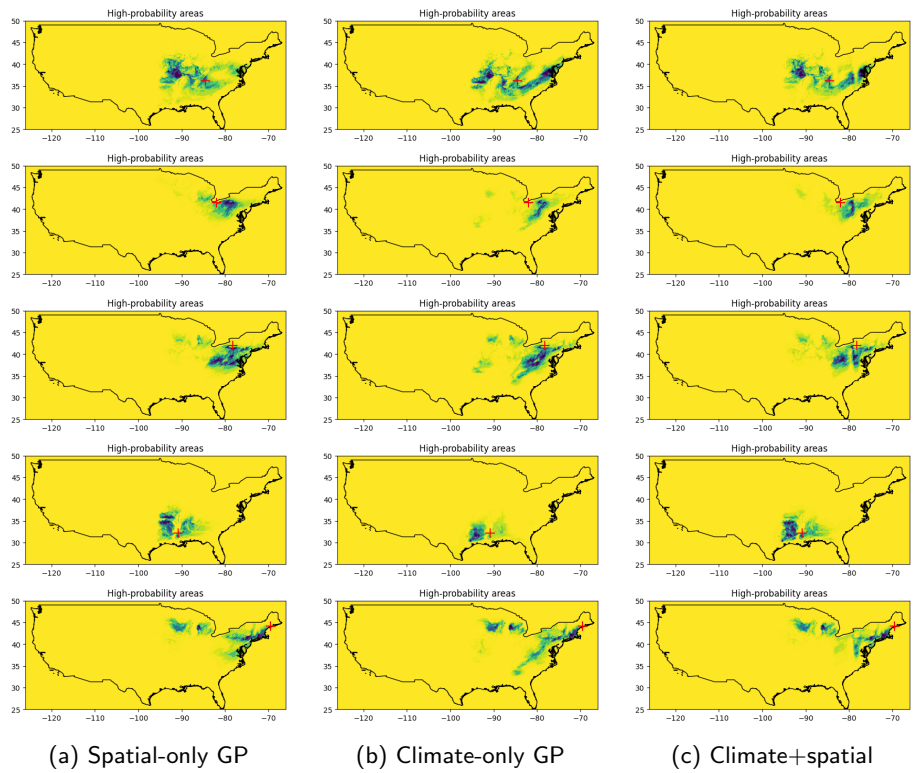


Figure 3: Spatial predictions from the three models for 5 points from the test set using the density prior. Darker shades denote areas with higher probability mass and the red cross indicates the actual location of the tree.

357 high predictive variance and sampling in those areas results in a visible improvement.  
 358 The highest decrease in predictive variance was observed for  $\delta^2\text{H}$  while the lowest  
 359 decrease was observed for  $\delta^{18}\text{C}$ . Most of the chosen locations are close to, but not  
 360 at the boundary of the allowed sampling area.

