

1 **Toward a metabolic theory of life history**

2
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14 15 **Significance**

16 Data and theory reveal how organisms allocate metabolic energy to components of the life
17 history that determine fitness. In each generation animals take up biomass energy from the
18 environment and expended it on survival, growth, and reproduction. Life histories of animals
19 exhibit enormous diversity – from large fish and invertebrates that produce literally millions of
20 tiny eggs and suffer enormous mortality, to mammals and birds that produce a few large
21 offspring with much lower mortality. Yet, underlying this enormous diversity, are general life
22 history rules and tradeoffs due to universal biophysical constraints on the channels of selection.
23 These rules are characterized by general equations that underscore the unity of life.

24 25 **Abstract**

26 The life histories of animals reflect the allocation of metabolic energy to traits that determine
27 fitness and the pace of living. Here we extend metabolic theories to address how demography
28 and mass-energy balance constrain allocation of biomass to survival, growth, and reproduction
29 over a life cycle of one generation. We first present data for diverse kinds of animals showing
30 empirical patterns of variation in life history traits. These patterns are predicted by new theory
31 that highlights the effects of two fundamental biophysical constraints: demography on number
32 and mortality of offspring; and mass-energy balance on allocation of energy to growth and
33 reproduction. These constraints impose two fundamental tradeoffs on allocation of assimilated
34 biomass energy to production: between number and size of offspring, and between parental
35 investment and offspring growth. Evolution has generated enormous diversity of body sizes,
36 morphologies, physiologies, ecologies, and life histories across the millions of animal, plant and
37 microbe species, yet simple rules specified by general equations highlight the underlying unity of
38 life.

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40 \body

41 42 **Introduction**

43 “*The ‘struggle for existence’ of living beings is not for the fundamental constituents of food ...*
44 *but for the possession of the free energy obtained, chiefly by means of the green plant, from the*
45 *transfer of radiant energy from the hot sun to the cold earth.*”

46 physicist Ludwig Boltzmann (1)

47
48 *“In the struggle for existence, the advantage must go to those organisms whose energy-capturing*
49 *devices are most efficient in directing available energies into channels favorable to the*
50 *preservation of the species.”*

51 theoretical biologist Alfred Lotka (2)

52
53 Energy is the staff of life. The life history of an organism is the constellation of Lotka’s
54 “channels”: traits that determine fitness by affecting growth, survival and reproduction. There is
55 enormous diversity of life histories: from microscopic unicellular microbes with lifespans of
56 minutes to whales and trees with lifespans of centuries; from giant fish, clams, and squids that
57 produce literally millions of miniscule offspring to some birds and bats that fledge a few
58 offspring as large as their parents. Life history theory has made great progress by analyzing
59 tradeoffs between traits, such as number vs. size of offspring, current vs. future reproduction,
60 male vs. female offspring, and sexual vs. asexual reproduction (e.g., 3–9). But life history theory
61 has been slow to use metabolic energy as the fundamental currency of fitness. Organisms are
62 sustained by metabolism: the uptake, transformation, and expenditure of energy. Fitness depends
63 on how metabolic energy is used for survival, growth and reproduction.

64
65 The millions of species exhibit an enormous variety of anatomical structures, physiological
66 functions, behaviors and ecologies. Studies of biological scaling and metabolic ecology have
67 revealed unifying patterns and processes, such as effects of body size and temperature on energy
68 use, abundance, and species diversity. We present new theory to show how energy metabolism
69 has shaped the evolution of life histories. Underlying the spectacular diversity of living things
70 are universal patterns due to two fundamental constraints: 1) a demographic constraint on
71 mortality so that, regardless of the number offspring produced, only two survive to complete a
72 life cycle of one generation; and 2) a mass-energy balance constraint so that over a lifespan in
73 each generation all of the energy acquired by assimilation from the environment is expended on
74 respiration and production, and energy allocated to production exactly matches energy lost to
75 mortality. Consequently, at steady state, by the time parents have reproduced and died, their
76 energy content has been exactly replaced by the energy content of their surviving offspring. New
77 theory incorporating these constraints accounts for the schedules of survival, growth and
78 reproduction and predicts the fundamental tradeoffs between number and size of offspring and
79 between parental investment and offspring growth.

81 **Theories of resource allocation in life history**

82 Much life history theory traditionally focused on tradeoffs that affect resource allocation to
83 survival, growth and reproduction: e.g., between number and size of offspring in a clutch or litter
84 or of offspring produced over a lifetime; between semelparous and iteroparous reproduction (i.e.,
85 ‘big bang’ or ‘one shot’ vs multiple reproductive bouts); between determinate vs indeterminate
86 growth; and between fast or ‘r-selected’ vs slow or ‘K-selected’ lifestyles (i.e., rapid maturation
87 and high fecundity vs slow development and low fecundity). While these theories implicitly
88 recognize that life history traits are constrained by some ‘limited resource’, they rarely impose
89 mass-energy balance or other biophysical constraints to explicitly identify the resource and
90 quantify its allocation.

91

92 Consider, for example, the tradeoff between number and size of offspring which is readily
93 apparent across animals with contrasting life histories. At one extreme are large fish and
94 invertebrates, which produce literally millions of tiny, externally-fertilized eggs that hatch into
95 independent larvae and feed themselves as they grow to maturity. At the other extreme are bats
96 and altricial birds, which are nourished until they are close to adult size. Clearly parents which
97 produce tiny offspring must produce many of them to offset the mortality as they grow. In
98 contrast, parents which produce large offspring can produce fewer of them, because they suffer
99 less mortality due to their more developed state and shorter time to maturity.

100
101 Most life history theories assume that a female invests a constant fraction of her energy content
102 or body mass in offspring and they predict a simple linear tradeoff between number and size of
103 offspring. But the various theories and models make somewhat different predictions, depending
104 on whether the tradeoff operates within a single clutch or over a lifetime, and on how it is
105 affected by schedules of growth and mortality (e.g., 5–14). For example, one theory and some
106 data suggest that “lifetime reproductive effort” is constant: a female invests approximately the
107 same fraction of her body mass in offspring, regardless of her absolute size (4, 5, 13, 14).
108 However recent empirical studies show that investment in offspring increases with the size of
109 parent in large teleost fish (15) and terrestrial vertebrates (16). Here we provide a theoretical
110 explanation for these patterns.

111 **Empirical patterns of biomass allocation to growth, survival and reproduction**

112 To more comprehensively analyze allocation of metabolic energy to offspring, we compiled a
113 database for 36 species of animals encompassing a wide range of sizes and taxonomic and
114 functional groups. For most species it is difficult to ensure that these data are collected accurately
115 and consistently, because the majority of animals have indeterminate growth and iteroparous
116 reproduction: they continue to grow and reproduce after reaching maturity. So it is difficult to
117 determine the average number of offspring (N_O) and size of breeding adult (m_A) for a population
118 at steady state. Initially we avoided this problem by using a subset of the database for 17
119 semelparous species: i.e., “big bang” or “one-shot” reproducers, which grow to mature size,
120 produce a single clutch or litter, and then die, thereby providing more reliable data on body and
121 clutch sizes (SI Appendix, Table S1). We then analyzed an expanded dataset that includes an
122 additional 19 iteroparous species. The dataset includes a wide diversity of taxa and
123 environments, from marine, freshwater, and terrestrial invertebrates to fish, lizards, birds and
124 mammals; they exhibit many orders of magnitude variation in number of offspring, N_O , and body
125 mass of offspring, m_O , and parent, m_A . Figs. 1 and 2 plot number of offspring, N_O , and lifetime
126 reproductive investment, L , as functions of relative size of offspring at independence, $\mu = \frac{m_O}{m_A}$, on
127 logarithmic axes.
128

129
130 *Number of offspring (N_O).*—We define N_O as the number of independent offspring (i.e., at the
131 end of parental investment) produced by an average female parent in one generation. Across the
132 36 species N_O varies negatively with μ (Fig 1; variables are defined in Table 1). Several aspects
133 of this empirical scaling are especially noteworthy. First, the naïve prediction of a linear tradeoff
134 is rejected; there is modest variation around the fitted regression line ($R^2 = 0.91$) and the
135 confidence intervals do not include -1. Second, the relationship is curvilinear when plotted on
136 logarithmic axes, so it is not a power law. Third, N_O depends on the relative size of offspring, μ ,
137 but not on the absolute sizes of the offspring, m_O , or the parent, m_A , which vary by more than 10

138 orders of magnitude, from insects weighing less than 0.01 g to whales more than 100,000,000 g
139 (Fig 1 and 2; SI Appendix, Table S1). Fourth, semelparous species tend to produce somewhat
140 fewer offspring than iteroparous species.

