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Title. Revisiting sigmoid functions in macro-ecology: mathematical definition and associated ecological properties.

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Abstract.

Introduction: Defining mathematical terms and objects is a constant issue in ecology; often definitions are absent, erroneous or imprecise.

Lack of a clear definition: Through a bibliographic review, we show that this problem appears in macro-ecology (biogeography and community ecology) where the lack of definition for the sigmoid class of functions results in difficulties of interpretation and communication.

Proposal of a clear definition: In order to solve this problem and to help harmonize papers that use sigmoid functions in ecology, herein we propose a comprehensive definition of these mathematical objects. In addition, to facilitate their use, we classified the functions often used in the ecological literature, specifying the constraints on the parameters for the function to be defined and the curve shape to be sigmoidal.

Ecological justifications: Finally, we interpreted the different properties of the functions induced by the definition through ecological considerations in order to support and explain the interest of such functions in macro-ecology.

Keywords. Biogeography; Nonlinear relationships; Curve shape; Species-area relationship; Species-resource relationship

MAIN TEXT.

1. Introduction

Using well-defined and uniform terms is a key point in science. Yet, one of the main criticisms that can be made in the science of ecology is the poor definition of terms and concepts or inconstant use within its community (Herrando-Pérez, Brook, & Bradshaw, 2017; Pickett, Kolasa, & Jones, 2007). Many concepts do not yet have a consensual definition, and communication is therefore difficult. Furthermore, loosely defined concepts can cause not only an unstable expression of a scientific concept, but can also result in inconsistencies within the concept itself (e.g. Gosselin 2001). This is why many articles have tried to highlight this problem and to establish precise definitions - i.e. “ecological niche” (Araújo & Guisan, 2006) or “ecological function” (Jax, 2005). However, the problem is not restricted to ecological concepts; it also concerns ecological domains (i.e. “ecological engineering”, cf. Gosselin, 2008) or certain terms and concepts used in ecology and borrowed from other sciences. This is the case for mathematical terms as, for example, the notions of extinction or demographic stochasticity (clarified in Gosselin, 1997 or Lebreton, Gosselin, & Niel, 2007). Reflections on mathematical definitions make it possible to conceptualize possibilities not yet foreseen (e.g. the importance of dependence between individuals within demographic stochasticity or uncertainty in McCarthy, Franklin, & Burgman, 1994). In the present paper, we deal with the term "sigmoid" and propose a definition to overcome imprecision problems. Hereafter, we will call “sigmoid” the curve shape that can be represented by different functions, and the “sigmoid class of functions”, the class that contains these functions.

Ecologists often study relationships between two ecological variables (e.g. a biodiversity metric as a function of an environmental variable/predictor). Although, the most often considered form of these relationships is linear, nonlinear forms have also been used (power, exponential etc.), including sigmoidal forms. In ecology, sigmoidal relationships are generally implicitly used in binomial regressions. However, in the field of macro-ecology and, in particular, in the study of species-area relationships (SARs), explicit sigmoidal forms occur fairly often. Indeed, a sigmoidal shape is very likely to emerge when species richness is related to the area in which the species were sampled (Preston, 1962). Many sigmoidal functions have been developed and used in a SAR context; however, they can also be applied to the study of relationships between biodiversity and a resource gradient other than available habitat area (species-resource relationships, or SReRs). Furthermore, the sigmoidal form of a relationship may prove useful for decision-making in forest or conservation management. Indeed, certain characteristics of the curve can provide management targets like the inflection point or the upper asymptote (Ranius & Jonsson, 2007).

In recent years, numerous articles have been published which review the use of nonlinear functions, including sigmoids, in the field of biogeography and especially for SAR-type relationships (Dengler, 2009; Tjørve, 2003, 2009; Williams, Lamont, & Henstridge, 2009). Unfortunately, no clear definition of the term sigmoid was provided in these publications.

Despite the frequent use of sigmoidal functions, in most cases, there is no proper, accessible definition of what exactly is meant by a “sigmoidal” shape. Classically defined as an S-shape, the sigmoid may seem clear and that is the reason why it is so rarely defined. Yet, the precise characteristics of these curves are not formalized or made explicit. This absence of a clear definition results in a lack of harmonization between papers in ecology, and inconsistencies between articles, or even within one and the same article can ensue. For example, although most definitions include the presence of an upper asymptote (e.g. Tjørve, 2003; Veech, 2000), Mashayekhi, MacPherson, & Gras (2014) define one of their functions (Persistance2) as sigmoidal though it does not have an upper asymptote; this contradicts the general idea of a sigmoid. There is therefore a need to more explicitly define the sigmoidal class of shapes.