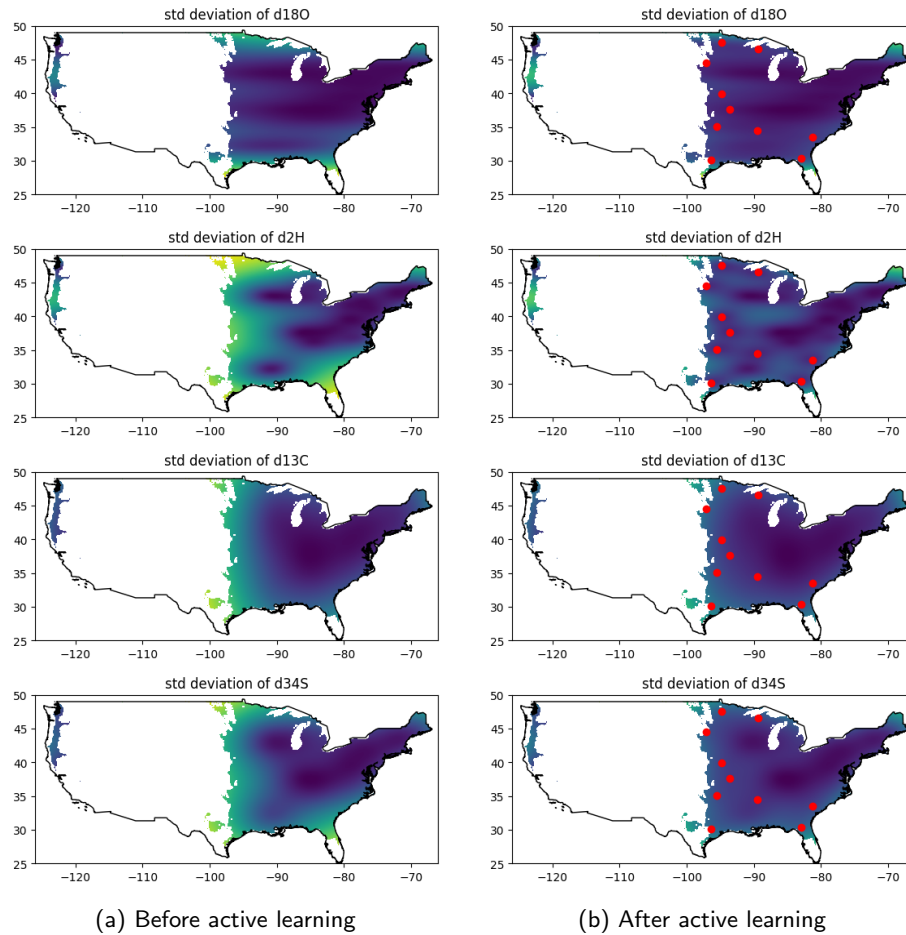


Figure 4: Maps showing predictive standard deviations for the four isotopes before and after adding  $n_s = 10$  sample locations proposed by our active learning method for the spatial-only model. Standard deviations are only shown within the allowed sampling area, which is the union of ranges for the species in our data set. The red dots show the proposed locations. Our method proposes locations in areas with high predictive variance, particularly for  $\delta^2\text{H}$  and  $\delta^{34}\text{S}$ . Adding the proposed locations leads to a marked reduction of variance in the neighboring areas.

361 To investigate the efficiency of our active learning procedure, we compared  
 362 isoscape variances resulting from active learning with those resulting from adding

363 the same number of points sampled randomly from the allowed sampling area. We  
364 generated  $n_r = 100$  such variance maps and compared the average variance (across  
365 the allowed sampling area) of those maps with the maps in Fig. 4. Fig.S1 shows the  
366 average predictive variances as a function of the number of points added for both  
367 random and active learning sampling strategies. We see that our active learning  
368 strategy results in substantially faster decrease in predictive variances. After adding  
369 10 samples, the reduction in variance achieved by our active learning method is  
370 between 64% ( $\delta^{13}C$ ) and 86% ( $\delta^{18}O$ ) greater than the average reduction achieved  
371 by the same number of random samples.

## 372 4 Discussion

### 373 4.1 Timber origin estimation

374 To halt illegal logging, to enforce timber regulations and to protect biodiversity in  
375 forested landscapes, we need to be able to accurately estimate the timber harvest lo-  
376 cation. Although several examples exist of applying SIRA for timber origin questions  
377 [22, 47, 27], these approaches do not take full advantage of (1) atmospheric and  
378 species distribution datasets available or (2) state-of-the-art probabilistic machine  
379 learning models. In this work we present a new computational pipeline which aims at  
380 taking advantage of both. The accuracy of our models depends on the specific mod-  
381 elling approach being used and the data sets incorporated. Using prior information  
382 about species distribution results in a considerable increase in accuracy regardless  
383 of which model is used by all metrics considered. The impact of adding species  
384 distribution data appears to be greater for the spatial-only model than models that  
385 use atmospheric information. This could be due to climate patterns influencing both  
386 species distributions (habitat suitability) and the values of the atmospheric variables  
387 that we incorporated in our models, which renders species distribution information  
388 more redundant once atmospheric variables have been included in the model.