141 *Lifetime reproductive effort (L)*.—We define parental investment, I , of energy or biomass in
142 offspring over one generation as

$$143 \quad I = N_O m_O \quad (1)$$

144 where N_O is the average number of independent offspring produced over an average lifetime and
145 m_O is the average mass of an offspring at independence. So I is the sum of the biomass in
146 gametes and nutrition invested in offspring by the parent. It can be normalized by adult mass to
147 give the lifetime reproductive investment:

$$148 \quad L = \frac{I}{m_A} = N_O \frac{m_O}{m_A} = N_O \mu \quad (2)$$

149 This dimensionless parameter is conceptually identical to Charnov's "lifetime reproductive
150 effort" (4, 5, 14). Across the 36 species, L varies widely with μ (Fig. 2). Several aspects are
151 noteworthy. First, L is not constant: it varies about 3000-fold (from 0.004 to 11.6, or from -6 to 2
152 on the natural log scale), and confidence intervals for the linear regression do not include zero.
153 Second, the magnitude and pattern of variation are consistent with the curvilinear tradeoff shown
154 in Fig. 1. Third, lifetime reproductive investment varies with relative offspring size, μ , but is
155 independent of the absolute size of the parent, m_A (Fig. 2; SI Appendix, Fig S1).

156
157 Our data do not support traditional life history theories that predict a linear tradeoff between
158 number and size of offspring or a constant lifetime reproductive effort: i.e., N_O and L do not vary
159 with relative offspring size, μ , as simple power laws with exponents of -1 and 0, respectively.
160 The U-shaped pattern of L as a function of μ is consistent with recent studies of vertebrates: 1)
161 the left-hand side with the increase in L with decreasing offspring size in large teleost fish which
162 produce enormous numbers of miniscule offspring (15); 2) and the right-hand side with
163 increasing L corresponds with increasing offspring size in terrestrial vertebrates that produce
164 fewer larger offspring (16).

165 166 **New theory**

167 We now present new theory that quantifies how organisms allocate metabolic energy to the
168 components of the life history. Adaptive traits have evolved by natural selection because they
169 promote the components of fitness – survival, growth and reproduction. On average, however,
170 species have equal fitness because at steady state parents exactly replace themselves with
171 offspring each generation, birth rates equal death rates, and populations remain constant (17).
172 Our theory is based on this equal fitness paradigm and its assumption of steady-state non-
173 growing populations. It is formulated explicitly for sexually reproducing animals, and – like
174 most life history and demographic theory – it is formulated for the female parent, which usually
175 makes the largest direct resource investment in reproduction, both gametes and any post-
176 fertilization nutrition (e.g., pregnancy and feeding).

177 178 **Energy and fitness**

179 Even though the life history traits that determine fitness, such as fecundity and lifespan, vary by
180 many orders of magnitude, all organisms pass a near-equal quantity of biomass energy
181 (approximately 22.4 kJ/g dry weight) to surviving offspring each generation. This equal fitness
182 paradigm (Brown et al. 2018) is defined by the seminal equation $E = BGQF$, where E is
183 energetic fitness, B is mass-specific rate of biomass production, G is generation time, and Q is

184 energy density of biomass. Since Q is nearly constant (≈ 22.4 kJ/g dry weight; see also (15),
 185 fluxes and stocks can be measured in units of mass and this equation becomes

$$186 \quad E = BGF = 1 \quad (3)$$

187 where F , the fraction of production that is passed through to surviving offspring, is also
 188 relatively constant, varying from approximately 0.1-0.5. So E is lifetime mass-specific biomass
 189 production, and at steady state $E = 1$, because a parent exactly replaces its own biomass with
 190 one surviving offspring that successfully breeds in the next generation. The steady state
 191 assumption is robust and realistic. Temporary deviations occur, but species persist because of
 192 compensatory ecological and evolutionary processes such as density dependence (e.g., 18) and
 193 Red Queen coevolution (e.g., 19).

194
 195 The equal fitness paradigm (eq 3) expresses the fundamental tradeoff between biomass
 196 production, B , and generation time, G : organisms that produce little biomass have short
 197 generations and vice versa (17). But it does not indicate how metabolic energy is allocated to the
 198 life history traits of survival, growth and reproduction to affect fitness. These allocations are
 199 subject to two powerful constraints: i) demography and ii) mass-energy balance. They are
 200 fundamentally “biophysical” because they can be parameterized in units of mass and energy.

201
 202 ***Demographic constraint: mortality and parental investment***

203 *Mortality as a function of age.*—Mortality of offspring over ontogeny is necessarily related to
 204 ontogenetic growth. The smaller the relative size of offspring at independence and the longer
 205 they take to grow to maturity, the greater their mortality. At steady state in generation time, G ,
 206 the number of offspring remaining alive decreases from $N = N_0$ of body mass m_0 at the end of
 207 parental nutritional input, to $N = 2$ at maturity with body mass m_A . Empirical evidence clearly
 208 shows that mortality rate decreases over ontogeny (e.g., 20–22). In animals, such as large teleost
 209 fish and invertebrates which produce enormous numbers of very small offspring, mortality is
 210 very high initially and decreases rapidly with age as the individuals grow to larger, less
 211 vulnerable sizes (Fig. 3). Even in birds and mammals, which produce a few relatively large
 212 offspring, mortality is higher for the smaller, younger, less experienced individuals.

213
 214 We derive the schedule of mortality as a function of offspring age and body mass starting with a
 215 very general von Bertalanffy-type model of ontogenetic growth (23, 24). This model, based on
 216 the scaling of metabolism as body size increases over ontogeny, also gives an expression for
 217 generation time

$$218 \quad G = G_0(m_A^{1/4} - m_0^{1/4}) \quad (4)$$

219 where G_0 is the normalization coefficient with a unit of [time/mass^{1/4}], and the 1/4-power mass-
 220 scaling exponents reflect the canonical quarter-power allometries (e.g., 25–29). We assume that
 221 over ontogeny the mortality rate, D , can be expressed as a function of adult mass m_A , offspring
 222 mass m_0 , and age x :

$$223 \quad D(x) = D_A m_A^{-1/4} + D_J m_0^{-1/4} \cdot e^{-(b/G)x} \quad (5)$$

224
 225 where D_J and D_A are coefficients for initial (juvenile) and adult mortality respectively, and b is a
 226 unitless constant such that b/G quantifies how fast the mortality rate decreases exponentially
 227 with age x (Fig. 3). We fit the mortality rate of two species (Fig. 3) with a general equation
 228 $y = \alpha + \beta x e^{-\gamma x}$. This equation has three constant parameters, each corresponding to the
 229 coefficients in the mortality rate function (Eq. 5). i.e., $\alpha = D_A m_A^{-1/4}$, $\beta = D_J m_0^{-1/4}$, and $\gamma = b/G$.

230 The nonlinear fitting gives the values of α , β , and γ . But the values of D_A , D_J , and b for a given
 231 species would require the knowledge of m_A , m_o , and G for that species. Nonetheless, the purpose
 232 of Fig. 3 is to show that mortality rate exponentially decays over ontogeny, and Eq. 5 captures
 233 the key features of it. Thus, the exact values of D_A and D_J are not necessary.

234
 235 We address two important points about parameter b in eq 5. First, since b is assumed to be a
 236 constant, the exponential decay is controlled by generation time, G . Intuitively, if G is large the
 237 mortality rate decreases slowly, because the animals grow slowly and hence reach adult size at
 238 an older age. For example, painted turtle (Fig. 3A) matures at age 6~10 years, $b/G \approx 0.48/\text{y}$, and
 239 $b \approx 5$. The generation time of humboldt squid is less than a year, and the fitted value of b/G is
 240 13.1, thus b is larger than 13. Second, b must be >1 . Equation 5 indicates that at maturity when
 241 age $x = G$, the exponential decay term, $e^{-b/Gx}$, becomes e^{-b} , and b must be sufficiently large so
 242 that e^{-b} is almost zero; then $D \approx D_A m^{-1/4}$, in agreement with empirically observed scaling of
 243 adult mortality rate (e.g., 20, 30). See SI Appendix for derivation.