Our first goal is to assess the use of the term sigmoid in biogeography studies and highlight the lack of a clear definition. Then, we propose a definition of the term so that its use in the literature is harmonized and no longer confusing. Finally, we justify the definition in relation with ecological theory and we highlight the implications and advantages of this new definition. The two underlying questions are: what characteristics should sigmoid curves exhibit? What functions can be included in the sigmoid class?

2. An obvious lack of a clear definition

The word "sigmoid", composed of "sigma" and "eidos" (*sigmoeidḗs* in ancient Greek), means something that has the form of the capital letter sigma (Σ). The term sigmoid is more generally defined as an S-shaped curve. Yet these descriptions, in addition to being vague, are not accurate since the form of an S (or a Σ) is impossible in mathematical curves described by functions. In fact, if we apply an S form to mathematical curves, we notice that we obtain two or three values of f(x) for one x, which is impossible according to the very definition of a function. Moreover, the representation of an S-shaped curve excludes forms that should logically be part of sigmoid curves such as decreasing sigmoid curves.

Given this intrinsic difficulty with the notion of sigmoid, we investigated how authors in ecology have used and define this term. We selected an ecological domain where sigmoid functions are often explicitly used to describe relationships: biogeography with species-area relationships (conventionally abbreviated as SARs) and species response to ecological gradients within species-resource relationships (abbreviated here as SReRs).

In June 2017, we searched articles accessible via Scopus for a combination of keywords related to sigmoid curves and to the above-mentioned domains of ecology. In some papers, the term sigmoid is not mentioned even if sigmoidal functions are used. Our sigmoid keywords therefore covered a wide range of meanings: we searched for “sigmoid” OR “nonlinear” OR “logistic”. We combined these keywords with other keywords related to the targeted ecological aspect: “SAR” OR “species-area” OR “species-resource” or “biogeography”.

Among the search results, we selected the papers where, according to the title and the abstract, the authors either used sigmoid functions or were interested in a sigmoidal form of relationship.

The 36 selected papers (see Appendix S1 in Supporting Information for references) were sorted according to the three possibilities: (i) papers that did not use a sigmoid family term; (ii) papers that used a sigmoid family term but did not define it; and (iii) papers that either entirely or partly defined the sigmoid.

As Table S1.1 (see Appendix S1 in Supporting Information) shows, sometimes authors use sigmoidal function without ever specifically referring to the sigmoid family (13.9 %), but this number may be underestimated due to the difficulty of finding such papers. Most of the time, the authors use a word from the sigmoid family to define their functions (“sigmoid” or “sigmoidal”), but they do not define what they mean by these terms (72.2 %). What is quite surprising is that some authors create new sigmoid functions and state that their functions have a sigmoidal form, but they never evoke the characteristics implied by this form and included in their function (e.g. Kobayashi, 1976).

Finally, only a few authors take the time to define a sigmoid (13.9%), but typically the definition is fragmented or the functions imprecisely characterized, thus giving the impression of an incomplete definition. Sometimes definitions can even be confusing or contradictory.

Preston (1962) proposed a descriptive definition of the shape of the sigmoid curve, which gives us an idea of the form but without specifying its properties: “it began at a low slope, steepened considerably, and then became less steep”.

Tjørve (2003; 2009) does not give a complete definition of the sigmoid curve, but does mention some of its characteristics when describing the functions he considers in his study. In Tjørve’s papers (2003; 2009), the characteristics common to all sigmoid functions include: (i) the presence of an upper asymptote; (ii) a lower j-shape (probably implying a lower asymptote); and (iii) the presence of an inflection point. Tjørve (2003; 2009) also mentions two characteristics which vary among different sigmoid functions: symmetry around the inflection point, which may or may not exist; and the positions of the inflection point and of the asymptote.

