Phylogenetic multilevel meta-analysis: A simulation study on the importance of modeling the phylogeny

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Statement of authorship: SN provided contextual and literature review support, WV provided the code to run the simulation, all authors contributed to the manuscript.

Data accessibility statement: No new data were used in this study. The material to reproduce the results are available at: https://osf.io/ms8eq/.

Running Title: Phylogenetic multilevel meta-analysis

Keywords: phylogenetic meta-analysis, multilevel models, mixed-effects models, model efficiency, random-effects variance estimation, comparative analysis.

The type of article: Letters

The number of words in the abstract: 132

The number of words in the main text: 5389

The number of references: 44

The number of figures: 4

The number of tables: 3

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Introduction

Meta-analysis encompasses an array of methods for synthesizing information from primary studies examining some phenomenon of interest and evaluating the consistency of their results (Glass, 1976; Hedges & Olkin, 1985; Cooper *et al.*, 2009; Senior *et al.*, 2016). Although these methods have been mostly developed in the medical and social sciences (Egger *et al.*, 2001; Sutton & Higgins, 2008; Cooper *et al.*, 2009), ecologists and evolutionary biologists have successfully adopted these techniques for conducting research syntheses in their respective fields (Gurevitch *et al.*, 2001; Koricheva *et al.*, 2013; Gurevitch *et al.*, 2018). However, meta-analyses in ecology and evolution typically have several features that require special attention so that trustworthy evidence can be obtained.

To start, meta-analyses in these fields often incorporate data from multiple species which share an evolutionary history, known as phylogeny (Arnqvist & Wooster, 1995; Gurevitch & Hedges, 1999; Chamberlain *et al.*, 2012). As a result, the samples (and the effect sizes obtained from these samples) are not independent which violates the independence assumption underlying conventional meta-analytic models. For example, the standard fixed- and random-effects models (Hedges & Olkin, 1985; Hedges & Vevea, 1998), often used for ecological meta-analyses (Nakagawa & Santos, 2012), assume independence among the effect sizes and therefore do not account for the phylogeny (Chamberlain *et al.*, 2012; Noble *et al.*, 2017). This issue was first addressed by two meta-analytic methods proposed by Adams (2008) and Lajeunesse (2009) which incorporate phylogeny into the fixed- and random-effects models, respectively.

Chamberlain *et al.* (2012) empirically investigated how the inclusion of phylogeny affects the estimate of the overall mean based on data from 30 meta-analyses in ecology and evolution. They showed that the estimate of the overall mean did not change considerably in most of the meta-analyses, especially when using a random-effects model for the analysis. However, a substantial portion of the meta-analyses, which reported significant results before, produced non-significant results when the phylogeny was incorporated into the model.

Therefore, including phylogeny might be an important factor to reduce Type I error rates and to obtain an accurate reflection of the uncertainty of meta-analytic estimates.

Although Chamberlain *et al.* (2012) is the most extensive study to date to examine the effects of phylogeny in meta-analysis, there are two potential limitations. First, their work was based on4925 available meta-analyses. To investigate the issue of phylogeny more broadly, we require a simulation study to explore a wider parameter space and under controlled conditions. Second, Chamberlain *et al.* (2012) did not address the fact that ecological and evolutionary studies usually report multiple effect sizes per study, which leads to yet another source of non-independence in a meta-analysis (Nakagawa & Santos, 2012; Noble *et al.*, 2017). Although past and current meta-analyses have sometimes avoided this issue by selecting a single effect size from each study or by collapsing multiple effect sizes into one, these procedures can lead to a severe loss of information (Nakagawa & Santos, 2012).

As an alternative, Hadfield & Nakagawa (2010) proposed a mixed-effects model that accounts for the multilevel structure in the data via a study-level random effect (i.e., multiple effect sizes per study are nested within this random effect). In the same model, they include two additional random effects to estimate the non-phylogenetic and the phylogenetic variance. This way, among-species variance can be decomposed into two components, the one resulting from species similarities due to evolutionary history and the other from species similarities due to shared ecology and other factors (Lynch, 1991). Although the model by Hadfield & Nakagawa (2010) addresses two major statistical issues in ecological and evolutionary meta-analyses, the complexity of the model poses certain challenges.

Partitioning the among-species variance into its two components is a challenging endeavor, because both components are modeled using random effects at the species level, with the only difference that the phylogenetic component assumes that the random effects for different species are correlated according to a phylogenetic correlation matrix – which in turn is derived from a phylogenetic tree that is constructed based on the similarities and differences of species in terms of their (usually) genetic (but sometimes also physical)

characteristics (Felsenstein, 2004). This raises concerns about the identifiability of the corresponding variance components and potential bias in their estimates, issues that have also been raised outside the meta-analytic context when analyzing the data of primary studies that include multiple species (Paradis, 2012).