244
 245 *Number of offspring.*—Now we use eq 5, to derive $N(x)$, the number of offspring surviving to
 246 age x , as follows:

247 By definition: $\frac{dN(x)}{dx} = -N(x)D(x)$ and solving this differential equation gives

$$248 \quad N(x) = N_o e^{\frac{-D_J m_o^{-1/4}}{(b/G)} (1 - e^{-(b/G)x}) - D_A M^{-1/4} x} \quad (6)$$

249 Applying the demographic constraint that at steady state, when $x = G$ (generation time),
 250 $N(G) = 2$, this equation becomes $N_o = 2e^{D_A m_A^{-1/4} G} e^{D_J (1 - e^{-b}) / (b/G m_o^{1/4})}$. Since b is relatively
 251 large and e^{-b} is almost zero, eq 6 reduces to

$$252 \quad N_o = 2e^{D_A m_A^{-1/4} G} e^{D_J m_o^{-1/4} G/b} \quad (7)$$

253 Substituting eq 4, $G = G_o (m_A^{1/4} - m_o^{1/4})$, and $\mu = m_o/m_A$, $A = D_A G_o$, $J = \frac{D_J G_o}{b}$, we have

$$254 \quad N_o = 2e^{A(1-\mu^4)} e^{J(\mu^{-1/4}-1)} \quad (8)$$

255 Taking logarithms of both sides gives

$$256 \quad \ln[N_o] = \ln[2] + (A - J) + (J\mu^{-1/4} - A\mu^{1/4}) \quad (9)$$

257 Fitting eq 9 to the data in Table S1 (SI Appendix) accurately predicts the relationship between
 258 the number, N_o , and relative size, $\mu = \frac{m_o}{m_A}$, of offspring, accounting for 92% of the variation (Fig.
 259 4). The model with the two fitted parameters $A = 6.03$ and $J = 0.11$ captures the curvilinear shape
 260 of the tradeoff between number and size of offspring shown in Fig. 1.

261
 262 *Lifetime reproductive investment.*—It is now straightforward to predict how lifetime
 263 reproductive investment, $L = N_o \frac{m_o}{m_A} = \frac{l}{m_A} = N_o \mu$ (eq 2), varies with μ . Substituting into and
 264 following the derivation above gives

$$265 \quad L = \mu x e^{A-J} e^{J\mu^{-1/4} - A\mu^{1/4}} \quad (10)$$

266 and

$$267 \quad \ln[L] = \ln[\mu] + (A - J) + (J\mu^{-1/4} - A\mu^{1/4}) \quad (11)$$

268 Fitting the two parameters, $A = 6.05$ and $J = 0.01$, gives the predicted relationship shown in Fig.
 269 5 (red curve), which accounts for 38% of the empirical variation. The distinctly curvilinear

270 relationship indicates that lifetime reproductive effort is not constant and independent of
271 offspring size as suggested by Charnov and colleagues (4, 5, 13, 14, but see 21). It is consistent
272 with recent findings that larger fish with lower μ and larger terrestrial vertebrates with higher
273 μ invest proportionally more resources and produce a proportionally greater total biomass of
274 offspring (16, 31).

275

276 ***Mass-energy balance constraint***

277 The physical law of mass-energy balance powerfully constrains the uptake of energy from the
278 environment and its allocation to survival, growth and reproduction. A mass-energy balance
279 diagram for an individual animal over one generation at steady state is depicted in Fig. 6A.
280 Biomass is taken up from the environment in the form of food and allocated between respiration,
281 where the majority of the assimilated organic molecules are catabolized to produce ATP and pay
282 the metabolic costs of maintenance and the energy is ultimately dissipated as heat, and
283 production, where a relatively small fraction of assimilated molecules are repackaged into “net
284 new” biomass.

285

286 *Tradeoff between offspring growth and parental investment.*—The lifetime biomass production
287 of an individual animal, P , is the sum of individual growth plus parental investment, where
288 growth,

$$289 H = m_A - m_O \quad (13)$$

290 and parental investment, $I = N_O m_O$ (eq 1 and Fig 6A). We normalize by dividing by adult mass
291 to obtain an expression for relative or mass-specific lifetime individual production

$$292 P = \frac{H+I}{m_A} = \frac{m_A - m_O}{m_A} + \frac{N_O m_O}{m_A} = (1 - \mu) + L \quad (14)$$

293 It is straightforward to calculate empirical values of $P = \frac{m_A - m_O}{m_A} + \frac{N_O m_O}{m_A} = (1 - \mu) + L$ for the
294 36 animal species in the dataset (SI Appendix, Table S1). The result, shown in Fig. 7A, is that P
295 shows a U-shaped pattern similar to and reflecting the U-shaped variation in L (Fig. 2). It is also
296 straightforward to substitute the theoretically derived value of L from eq 10 and solve eq 14 to
297 predict P as a function of μ . Not surprisingly, because the expression for L was obtained by
298 fitting the mortality equation using the observed number of offspring, N_O , the prediction (Fig.
299 7A) closely resembles the empirical pattern. Individual lifetime production, P varies more than
300 one order of magnitude, from close to 1 in some insects, aquatic invertebrates, and fish with
301 intermediate values of μ , to >10 in birds and mammals with $\mu \approx 1$. Note, however, the secondary
302 peak > 1.5 in some fish and invertebrates with very low values of μ , where parental investment is
303 more than half of maternal body mass.

304

305 *Allocation of growth and parental investment to cohort production and energetic fitness.*—The
306 mass-energy balance diagrams show that mass-specific production of an individual, $P = \frac{H+I}{m_A} =$
307 $(1 - \mu) + L$ (Fig. 7A and eq 14), is not the same as the mass-specific lifetime biomass
308 production, $\frac{E}{F} = BG = C$, of the equal fitness paradigm (Fig. 6B and eq 3), because P does not
309 include mortality. The parameter $C = E/F$ is the mass-specific production of the entire cohort of
310 offspring produced by a parent, so the sum of the initial parental investment, L , plus the total
311 energy, W , accumulated as growth of all offspring up until they died, including the two that
312 replaced the parents.

313 So

314 $C = W + L$ (15)

315 where

316 $W = \sum_{x=0}^{x=G} N_d m_d$ (16)

317 and N_d is the number of offspring dying at age x and m_d is the mass of those offspring when
318 they died.

319

320 Unfortunately, we do not have good data on mortality or growth rates for the species in Table S1
321 (SI Appendix), so we cannot evaluate these predictions empirically. We can, however, use our
322 model for mortality to predict W and C as functions of μ . Following eq 8, the number of
323 offspring dying at age x is

324 $N_d(x) = N_0 S(x) D(x)$ (17)

325 where $S(x)$ is the survival rate and $D(x)$ is the mortality rate at age x . Converting age to mass, m_x ,
326 eq 16 becomes

327 $W = \int_{x_0}^{x_A} N_d(x) m(x) dx$ (18)

328 Solving and normalizing in terms of μ , we obtain W , as a function of μ as plotted in Fig. 7B.

329

330 Finally, lifetime cohort production is the sum of cohort growth plus lifetime parental investment,
331 $C = W + L$ (eq 15). These three variables are plotted in Fig 7B. Note the linear scale of the x-
332 axis. Over most of the range of relative offspring size, cohort growth, W , is constant = 2, but it
333 increases sharply as μ becomes very small ($<10^{-7}$). Cohort lifetime production has a distinctly
334 bimodal distribution, with a modest peak at $\mu \approx 0.1$ and a sharp increase when $\mu < 10^{-7}$.

335 Importantly, C is relatively constant over most of the range, varying by a factor of less than 3-
336 fold. These allocations are consistent with the equal fitness paradigm, which predicts that
337 lifetime cohort biomass production is relatively constant. The fraction $F = 1/C$, of lifetime
338 cohort production that survives pre-reproductive mortality and is passed on to the two-surviving
339 offspring in the next generation, is also relatively constant and within the range, from 0.5 in
340 asexual microbes to perhaps 0.1 in some sexual eukaryotes, as predicted by Brown et al. (17).

341

342 The bottom line is that demography and mass-energy balance tightly constrain allocation of
343 metabolic energy to the components of fitness: survival, growth and reproduction. The near-
344 constant lifetime cohort production reflects a tradeoff between growth and parental investment.
345 Most animals are of intermediate body size, produce very small offspring, and allocate much
346 more of their lifetime production to growth than to reproduction. The species that produce a few
347 relatively large offspring allocate most of their production to reproduction (parental investment).
348 The relatively few fish and invertebrate species of very large adult size that start life as
349 microscopic larvae press the limits set by the constraints; they make a sizeable parental
350 investment to produce millions of offspring and offset the very high initial mortality.

351

352 **Discussion**

353 *Applications, extensions, modifications.*—Living things are amazingly diverse. The species in
354 our analyses (SI Appendix, Table S1) – and the millions of animals, plants and microbes more
355 generally – differ enormously not only in body size, anatomical structure, and physiological
356 function, but also in life history traits such as generation time, mortality rate, number and size of
357 offspring, and kind and magnitude of parental care. Nevertheless, we show above that single
358 equations predict: the schedule of mortality over the life cycle (eq 7); the tradeoff between
359 number and relative size of offspring (eq 8); the allocation of biomass to parental investment (eq

360 10); and the tradeoff between growth and parental investment at the level of both an individual
361 (eq 14) and the entire cohort of offspring produced by a parent (eq 18). The theory predicts and
362 the data in Figs 1 and 2 show qualitatively similar patterns in both semelparous and iteroparous
363 species. There is, however, a modest quantitative difference as noted above and addressed below.