Furthermore, in addition to being incomplete, these "definitions" may present other problems that impede understanding. This is the case when mathematical terms characterizing a mathematical object, here the sigmoid curve, are incorrectly used. For example, some authors erroneously define their sigmoid functions as “convex” (e.g. Gentile & Argano, 2005; Tjørve, 2003, 2009). Indeed, in mathematics, a curve/function is “convex” if, for any two points A and B of the curve, the segment [AB] is entirely situated above the curve. Conversely, a concave function is the opposite of a convex function (f is concave if and only if –f is convex). A concave curve is therefore a curve for which, for any two points A and B of the curve, the segment [AB] lies entirely below the curve. Yet, some studies make no distinction between the two curves and use “convex” for both convex and concave forms (Tjørve 2012), then distinguish them with the mentions “downward” or “upward”. Usually, given the properties attributed to the curves defined as convex, the term concave, rather than convex, is clearly the correct term. For example, what Tjørve (2009) described as a "constantly decelerating" convex curve is actually concave, and what he defined as a "J-shape" would correspond to the convex part of the sigmoid curve. This error is common since convex and concave shapes are often respectively described as a hump and a hollow (from the definition of a convex set), which can lead to confusion. Therefore, though the study is very interesting, the discourse is blurred by terms that are confusing (as also pointed out by Dengler, 2009). Consequently, we suggest using mathematical definitions and terms, so that all researchers will refer to the same definition of sigmoid curves.

If one moves away from the literature in ecology, we find that few definitions are easily accessible even in statistical literature. Hill and Lewicki (2006) propose one such definition in their glossary: a sigmoid function is “an S-shape curve, with a near-linear central response and saturating limits” (p724). This definition, which includes the notion of an S-shape discussed above, make it possible to understand the general shape and to accept different forms, but they are not necessarily very clear on which forms are included or excluded when we speak of a sigmoid, and the properties of the functions are not precise. Menon, Mehrotra, Mohan, & Ranka (1996) also start by defining the sigmoid curves as S-shaped; then the authors define two sub-classes of sigmoids: (i) simple sigmoids are “odd, asymptotically bounded, completely monotone functions in one variable”; and (ii) hyperbolic sigmoids are “a proper subset of simple sigmoids and a natural generalization of the hyperbolic tangent”. Although detailed, notably when characterizing certain functions, they seem to have forgotten to mention the monotonic character that such a function should have. Moreover, the two defined classes do not integrate all the possible sigmoidal forms; for example, “odd” excludes asymmetric curves and curves that do not intersect the origin. Finally, concerning definitions easily accessible to the general public, dictionaries are not of much better help since, for example, the French dictionary *Le Petit Robert* defines a sigmoid as a "sinuous curve with two waves of growth separated by a point of inflection" (translated from French), a very confusing definition (“Le Petit Robert : Sigmoïde,” 2017).

To sum up, very few definitions of sigmoid functions are available in the ecological literature, and they are usually vague, or based on only certain characteristics, or can even contain errors. Therefore, it seems clear that the lack of a time-honored definition, or the use of unstable definitions, can lead to difficulties in producing studies and articles. This is particularly true for bibliographic research and for young researchers and students (PhD or Masters students) who are still forging their knowledge (Herrando-Pérez et al., 2017). It can also sometimes distort communication among collaborators. For example, within our own research group, differences of wording regarding the properties of different curves have surfaced, with misunderstandings of what is meant by “convex” and “concave”.

3. Proposal of a clear definition

Although the definition on Wikipedia is globally correct (“Wikipedia - Sigmoid function” n.d.), this website cannot be used as a reference since the page can be modified at any time, making the definition unstable. We have therefore decided to propose a definition, which is stable, understandable for ecologists, and as complete as possible (including as many cases as possible) in this paper. For this purpose, we first looked at the characteristics of the functions used in the literature (cf. Table 1).

Ultimately, a sigmoid curve is a curve described by a real-valued, univariate function (a function f of a unique real-valued variable x that takes real values y=f(x)), defined over the whole set of real numbers, and which is continuous, infinitely differentiable, monotonic (always either increases or decreases), has at least one inflection point and is bounded on the Y-axis. The term “inflection point” refers to the point where the curve shifts in convexity: from convex to concave or vice versa. The change in slope is continuous and should therefore be distinguished from the term “breakpoint” used by ecologists, which, although we did not find a precise mathematical definition, seems to refer to a non-continuous function (e.g. in change point models, Muggeo, 2003; Quandt, 1958).