389 Within timber tracing literature, our method bears the most resemblance to the  
390 work of Watkinson *et al.* [47], which uses linear regression to predict isoscapes based  
391 on atmospheric data. Their approach assumes a constant variance across the study  
392 area. In contrast, our method estimates the predictive variances based on the spatial  
393 covariance structure learned from the reference data, which enables us to translate  
394 differences in sampling density across regions into varying levels of confidence in  
395 isoscapes across space. Like Watkinson *et al.* [47], our method assumes a linear  
396 relationship between atmospheric predictors and isoscapes, but our GP formulation  
397 implicitly integrates over plausible values of regression parameters, which should lead  
398 to more robust predictions compared to standard linear regression. In addition, our  
399 approach makes use of species distribution data, which yields substantially improved  
400 predictions compared to uninformative priors. Finally, our approach enables us to  
401 propose locations for further sample collection that maximize the utility of the  
402 samples.

403 Estimating the spatial covariance structure has recently attracted attention in  
404 animal stable isotope studies. Ma *et al.* [35] recently proposed a method that uses

405 probabilistic precipitation isoscapes derived from a GP [13], which are then cali-  
406 brated to produce isoscapes for the species of interest. St. John Glew *et al.* [42]  
407 introduced a model combining spatial and environmental effects using a novel likeli-  
408 hood approximation for isoscape estimation, though the main focus of their work is  
409 isoscape modelling, not origin estimation. These approaches differ from ours in that  
410 1) they rely on Laplace approximations for isoscape estimation rather than exact  
411 likelihood maximization; 2) they use ordinary least-squares regression to account for  
412 atmospheric predictors, whereas our method uses a Bayesian approach via a linear  
413 covariance term; and 3) they do not aim to actively improve isoscapes through  
414 additional sampling. A common feature between these models and ours is using a  
415 grid to compute the posterior distribution of origins, which was first considered by  
416 Wunder [52].

417 Our current best performing model can estimate the origin of harvest location  
418 for *Quercus* species to 520 km across the (north-)east of the United States. Future  
419 field expeditions will lead to an improvement, especially if the identified priority  
420 locations are targeted (see 4.2). The presented model will be adapted to other  
421 use cases, with mainly a focus on tropical species on which the logging pressure is  
422 significant and which might be endangered.

## 423 **4.2 Guiding future collection efforts**

424 We expect that our models will be more accurate once more timber samples be-  
425 come available. The size of the current data set of wood samples available to this  
426 study ( $n=87$ ) is quite small relative to the area of contiguous United States, which  
427 inevitably results in large predictive variance in many areas. In addition to reducing  
428 uncertainty about undersampled areas, larger data sets (in the range of hundreds to  
429 thousands of samples collected from across the US) should also enable researchers  
430 to use more complex GP models, including models with heterogeneous noise [3], or  
431 deep GP models where the covariance function is modelled by a neural network [16].

432 Under the World Forest ID Programme [21], tens of thousands of tree samples  
433 are being collected globally, and are being analysed by different techniques, including  
434 SIRA. Our active learning approach can be used to inform future sample collection  
435 efforts and increase model accuracy that can be achieved within a fixed sampling  
436 budget. This will be especially important in tropical regions, where reaching sam-  
437 pling sites can be difficult, time intensive and expensive. A good sampling design  
438 can substantially improve model performance [12], and our method can be used to  
439 adapt sampling efforts as more data is analysed. Our current approach focuses on  
440 minimizing predictive variances without considering the impact of newly sampled  
441 points on model parameters. Extending our approach to *non-myopic* sampling [31],  
442 which considers the impact on model parameters, would constitute an interesting  
443 future research direction. Another avenue for improving our approach would be  
444 to augment our IG criterion to reflect the varying cost of collecting samples as a  
445 function of the time and financial cost of reaching the desired sampling location.