364
365 Moreover, these interrelationships among dimensionless life history traits are even more general
366 than the underlying rates and times, which vary with body size, temperature, other intrinsic
367 (biological) traits, and extrinsic (environmental) conditions. For example, there are substantial
368 differences in production and mortality rates and in generation times, even between species with
369 comparable body sizes and temperatures (e.g., in mammals between short-lived rodents and
370 long-lived primates, and in insects between species with multiple generations per year and the
371 accurately named seventeen-year cicada). Despite such variation, fundamental life history
372 tradeoffs are always preserved because no organisms are exempt from the universal biophysical
373 laws.

374
375 Our theory shows that much of this variation is the consequence of two biophysical constraints:
376 1) demography, whereby the number of offspring decreases from $N = N_0$ at independence to
377 $N = 2$ at maturity; and 2) mass-energy balance, whereby relatively constant lifetime mass-
378 specific biomass production is partitioned between growth and parental investment. These two
379 constraints are almost tautologies, but together with scaling of metabolic traits with body size
380 and temperature (e.g., 25–27, 29, 32, 33), they powerfully constrain the life histories of all
381 organisms. So, our theory should apply, with at most minor adjustments, not only to animals as
382 documented here, but also to plants and unicellular microbes (which are not considered explicitly
383 here, but see (30, 34–36).

384
385 Our theory predicts much of the variation in life history traits across a diverse array of animal
386 species with respect to phylogeny, body size, anatomy, physiology, behavior and ecology (Figs.
387 1 and 2; SI Appendix, Table S1). Some of the unexplained variation and deviations from
388 theoretical predictions may be explained by questionable data, but some of this variation is
389 undoubtedly due to characteristics of real species that do not exactly match the simplifying
390 assumptions of the theory. For example, semelparous species tend to produce fewer offspring
391 than iteroparous species of similar body mass and relative offspring size (Fig. 1). More detailed
392 models that incorporate variation with age in fecundity as well as mortality should account for
393 this pattern, because iteroparous species with indeterminate growth have successive bouts of
394 reproduction with increasing numbers of offspring as they grow older and larger. The interesting
395 decrease in parental investment and cohort production as size of offspring approaches size of the
396 parent (i.e., in birds and mammals where $\mu = 0.1-1.0$: Figs. 4, 5, 7) may be due to
397 oversimplification: failure to include non-nutritional parental care that may affect offspring
398 mortality.

399
400 Other modifications can address additional complications, such as asexual reproduction, different
401 investments of male and female parents, and effects of parental care on offspring survival. It
402 should be challenging but informative to apply the theory to organisms with complex life cycles,
403 such as parasites in which different ontogenetic stages within a single generation infect different
404 hosts, have different schedules of growth and mortality, and exhibit both sexual and asexual

405 reproduction. In such cases more detailed analyses will be required to develop and test
406 quantitative predictions.

407
408 *Ecological and evolutionary implications.*—The theory presented above is one example of how
409 incorporating energetics and metabolism can contribute to a unified conceptual framework for
410 ecology and evolution. The disciplines of demography, behavior, and population and community
411 ecology have traditionally used numbers of individuals as the primary currency for their
412 empirically studies and theoretical models. In contrast, physiology and ecosystem ecology have
413 long used energetic currencies, such as scalings of rates and times with body size and
414 temperature. The result is that these disciplines have remained specialized, with only limited
415 cross-fertilization. But individual organisms are composed of energy and matter and their
416 structures and dynamics must obey the fundamental biophysical laws. So there is great potential
417 to use metabolism to link patterns and processes across levels of biological organization from
418 individual organisms to ecosystems.

419
420 Consider, for example, the paradigmatic “biomass spectrum” and distribution of body sizes in
421 marine ecosystems (e.g., (37–44). In the pelagic zone, solar energy is captured and converted
422 into biomass by tiny unicellular algae; then it is passed to successively higher size-structured
423 trophic levels as larger predators consume larger prey, culminating in apex fish, bird, and
424 mammal predators. It has long been recognized that the trophic levels are comprised of a
425 combination of adults of some species and immature stages of species with larger adults at
426 higher trophic levels (e.g., 40, 42). For example, newly-hatched planktonic larvae of large fish
427 and invertebrates are about the same size as adult zooplankters, whose larvae in turn are about
428 the same size as unicellular algae and protists (43, 45). So far, however, life history theory has
429 rarely been combined with trophic ecology to elucidate how the dynamics of growth, mortality
430 and reproduction within and across species combine to affect the flows and stocks of energy and
431 biomass in ecosystems (but see (45, 46).

432
433 Another area ripe for unification is the role of key life history traits in evolutionary
434 diversification of lineages. Biological diversity and ecological dominance of particular
435 taxonomic, functional and phylogenetic groups often have been attributed to evolutionary
436 innovations in anatomy, physiology, ecology and behavior: e.g., teleost jaw, amniote egg,
437 endothermy, primate brain. Life history traits have received less attention, but have arguably
438 been equally important. For example, special features of the life histories of teleost fish,
439 passerine birds, and placental mammals have undoubtedly played important roles as these three
440 lineages diversified spectacularly after the Cretaceous-Tertiary extinctions. In the pelagic zone,
441 large teleosts fish, which produce literally millions of microscopic eggs largely replaced sharks
442 and rays, which produce a few large eggs or live-born offspring ((41, 47). A suite of interrelated
443 traits – endothermy, parental nutrition and care, and production of a few large independent
444 offspring – are associated with the ecological dominance of birds and mammals in terrestrial
445 environments. The ecological dominance of avian and mammalian predators in cold, high-
446 latitude marine pelagic marine environments may owe as much to the role of endothermy and
447 large offspring size in reducing mortality and generation time as to the effects of endothermy and
448 associated physiology and behavior in facilitating the capture of slow ectothermic prey (but see
449 48, 49). Recently, Morrow et al. (16) have shown that dimensionless life history variables can be
450 used to define a multidimensional life history space, within which the different classes of

451 terrestrial vertebrates occupy discrete, largely non-overlapping sub-spaces. There is abundant
452 scope to investigate the role of energetics in both constraining and facilitating the filling of life
453 history spaces, ecological niches, and evolutionary lineages.

454
455 *Universality underlying biodiversity.*—The most fundamental features of life present a
456 challenging paradox. On the one hand, living things are amazingly diverse. The millions of
457 animal, plant and microbe species vary enormously in body size, anatomical structure,
458 biochemical, physiological and behavioral function, and ecological relations. On the other hand,
459 underlying all of this variety are universal patterns and processes shared by all species. Many of
460 these reflect the single origin of life and the unique biological network of metabolism that takes
461 physical energy and materials from the environment and convert them into living, self-
462 perpetuating biomass. Shared physical-chemical-biological processes at molecular and cellular
463 levels of organization are reflected in common themes of structure and function at whole-
464 organism, population and ecosystem levels, and common patterns of evolution and biodiversity.
465 So, for example, rates and times of biological processes vary by many orders of magnitude with
466 body size and temperature, but the variation is severely limited by scaling laws (e.g.
467 25–27, 29, 32, 33).

468
469 The near-tautological equal fitness paradigm calls attention to an even more universal attribute of
470 living things: all species that persist have nearly equal energetic fitness. At steady state each
471 parent allocates an identical quantity of energy (~22.4 kJ per gram dry weight per generation) to
472 produce a surviving offspring (17). The present theory shows how this equal fitness paradigm
473 emerges from two universal biophysical constraints: 1) a demographic constraint on mortality,
474 such that no matter the number and size of offspring produced, only one survives to replace each
475 parent; and 2) a mass-balance constraint on metabolism, such that energy acquired by
476 assimilation is allocated between offspring growth and parental investment so as to produce one
477 surviving offspring per parent. Our theory accounts for the classic tradeoffs between number and
478 sizes of offspring and between growth and reproduction. Extensions of our theory should account
479 for much of the variation in life history traits across all organisms.

480

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485

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487

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Figure legends

Figure 1. Plot on logarithmic axes of number of offspring (N_o) as a function of relative offspring size, $\mu = \frac{m_o}{m_A}$, for 36 animal species. The regression fits a power law scaling relation, $N_o = 0.24 \mu^{-0.83}$ (solid black line; $R^2 = 0.91$). The 95% confidence interval (-0.92, -0.74) of the slope does not contain the -1 predicted for a simple linear tradeoff (dashed gray line). Moreover, the relation is curvilinear on logarithmic axes as indicated by statistical LOESS (LOcally Estimated Scatterplot Smoothing) fit to the data (solid blue line), indicating a deviation from power law scaling.