Its inherent features imply that the sigmoid curve: (i) has an upper and a lower asymptote if (x) varies over the set of real numbers; (ii) can increase (starting with the lower asymptote and finishing with the upper asymptote, with a positive slope between them) or decrease (starting with the upper asymptote and finishing with the lower asymptote, with a negative slope between them, Fig. 1.b); and (iii) can be symmetrical or not around the inflection point or points (Fig. 1.c).

We extend the definition given above to two other cases where the explanatory variable (x) is defined on the set of real positive numbers ($x \geq 0$) and: (i) f(x) is a function of (x) over the entire set of real numbers and has a sigmoid curve; or (ii) the above definition for the sigmoid curve applies to f(x) as a function of ($x \geq 0$) except for the requirement that f(x) defined over the entire set of real numbers. Indeed, in island biogeography, the function never occurs with negative x-values (since area cannot be negative). In this case, the sigmoid curve has only one of the two asymptotes. Even after extension, however, our definition does not include the case where (x) is bounded on both sides and therefore possesses neither of the two asymptotes (see He & Legendre, 2002). Note that f(x) as a function of (x) can have a sigmoidal form without f(x) as a function of log(x) or f(exp(x)) as a function of (x) being true, and vice versa.

The class of sigmoid functions includes the functions which, for the given parameters, meet the above definition. The same function may or may not belong to the sigmoid class depending on the value of its parameters. To return to a previous example, the Chapman-Richards function belongs to the sigmoid class if $c>1$. For other values of c, the function does not belong to the sigmoid class.

The sigmoid class can be divided into two sub-classes: (i) simple sigmoids, containing the functions that give curve shapes with a single inflection point, and (ii) multiple sigmoids containing functions that give curve shapes with several inflection points (i.e. a double sigmoid could fit the phenomenon described in Figure 6 in Lomolino, 2000). There must always be an odd number of inflection points in order to keep the two asymptotes on the Y-axis.

Based on the definition of the sigmoid class that we propose above, we inventoried the classical SAR or SReR functions selected from the review we conducted that belong to the simple sigmoid class, at least for some parameter values (cf. Table 1). We also described their characteristics, placing special emphasis on the constraints imposed on the parameter values or explanatory variable to ensure that the function is mathematically defined, is suitable in macro-ecology and does indeed have a sigmoidal form. We also provide the coordinates of the inflexion point, so that readers can distinguish between functions that are sigmoidal only when the whole set of real values for the explanatory variable is considered (i.e. functions with a negative abscissa value of the inflexion point) and those that are sigmoidal even when the abscissa values are positive.

Another class of functions that is close to the sigmoid class is the class of inverse sigmoid functions. These are bounded on the X-axis and do not have an asymptote over the Y-axis (Fig. 1.d). These functions have no biological reality in SReR and SAR and are not members of the sigmoid class as we define it. Other curves defined as sigmoid by some authors do not meet the requirements of our definition either, for example, “sigmoid curves […] free of upper asymptotes” (Tjørve, 2012).