Even if the parameters can be estimated unbiasedly, the complexity of the model poses a threat to the convergence of optimization algorithms (Bates *et al.*, 2015). Accordingly, Nakagawa & Santos (2012) suggested that model fitting may only be feasible with larger datasets, which would limit the applicability of the model in practice. To avoid these problems, some ecological and evolutionary meta-analyses have been carried out using a simplified model without the non-phylogenetic random effect and that therefore accounts for the species variance only via the phylogenetic component (e.g., Garamszegi *et al.*, 2012; Moore *et al.*, 2015). However, the consequences of doing so, and the performance of the more complex model, has yet to be evaluated in a simulation study.

Here we aim to investigate the performance of models for conducting a phylogenetic multilevel meta-analysis in a comprehensive simulation study. We simulate studies that report multiple effect sizes and use several models that vary in their complexity, starting from a simple model (including only a random effect at the level of the effect sizes) to the most complex model which incorporates a study-level random effect and two among-species random effects. Further, we generate specific conditions to test the performance of the most complex model when the phylogenetic relationships are weak and to examine the consequences of removing the non-phylogenetic component. Before we fully explain our simulation design, we first introduce the different meta-analytic models in further detail.

Meta-Analytic Models

To conduct a meta-analysis, the phenomenon of interest (e.g., the size of a treatment effect or the strength of the association between two variables) needs to be quantified in terms of

an effect size estimate for each study to be included in the analysis. Note that we use the term 'study' broadly here, as a single study may contribute multiple effect size estimates (e.g., for multiple species, subgroups, different treatments), but for the moment we assume that each study contributes a single estimate to the meta-analysis.

The specific effect size measure to be used in a meta-analysis depends on the phenomenon of interest and the information reported in the primary studies (Nakagawa & Santos, 2012). For example, raw or standardized mean differences and response ratios (Hedges *et al.*, 1999) are typically used to quantify group differences or treatment effects based on quantitative variables, correlation coefficients (or Fisher r-to-z transformed values thereof) reflect the (linear) relationship between two variables, while (log-transformed) odds/risk ratios and risk differences (calculated from 2×2 contingency tables) indicate group differences (e.g., treated vs. untreated, exposed vs. non-exposed) with respect to dichotomous dependent variables (e.g., cured vs. not cured, diseased vs. not diseased). For all of these measures, we can also derive an equation to compute the sampling variances of the effect size estimates, that is, the variability in each estimate that would be expected under repeated sampling of new study units under identical circumstances (Nakagawa & Cuthill, 2007; Cooper *et al.*, 2009; Borenstein *et al.*, 2011).

Regardless of the specific measure used in a meta-analysis, let y_i denote the effect size estimate for the ith study (with $i = 1, \ldots, N_{studies}$) and v_i the corresponding sampling variance. The most basic model that can be considered for synthesizing the estimates is the fixed-effects model, which is given by

$$y_i = \mu + e_i, \tag{1}$$

$$\mathbf{e} \sim N(\mathbf{0}, \mathbf{V}), \tag{2}$$

where μ is the overall mean, e_i is the sampling error for the ith study, $\mathbf{e}$ is a $1 \times N_{studies}$ column vector with the e_i values (which are assumed to be normally distributed with mean

0 and variance v_i), $\mathbf{0}$ is a column vector of zeros, and $\mathbf{V}$ is an $N_{studies} \times N_{studies}$ matrix with the v_i values along the diagonal.

The fixed-effects model assumes that the studies included in a meta-analysis share a single common true effect. This assumption, however, is rarely met in multi-population and multi-species meta-analyses of ecology and evolution studies (Gurevitch & Hedges, 1999; Higgins *et al.*, 2009). The random-effects model addresses this potential 'heterogeneity' among the true effects by adding a random effect corresponding to each estimate and is given by

$$y_i = \mu + u_i + e_i \tag{3}$$

$$\mathbf{u} \sim N(\mathbf{0}, \sigma_u^2 \mathbf{I}_u), \tag{4}$$

where u_i is the random effect corresponding to the ith estimate, $\mathbf{u}$ is a $1 \times N_{studies}$ column vector with the u_i values (which are assumed to be normally distributed with mean 0 and variance σ_u^2), and $\mathbf{I}_u$ is an $N_{studies} \times N_{studies}$ identity matrix.

Although the models above are suitable for conducting a meta-analysis in many circumstances, they do not account for the multilevel structure that arises when (at least some of) the studies provide multiple effect size estimates (e.g., when the same experiment was conducted several times under varying circumstances within the same study) and they do not account for phylogenetic dependence (when studies are conducted with multiple species that differ in similarity due to differences in their shared evolutionary history).