Figure 2. Plot on logarithmic axes of lifetime reproductive investment ($L = N_o \frac{m_o}{m_A} = N_o \mu$) as a function of relative offspring size, $\mu = \frac{m_o}{m_A}$, for 36 animal species. The fitted regression gives a power law scaling relation, $L = 0.24 \mu^{0.17}$ (solid black line; $R^2 = 0.30$) with the 95% confidence intervals (0.08, 0.26), so the slope is significantly different from the zero predicted for a simple linear tradeoff (dashed gray line), and lifetime reproductive effort is far from constant (it varies about 3000-fold: from -6 to 2 on the natural log scale). Moreover, the relationship is curvilinear on logarithmic axes as indicated by statistical LOESS (LOcally Estimated Scatterplot Smoothing) fit to the data (solid blue line), consistent with Fig. 1 and indicating deviation from power law scaling.

Figure 3. Exponential decay of mortality rate as a function of age, x , obtained by fitting eq 5 to data for: (A) painted turtle (*Chrysemys picta*), (B) Humboldt squid (*Dosidicus gigas*). Data from Halley et al. (22).

Figure 4. The model accurately predicts the curvilinear shape of the tradeoff between number of offspring, N_o and relative offspring size, $\mu = \frac{m_o}{m_A}$. The equation $\ln [N_o] = \ln[2] + (A - J) + J\mu^{-1/4} - A\mu^{1/4}$ with the two fitted parameters $A = 6.03 \pm 0.5$ and $J = 0.11 \pm 0.01$ (red curve) accounts for 92% of the variation. Icons are not drawn to scale and are not included for all species.

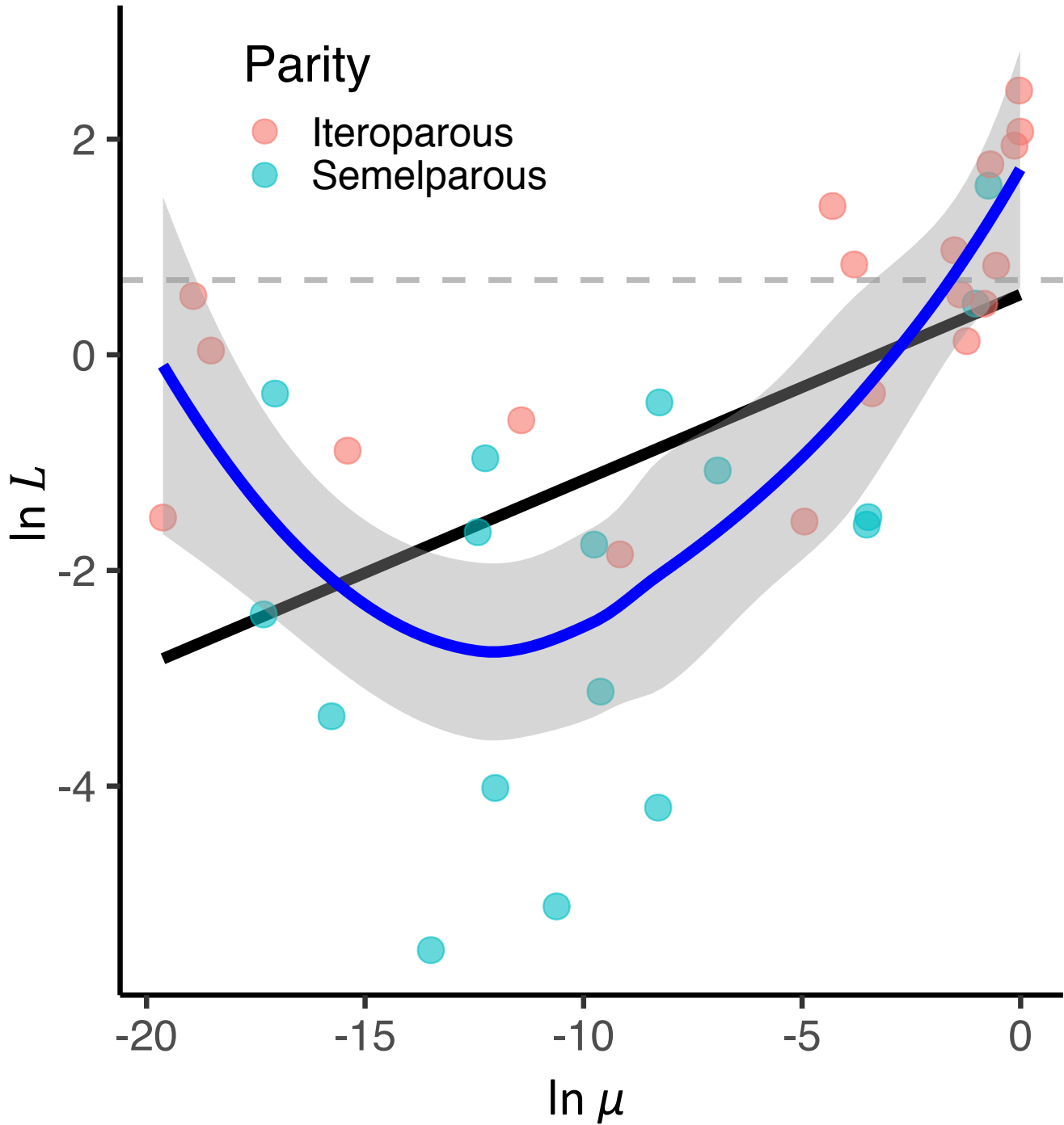
Figure 5. The model predicts the curvilinear form of the relationship between lifetime reproductive investment, L , and relative offspring size, $\mu = m_o/m_A$. The equation $\ln[L] = \ln[\mu] + (A - J) + (J\mu^{-1/4} - A\mu^{1/4})$ with fitted parameters $A = 6.03 \pm 0.5$ and $J = 0.11 \pm 0.01$ (red curve) accounts for 38% of the empirical variation. This curvilinear relationship is consistent with the relationship between number of offspring and μ shown in Figs. 2 and 4. It is not consistent with previous theory which predicts that lifetime reproductive effort is constant across species. Icons not drawn to scale and are not included for all species.