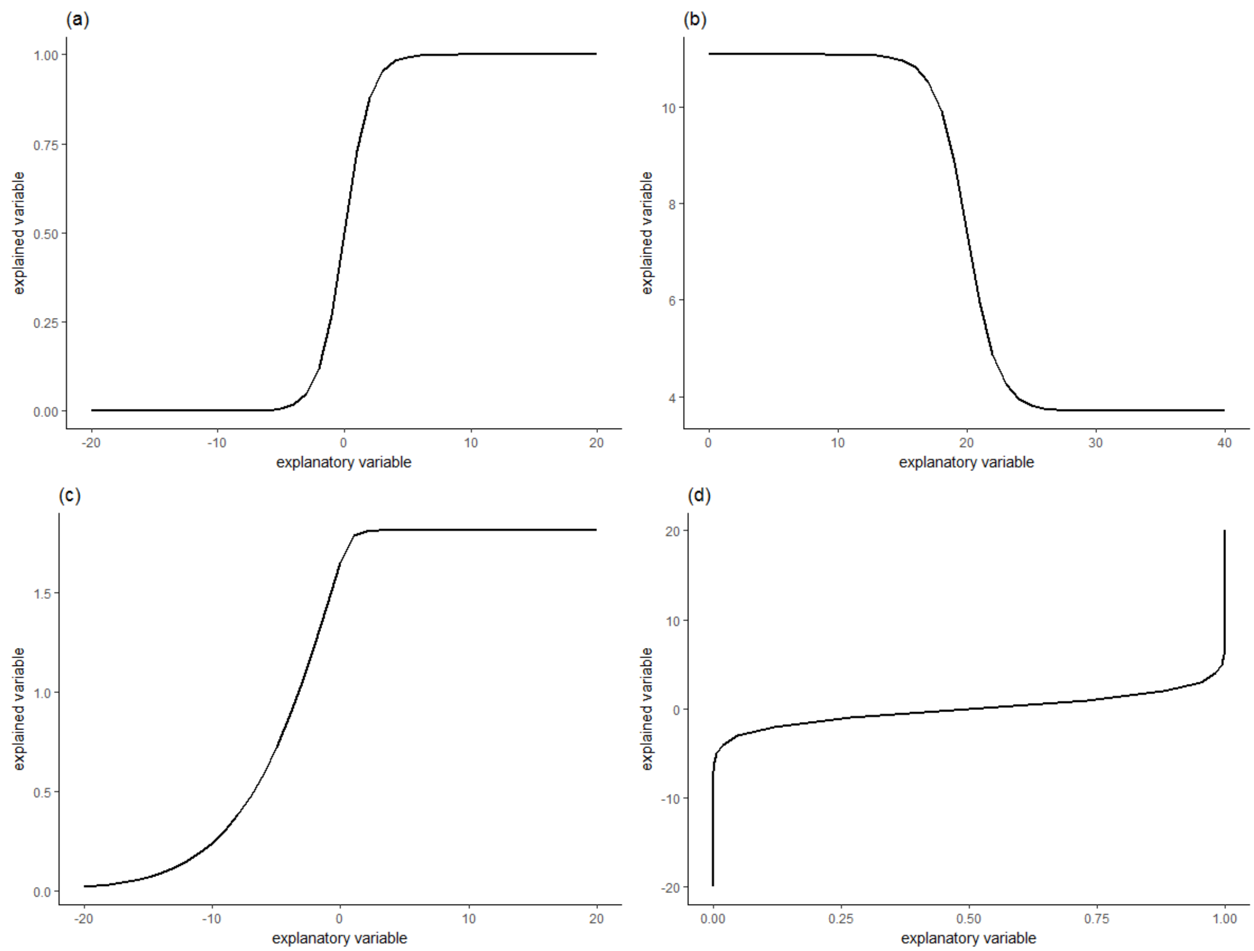
Table 1: Some characteristics of sigmoidal functions present in the SAR and SReR literature (see Appendix S1 in Supporting Information for references).

	Formula	Constraints on parameters to be defined and relevant to macro-ecology	Further constraints required to be in the sigmoid class	Inflection point	Symmetry around the inflexion point	Lower asymptote	Intersects origin	Direction of the relationship
Common logistic	f(x) = a/(1 + exp(-b*x + c))	a>0	/	x=c/b y=a/2 In other terms y=50% of the upper asymptote	Point symmetry	Zero	No	Increasing (if b>0) or decreasing (if b<0)
Gompertz	f(x) = a*exp(-exp(-b*x+c))	a>0	/	x=c/b y=exp(-1)*a In other terms y=36.8% of the upper asymptote	Asymmetric	Zero	No	Increasing (if b>0) or decreasing (if b <0)
Extreme value	f(x) = a*(1-exp(-exp(b*x+c)))	a>0	/	x=-c/b y=[1-exp(-1)]*a In other terms y=63.2% of the upper asymptote	Asymmetric	Zero	No	Increasing (if b >0) or decreasing (if b<0)
Champan-Richards	f(x) = a*(1-exp(-b*x))^c	a>0, x≥0, c>0, b>0	c>1	x= log(c)/b y= a*(1-1/c)^c)	Asymmetric	/ (irrelevant since x is non-negative)	Yes	Only increasing