## 446 **5 Conclusion**

447 The accurate estimation of geographic origin of globally traded wood products  
448 is a critical step in combating illegal logging and associated trade, by supporting  
449 authorities' ability to verify claims made by traders at any supply chain node. In this  
450 work we presented a novel analytical pipeline that brings together and incorporates  
451 multiple data types and algorithms. This methodology is able to accurately predict  
452 timber product origin and can be used to optimize future field sampling to further  
453 increase accuracy and precision. We hope that this work will inspire more efforts  
454 to expand reference collections of wood samples, such as under the auspices of the  
455 World Forest ID Programme (<https://worldforestid.org/>), and that governments  
456 and companies will more routinely use the technological tools at their disposal to  
457 have more oversight over their supply chains and promote a more sustainable use  
458 of natural resources.

## 459 **6 Conflict of interest statement**

460 The authors declare that they have no conflicts of interest.

## 461 **7 Author contributions**

462 Jakub Truszkowski, Victor Deklerck and Alexandre Antonelli jointly conceived the  
463 project. Jakub Truszkowski designed the methodology, implemented the algorithms,  
464 performed most of the analyses and wrote parts of the manuscript. Roi Maor  
465 selected and pre-processed some of the data sets, designed the main figure and wrote  
466 parts of the manuscript. Raquib Bin Yousuf, Subhodip Biswas, Naren Ramakrishnan  
467 and John Simeone wrote some of the software code and performed initial data  
468 explorations. Scot McQueen selected the data sets for species distribution models.  
469 Caspar Chater, Peter Gasson, Marigold Norman and Jade Saunders wrote parts  
470 of the manuscript. Victor Deklerck directed the project and wrote parts of the  
471 manuscript. All authors gave final approval for submission.

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## Supplementary material

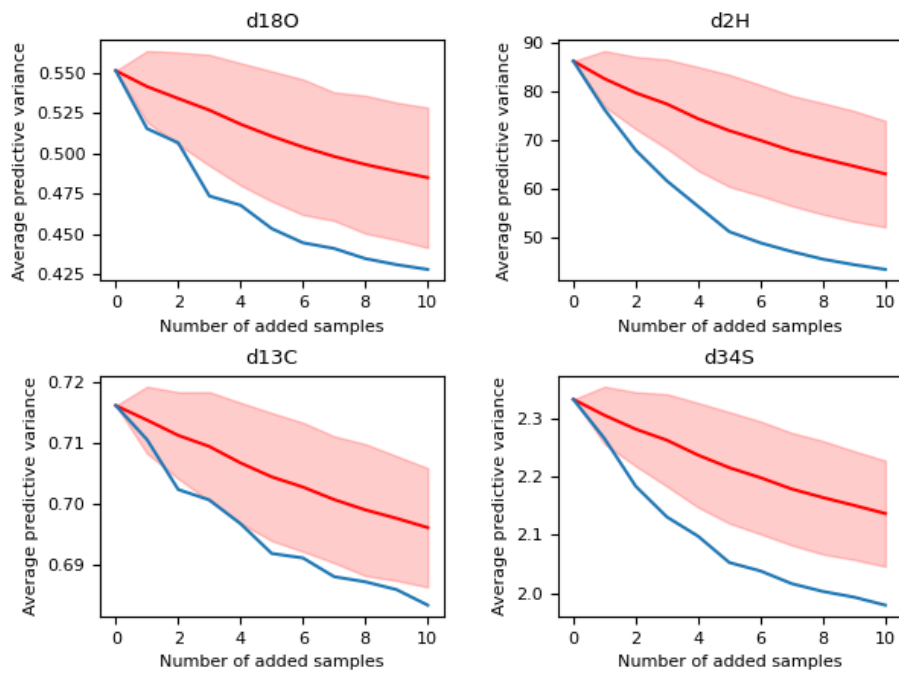


Figure S1: Average predictive variances for  $\delta^{18}O, \delta^2H, \delta^{13}C$  and  $\delta^{34}S$  as a function of the number of samples added to the base training data set; blue - active learning strategy; red - random sampling (shaded area denotes values within two standard deviations of the mean across  $n_r = 100$  simulations).