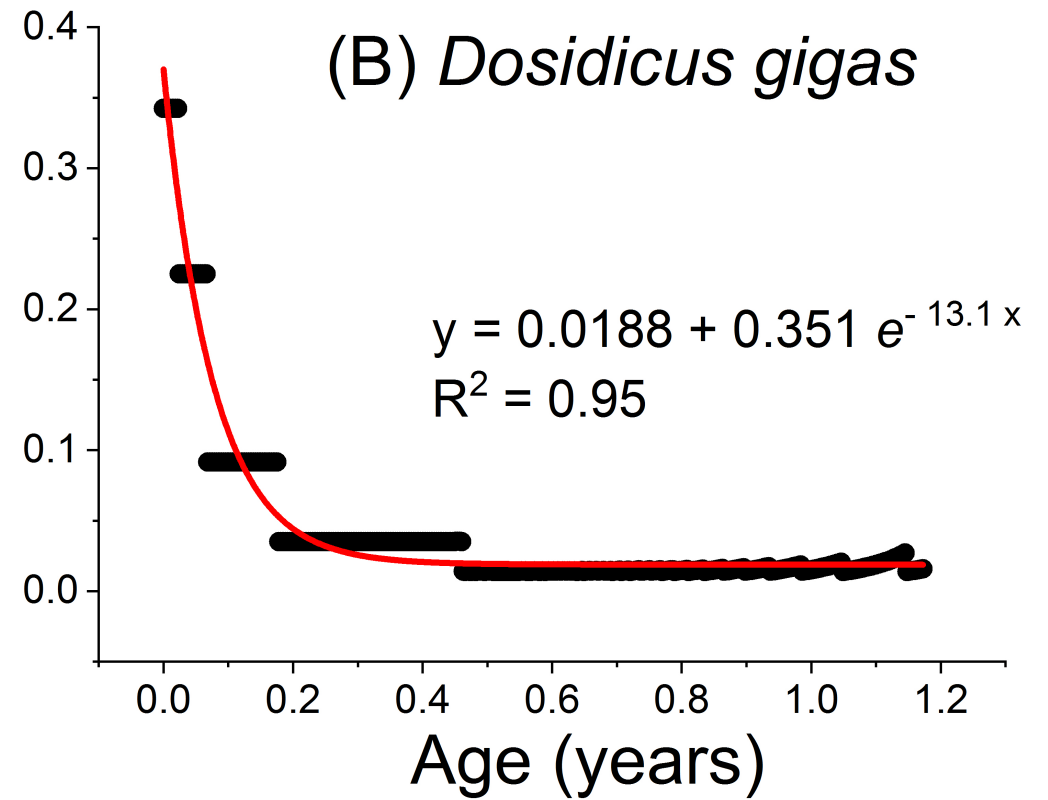
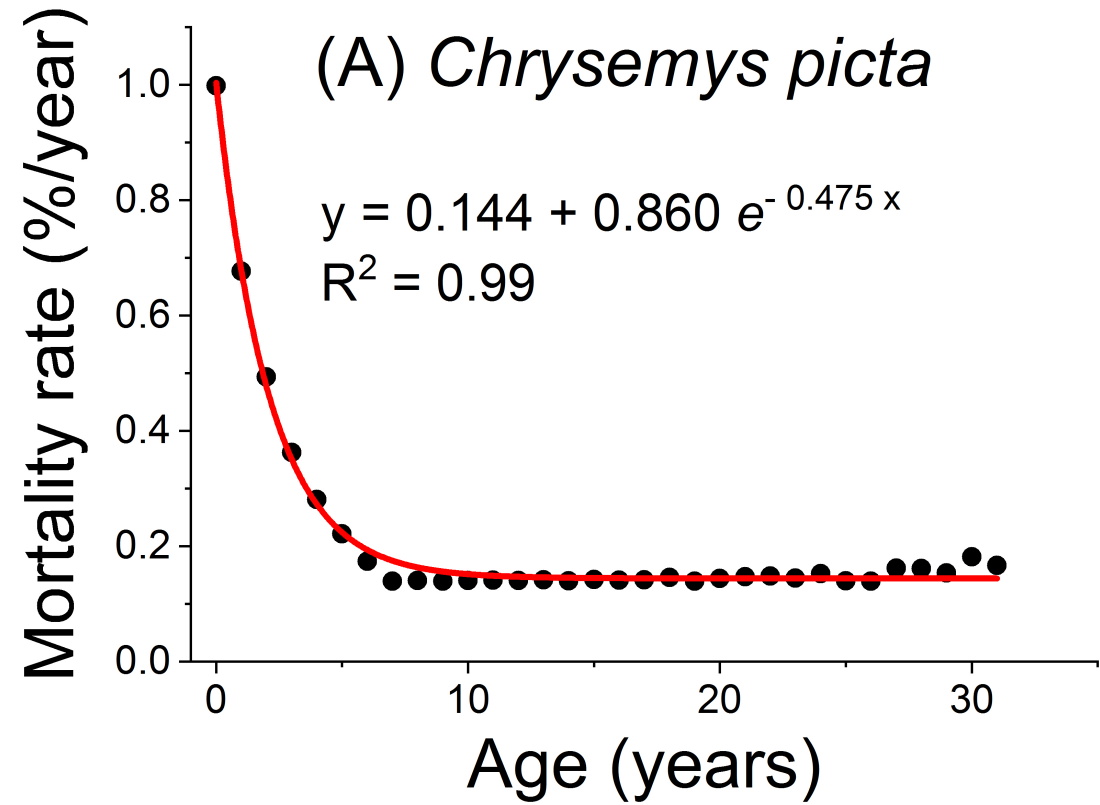
Figure 6. A) Mass-energy balance for an individual animal over one generation, so lifetime individual production, P , is assimilation minus respiration and is divided between growth and

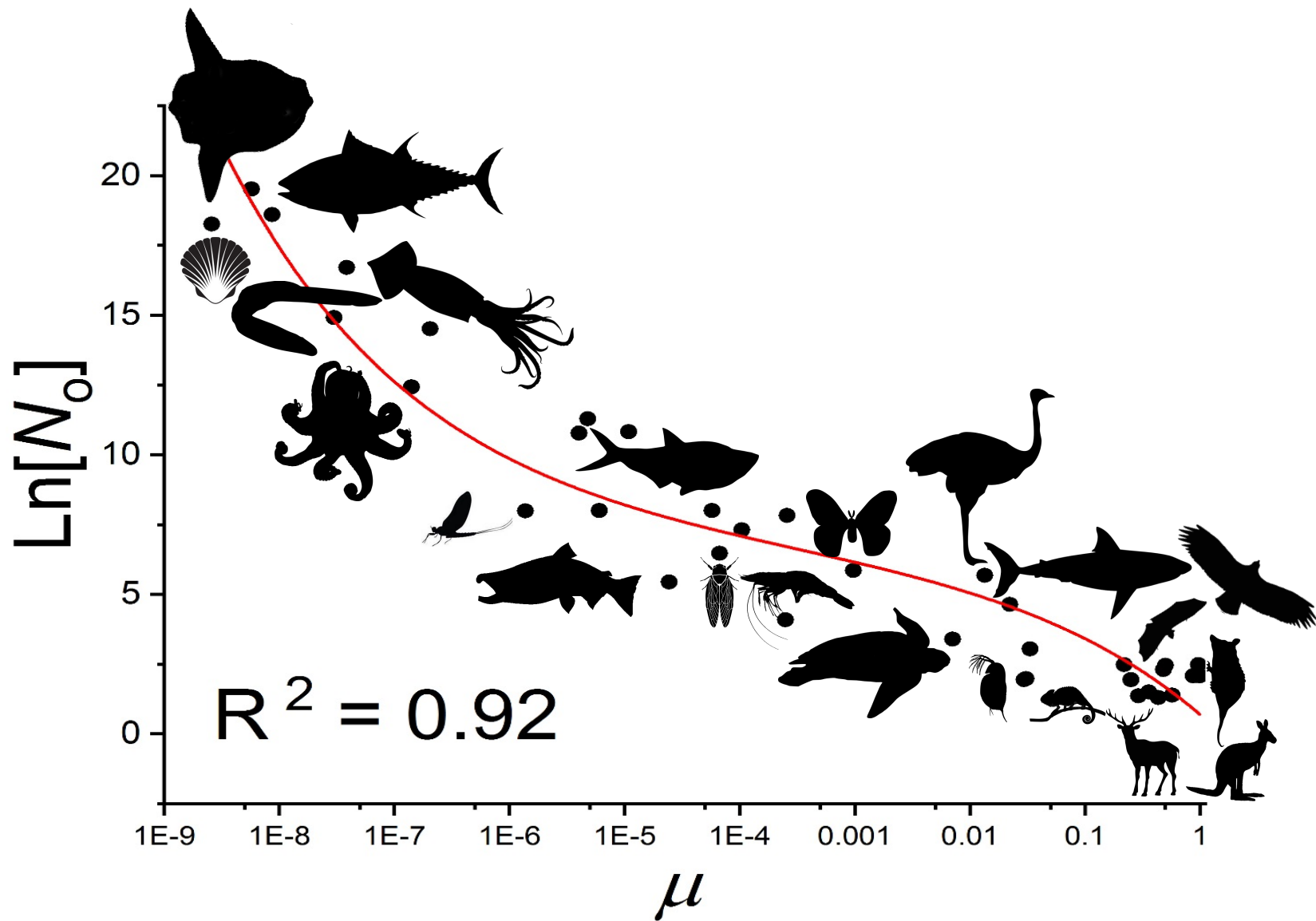
parental investment. B) Mass-energy balance for the cohort of offspring produced by a female parent in one generation, so lifetime cohort production, C , includes the biomass accumulated by growth of all offspring when they died, including the two that replaced their parents.