Cumulative Weibull distribution	f(x) = a*(1-exp(-b*(x^c)))	a>0, b>0, x≥0	c<0 or c>1	x=((c-1)/(b*c))^(1/c) y=a*(1-exp(-1+1/c))	Asymmetric	/ (irrelevant since x is non-negative)	Yes (if c>0)	Increasing (if c>0) or decreasing (if c<0)
Morgan-Mercer-Flodin (MMF)	f(x) = a*(x^c)/(b+(x^c))	a>0, b>0, x≥0 (with f(0)=a if c<0 to be continuous)	c>1 or c<(-1)	x=((c-1)*b/(c+1))^(1/c) y=a*(1/2-1/(2*c))	Asymmetric	/ (irrelevant since x is non-negative)	Yes	Increasing (if c>0) or decreasing (if c<0)
Cumulative beta-P distribution	f(x) = a*(1-(1+(x/c)^d)^(-b))	a>0, x≥0, c>0, b>0	d>1 or d<(-1/b)	x=c*((-d+1)/(-b*d-1))^(1/d) y=a*(1-(1+(-d+1)/(-b*d-1))^(-b))	Asymmetric	/ (irrelevant since x is non-negative)	Yes (if increasing, d>0)	Increasing (if d>0) or decreasing (if d<0)

Note that models II and III in Huisman, Olff & Fresco (1993), denoted as f(x)=M(1/(1+exp(a+b*x))) and f(x)=M*(1/(1+exp(a+b*x)))*(1/(1+exp(c))), are particular cases of the Common Logistic Function with, respectively, parameter (a) not estimated, and with parameter (a) estimated but with a given maximum value. The Archibald Logistic Function, denoted as f(x)=a/(b+c^x), is equivalent to the Common Logistic Function with (b), (c) and (a) in the Common Logistic Function, respectively equal to (-log(c)), (-log(b)), (a/b) in the Archibald Logistic Function. The He-Legendre Function, denoted as f(x)=a/(b+(x^(-c))), is equivalent to the Morgan-Mercer-Flodin Function with (a) and (b) of the MMF respectively equal to (a/b) and(1/b) in the He-Legendre Function. The type III Holling function, denoted as* $f(x)=ax^2/(b^2 + x^2)$*, is equivalent to the MMF, with (c) and (b) in the MMF respectively equal to (2) and (b^2) in the Holling III Function.*

Figure 1: Some possible forms of sigmoids and inverse sigmoids. (a) Simple logistic function, (b) decreasing sigmoid, (c) asymmetric increasing sigmoid, and (d) increasing inverse sigmoid.



4. Ecological justifications and implications of sigmoid curve characteristics

Although some characteristics of the sigmoid definition are justified mainly by mathematical considerations, many can be related to ecological hypotheses or considerations. First, the presence of an inflection point can be related to the following statement by Lomolino (2000) when he describes the phenomenon underlying the use of sigmoid curves in SARs: “with richness remaining relatively low and apparently independent of area for the smaller islands, increasing rapidly to rise through an inflection point for islands of intermediate size, and then asymptotically approaching, or leveling off at the richness of the species pool for the largest islands”. Of course, many other fields of ecology are interested in models that can depict such phenomena (e.g. ecophysiology; Paine et al., 2012). Another field where sigmoid curves could be useful is the field of ecosystem functioning-biodiversity relationships, where curves adopting such patterns seem frequent (Cardinale et al., 2012). The continuity and differentiability of the curve are related to the existence of an inflection point, and allow us to clearly relate the curve to a mathematical function, that is, to speak of the convexity or concavity of the curve. Continuity and differentiability also allow us to formulate hypotheses not only on the mean value of the response variable, but also on the speed (first derivative) or acceleration (second derivative) of the relationship between the response variable and the gradient being studied.

The pattern depicted by Lomolino for SARs might have led us to define sigmoid curves only as increasing curves. Yet we expect that in some areas of ecology, the reversed situation might occur and that such patterns would indeed fall into the domain of the sigmoid curve. For example, a decreasing sigmoid was considered in species-isolation relationships by Hachich et al. (2015). More generally in ecology, the decreasing sigmoidal curve can be used in the case where the gradient studied has a negative effect on the response variable (e.g. Morante-Filho, Faria, Mariano-Neto, & Rhodes, 2015).

Second, the existence of asymptotes is also very much related to considerations from ecology. The upper asymptote, implying a threshold above which the mean of the response variable (y) cannot go, theoretically reflects the Liebig law of the minimum in ecophysiology and ecology (Austin, 2007; Paris, 1992). In this case, the limiting factor would first be the predictor studied, and an increase in this limiting factor would lead to an increase in the explained variable. Then, upon reaching the asymptote, the predictor would not be limiting anymore but rather another, unmeasured, environmental factor would be involved, which prevents the explained variable from increasing any further. Inversely, the presence of a lower asymptote implies that the mean of the response variable cannot be lower than this asymptote. The existence of such an asymptote can often be related to the conjunction of the monotonic relationship and of the nature of the variable itself: when the variable is non-negative, the values of the mean cannot be below zero. In studies focusing on the response of a single species, the lower asymptote is therefore usually zero (e.g. Huisman et al., 1993). However, when studying community response, often a lack of a resources does not necessarily imply a total loss of species richness (for example, when species are mobile). In such cases, a logistic function where f(x) is a function of log(x), whose lower asymptote is located at zero (y = 0), is not actually adapted (Godeau et al. In Prep.).

The third component of our definition is asymmetry of the curve. Symmetric sigmoid curves, like the logistic function, are widely used, but more for their ease of modelling than for their underlying ecological theory. Indeed, for bell-shaped curves, Austin (1976) stated: “there is no a priori reason to assume that organisms' responses should follow such a symmetrical curve”. Different phenomena can explain asymmetrical curves (Austin 1990 and Austin & Gaywood 1994 for phyto-ecology) and theoretically supported asymmetry can also appear with sigmoidal curves (plants with differing initial and final rates of injury in response to temperature stress; c.f. Lim, Arora, & Townsend, 1998).

Having clear definitions makes it possible to more clearly reflect on the underlying concepts and theories implied by the functions available, and to visualize the most appropriate form of curve to adopt according to the ecological context. After defining and reflecting on the lower asymptote and asymmetry, the researcher naturally questions the choice of link function in the context of binomial logistic regressions. Classically, users of such tools choose canonical link functions such as the logit or the probit function. These two functions belong to the sigmoid class but they are symmetric around the inflection point and they have pre-specified minimum and maximum asymptotes (respectively 0.0 and 1.0). However, the inherent properties of such link functions could have strong ecological limitations, which would restrict their use in some cases. For example, having a maximum of 1.0 (meaning almost sure presence) along the gradient does not reflect biological situations where, even if local habitat conditions are optimal for the organism, the organism could be absent (e.g. due to dispersal limitation inside a metapopulation; Hanski & Gilpin, 1997). Along the same lines, sigmoid and logistic functions are sometimes confused with each other, whereas the latter is nothing more than a particular type of sigmoid (e.g. Hunsicker et al., 2015). Such confusion may prevent researchers from considering other families of functions that fall into the sigmoid class without being logistic.

More generally, the shape of the curve must be well integrated in order to properly interpret the results. As put forward by Fattorini, Maurizi, & Giulio (2012), Medellín & Soberón (1999) used a sigmoid model on their data, and then, in order to ensure fit with a logarithmic model, they chose to exclude some of the data corresponding to the first part of the sigmoid curve (where the slope is smaller). Fattorini, Maurizi, & Giulio (2012) point out that Medellín & Soberón (1999) should not have manipulated the data and should have retained a model that fit the entire dataset, the data represented by the first part of the curve being just as important from an ecological point of view as the data represented by the rest of the curve. In fact, the first part of the curve could reflect various ecological mechanisms that deserve to be studied such as – to name but two – sampling problems or biological functions in action (i.e. limiting factors, exclusions, etc.). Through this example, it becomes obvious that, if the sigmoid curve shape and its implications are not acknowledged or defined well enough in the mind of the ecologist, he or she may end up missing important patterns or making wrong assumptions.

Having a well-established definition of the sigmoid curve and understanding the constraints imposed on the parameter values of the functions which produce sigmoid curves allow us to better apprehend under which conditions a sigmoid function is adapted when one wishes to apply it to a dataset. For example, in the case of the Chapman-Richards function, the curve obtained will be of sigmoid shape only when ($c>1$). For values of (c) that do not satisfy this condition, the curve will not be of sigmoid shape. A related issue concerns the constraints imposed on the values of (x), which are most often unstated. To keep the same example, the Chapman-Richards function is not defined for ($x<0$) (cf. Table 1); the function is therefore not relevant in cases where a sigmoidal form of relationship is applied to a dataset where (x) can be negative (e.g. where (x) is a temperature in degrees Celsius or a single latent resource axis). Another, more extreme, example combines these two limitations: the persistence2 function. In fact, this function is sigmoid only if ($x>0$), ($b=0$) and ($c>0$).