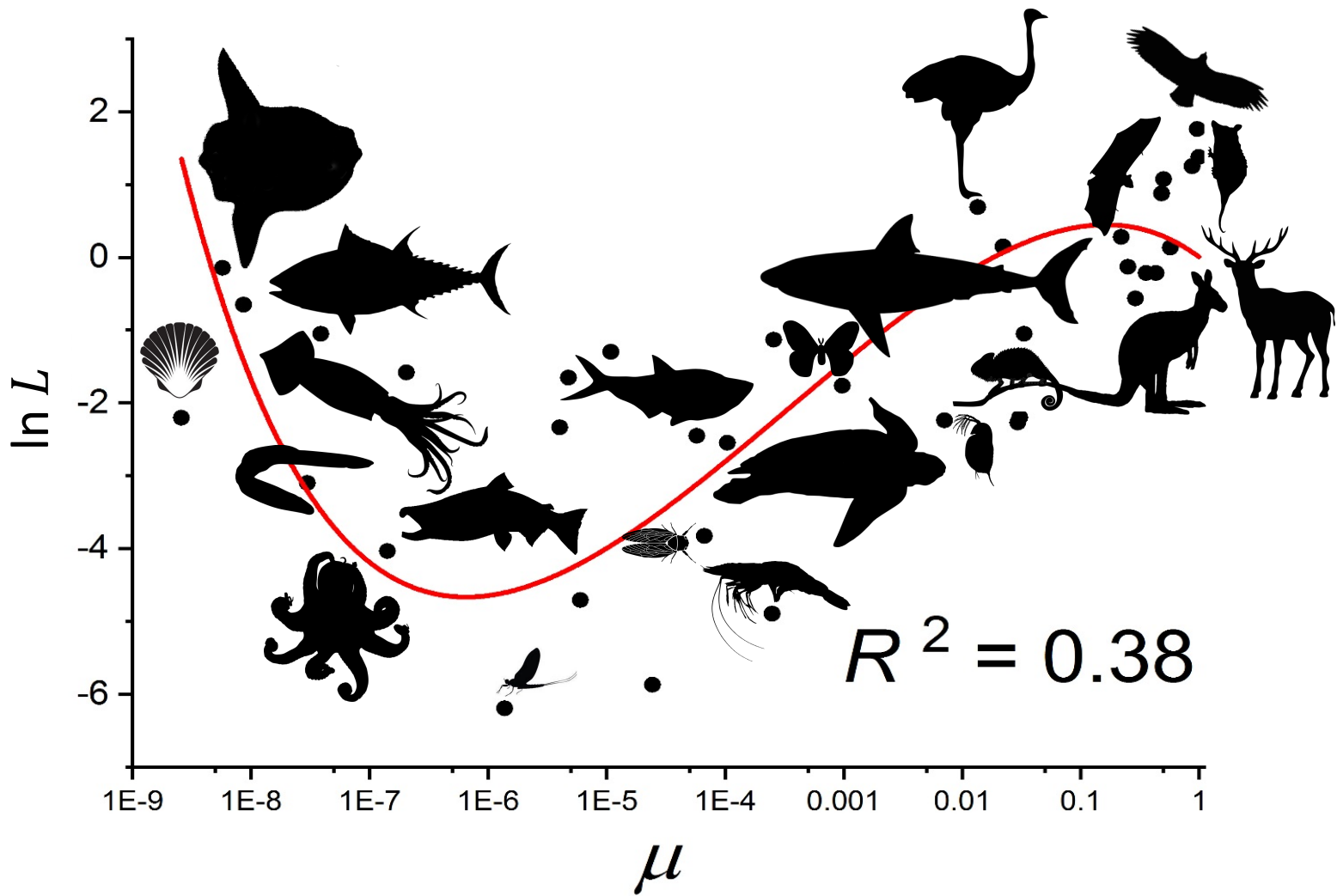
Figure 7. Mass-specific lifetime production of a single individual and of the cohort of all offspring of a parent, both as a function of relative offspring size, $\mu = m_o/m_A$. A) Empirical and theoretically predicted patterns of mass-specific lifetime production of an individual, $P = (1 - \mu) + L$, which is the sum of individual growth plus parental investment. The data points are the empirical values for the 36 animal species and the black curve is the theoretically predicted relationship based on the prediction of L (eq 10 and Fig.5). B) Lifetime biomass production, C , and its two components growth, W , and parental investment, L , for the cohort of all offspring produced by a parent. Note the linear scales of the Y-axes, so the variation in P and especially in C , W , and L is only a few fold.

Table 1. Model parameters (state variables) appearing in text.

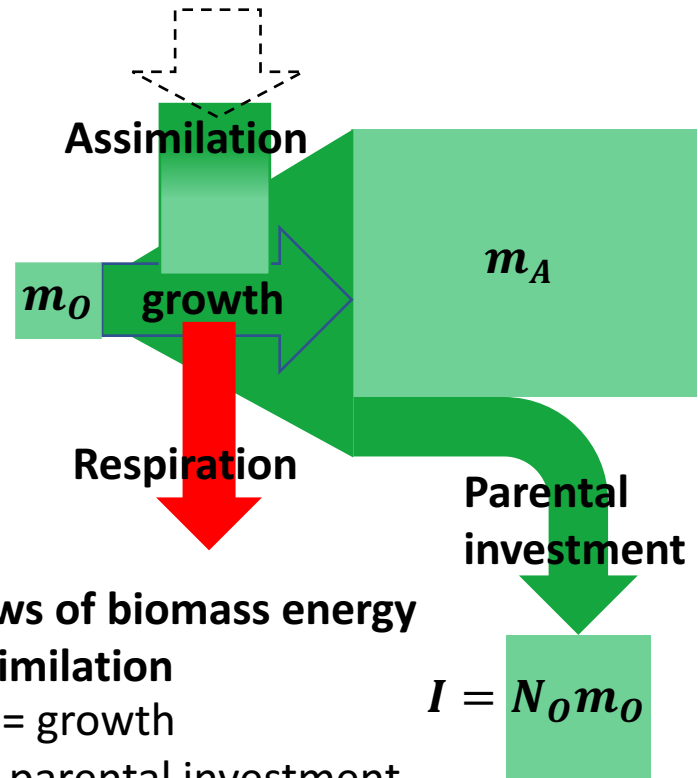








A) Mass-energy balance for an individual

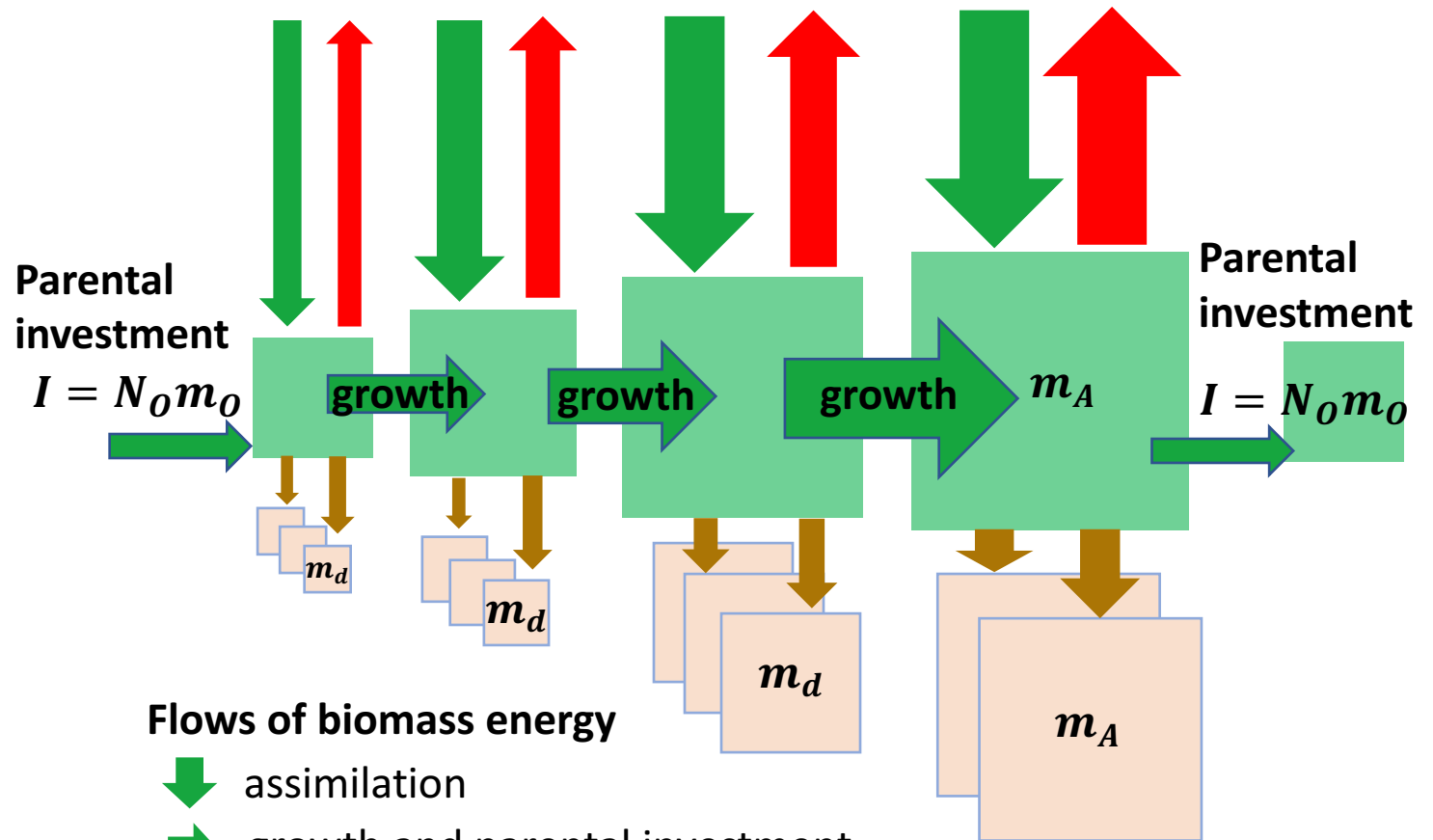


Flows of biomass energy
 Assimilation
 $H = \text{growth}$
 $I = \text{parental investment}$