5. Conclusion and perspectives

Our literature review points out the lack of a clear, stable, universally accepted definition of the sigmoid class of functions in ecology. Some aspects of sigmoid curves are typically ignored (symmetry, direction of the relation, etc.). We also found cases of misuse of convexity to define a curve or a function.

As Jeremy Fox stated “words are imprecise, and so purely verbal models and verbal arguments often are ambiguous or even invalid, even if apparently supported by empirical data (like Elton’s verbal arguments about why diversity and complexity beget stability). Mathematics has the virtue of forcing precise definitions of terms, precise and complete specification of assumptions, and rigorous derivation of conclusions” (Fox, 2011). It is therefore unfortunate to accept vague verbal definitions (such as “S-shape” or “J-shape”) when one is using a term derived from mathematics.

That is why we have proposed a definition that we hope will allow for better harmonization of what is meant by the term “sigmoid” when describing a curve or a function. In addition to clearly formulating the concept, our definition allows various functions to be united under the same banner (sigmoid class, presented in Table 1). This definition also excludes some functions that were previously considered to belong to the sigmoid family and which, in our opinion, should not be defined as such (sigmoid without an upper asymptote or inverse-sigmoid).

This new definition will quite naturally reveal the lack of some other functional shapes to fully represent the sigmoid class. In a future paper, we aim to develop a sigmoid function that incorporates the characteristics retained in this paper and is applicable to an SReR context. Such development of the sigmoid class might be of more general use in ecology, e.g. by broadening the scope of possibilities in binomial logistic regressions.

Finally, we hope that in future papers, authors who define a new sigmoid function, or use an already existing one, will take the time to specify the properties of the function and to clearly mention their implications and/or justifications in ecological terms.

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SUPPORTING INFORMATION.

Table S1.1: Papers in the SARs and SReRs domains that use functions with a sigmoidal form or that discuss about sigmoidal relationships, with precision about their use of a term.

Article reference	**Use "sigmoid" or "sigmoidal" word in the article**	**Define or describe sigmoid**
Bolgovics et al. 2016	YES	NO
Boomsma et al. 1987	YES	NO
Burbidge et al. 1996	NO	NO
Connor & McCoy 2001	YES	NO
Dengler 2009	YES	NO
Fattorini 2006a	YES	NO
Fattorini 2006b	YES	NO
Fattorini et al. 2012	YES	NO
Gao et al. 2016	YES	NO
Gentile et al. 2005	YES	NO
Hachich et al. 2015	NO	NO
He & Lengendre 1996	NO	NO
He & Lengendre 2002	YES	NO
Huisman et al. 1993	NO	NO
Kilburn 1963	YES	NO
Kobayashi 1976	YES	NO

Lomolino 2000a	YES	NO
Mashayekhi et al. 2014	YES	NO
Monteil et al. 2004	YES	NO
Natuhara and Imai 1999	YES	NO
Oksanen & Michin 2002	NO	NO
Panitsa et al. 2006	YES	NO
Preston 1962	YES	YES
Simaiakis et al. 2012	YES	NO
Stiles et al. 2007	YES	PARTLY
Tjørve 2003	YES	YES
Tjørve 2009	YES	YES
Tjørve 2012	YES	NO
Tjørve and Tjørve 2011	YES	NO
Tjørve and Turner 2009	YES	NO
Tjørve et al. 2008	YES	NO
Triantis et al. 2012	YES	NO
Turner & Tjorve 2005	YES	NO
Veech 2000	YES	PARTLY
Williams 1995	YES	NO
Williams et al. 2009	YES	NO
Total number : 36	Number of YES : 31	Number of YES or PARTLY : 5

"NO" in the second column is for papers that did not use a sigmoid family term. "YES" in the second column and "NO" in the third is for papers that used a sigmoid family term but did not

define. “YES” in the second column and “YES” or “PARTLY” in the third is for papers that either entirely or partly defined the sigmoid.

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