Flow of heat energy
 Respiration

Stocks of biomass
 $m_0 = \text{offspring}$
 $m_A = \text{adult}$
 $N_0 m_0 = \text{gametes}$

B) Mass-energy balance for a cohort



Flows of biomass energy
 ↓ assimilation
 → growth and parental investment
 ↓ death

Flow of heat energy
 ↑ Respiration

Stocks of biomass
 living
 dead

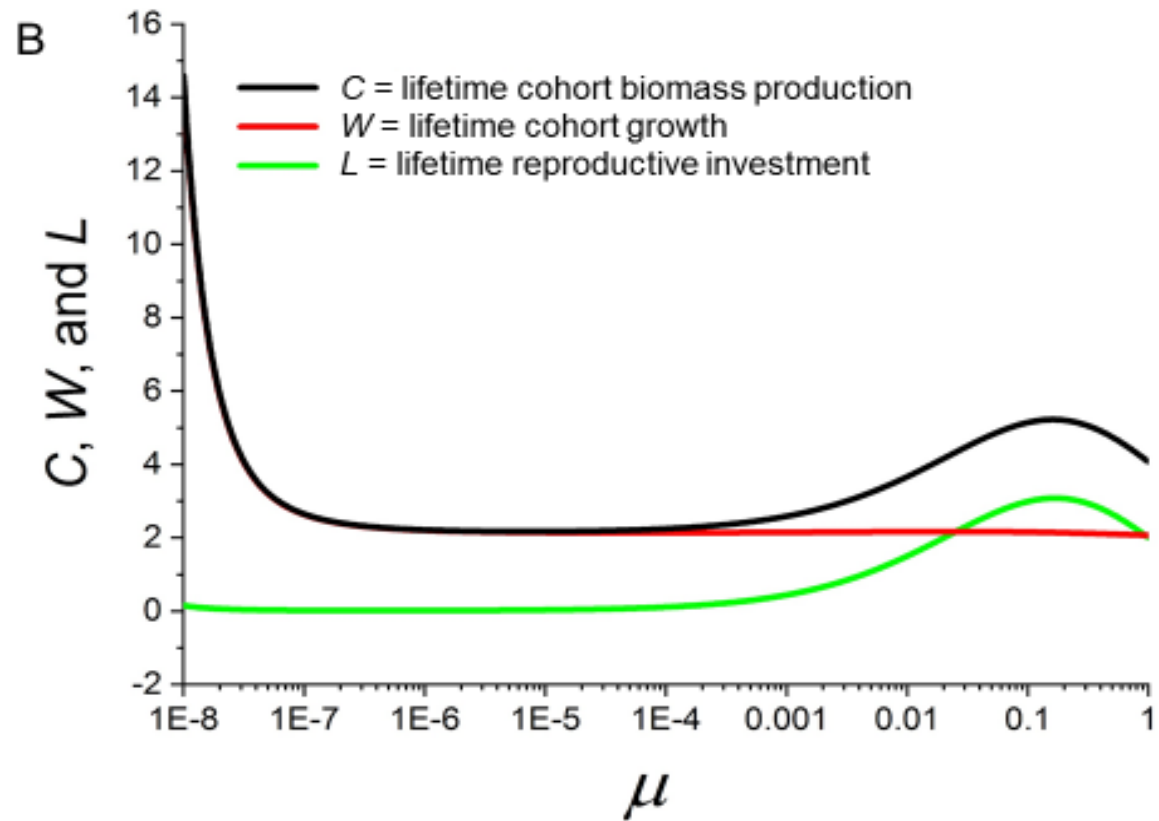
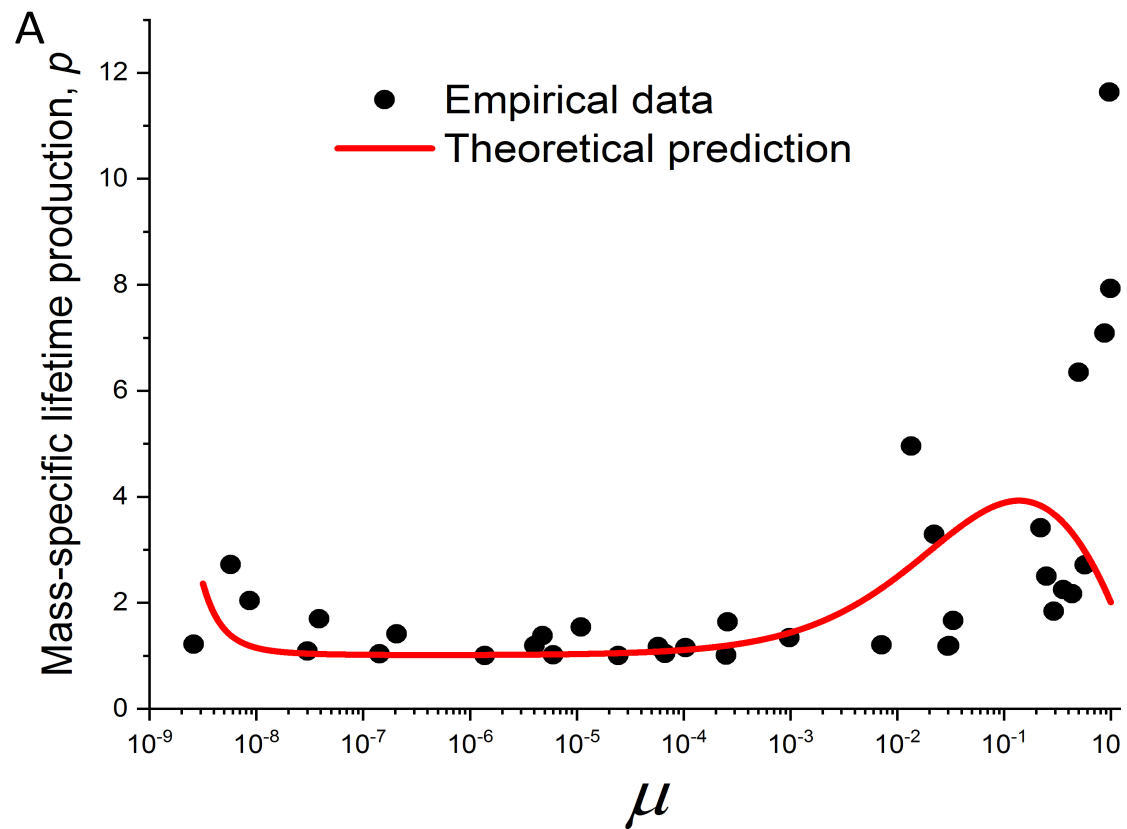


Table 1. Model parameters (state variables) appearing in text.

Symbol	Description	Units
E	Energetic definition of fitness $\approx 22.19 \text{ kJ g}^{-1}$ per generation	kJ g^{-1} per generation
B	Mass-specific rate of biomass production	y^{-1}
Q	Energy density of biomass	kJ g^{-1}
F	Fraction of production passed to next generation	dimensionless ratio
G	Generation time = $G_0 \left(m_A^{1/4} - m_O^{1/4} \right)$	y
x	Age (at time, t)	integer
m_O	Offspring mass at independence	g
m_A	Adult mass	g
μ	Relative offspring mass = M_O/M_A	dimensionless ratio
N_O	Lifetime # offspring	integer
I	Parental investment = $N_O m_O$	g
H	Individual growth = $(M_A - M_O)/M_A$	dimensionless (normalized by M_A)
P	Individual biomass production = (I) Investment + growth (H)	dimensionless (normalized by M_A and generation)
L	Lifetime reproductive investment = $\frac{I}{m_A} = \frac{N_O m_O}{m_A} = N_O \mu$	dimensionless (normalized by M_A and generation)
D	Mortality rate	y^{-1}
D_A	Adult mortality coefficient	dimensionless
D_J	Juvenile (initial) mortality coefficient	dimensionless
b	A constant that quantifies the decrease in mortality rate with age x	dimensionless constant
N_d	Number of offspring dying at age x	integer
W	Cohort growth: accumulated mass of all offspring of a parent, $W = \sum_{x=0}^{x=G} N_d m_d$	dimensionless (normalized by M_A and generation)
C	Cohort production: total production of all offspring of a parent = $I + W$	dimensionless (normalized by M_A and generation)