To address the first issue, we can use a multilevel meta-analytic model (Konstantopoulos, 2011; Nakagawa & Santos, 2012) which includes a random effect at the effect size level (as in model 3), but which now captures variability in the true effects within studies, and a random effect at the study level, which captures between-study variability. Let y_{ij} denote the jth effect in the ith study (with $j = 1, \ldots, N_i$, where N_i is the number of effect sizes reported in the ith study), v_{ij} the corresponding sampling variance, and let $N_{total} = \sum_{i=1}^{N_{studies}} N_i$ denote

the total number of effects. The model is then given by

$$y_{ij} = \mu + u_{ij} + s_i + e_{ij} \tag{5}$$

$$\mathbf{s} \sim N(\mathbf{0}, \sigma_s^2 \mathbf{I}_s), \tag{6}$$

where u_{ij} is a random effect corresponding to the jth effect size in the ith study, s_i is a random effect at the study level, $\mathbf{u}$ is now a $1 \times N_{total}$ column vector with the u_{ij} values, $\mathbf{s}$ is a $1 \times N_{studies}$ column vector with the s_i values (which are assumed to be normally distributed with mean 0 and variance σ_s^2), and $\mathbf{I}_u$ and $\mathbf{I}_s$ are $N_{total} \times N_{total}$ and $N_{studies} \times N_{studies}$ identity matrices, respectively. Finally, $\mathbf{e}$ is now defined as a $1 \times N_{total}$ column vector with the e_{ij} values and $\mathbf{V}$ is the corresponding (diagonal) variance-covariance matrix with dimensions $N_{total} \times N_{total}$.

To deal with multiple species (i.e., when the effect size estimates were computed based on a set of $N_{species}$ different species), we will need an additional index. Therefore, let y_{ijk} denote the jth effect in the ith study as before, but now let $k = 1, \ldots, N_{species}$ be the index that indicates for which species a particular effect size estimate was computed. Model 5 can then be extended to account for species-level variability as follows:

$$y_{ijk} = \mu + u_{ij} + s_i + n_k + e_{ij}, \tag{7}$$

$$\mathbf{n} \sim N(\mathbf{0}, \sigma_n^2 \mathbf{I}_n), \tag{8}$$

where n_k is a species-specific random effect, $\mathbf{n}$ is a $1 \times N_{species}$ column vector with the n_k values (which are assumed to be normally distributed with mean 0 and between-species variance σ_n^2), and $\mathbf{I}_n$ has dimensions $N_{species} \times N_{species}$. Note that n_k is a crossed random effect (e.g., Fernández-Castilla *et al.*, 2019) and not nested within studies and we therefore do not put subscript k on u_{ij}, s_i, or e_{ij}.

Model 7, however, does not account for phylogeny. For this, we can further extend the

model by including an additional species-level random effect (Hadfield & Nakagawa, 2010), but instead of assuming independence for different species (as for the n_k values), we allow the values of the random effect to be correlated according to a phylogenetic correlation matrix, which in turn is derived from a phylogenetic tree based on some model of evolution (such as Brownian motion) prior to the analysis (e.g., Lajeunesse, 2009; Felsenstein, 1985, 2004; Freckleton *et al.*, 2002). The model is then given by

$$y_{ijk} = \mu + u_{ij} + s_i + n_k + p_k + e_{ij}, \tag{9}$$

$$\mathbf{p} \sim N(\mathbf{0}, \sigma_p^2 \mathbf{A}), \tag{10}$$

where p_k denotes the phylogenetic random effect for the kth species, $\mathbf{p}$ is a $1 \times N_{species}$ column vector with the p_k values (which are assumed to follow a multivariate normal distribution with mean 0 and variance-covariance matrix $\sigma_p^2 \mathbf{A}$, where $\mathbf{A}$ is the $N_{species} \times N_{species}$ phylogenetic correlation matrix). Hence, the model includes a non-phylogenetic species-level random effect (i.e., the n_k values) to account for heterogeneity in the effects sizes due to differences between species unrelated to phylogeny (e.g., the influence of differences in the environments they live in) and a phylogenetic random effect (i.e., the p_k values) that captures dependencies in the effect sizes according to the similarities between species due to phylogenetic relatedness.

A concern with model 9 arises when the phylogenetic relationships are weak. In that case $\mathbf{A}$ starts to resemble $\mathbf{I}_n$, and hence σ_p^2 and σ_n^2 are confounded and may not be uniquely identifiable. This in turn may lead to bias in the estimates of the variance components. This concern, or the complexity of model 9 in general, has led some researchers to adopt a simplified model in their ecological and evolutionary meta-analyses where the non-phylogenetic variance component is removed. This leads to the model

$$y_{ijk} = \mu + u_{ij} + s_i + p_k + e_{ij}, \tag{11}$$

where all terms are as explained before. Whether such a simplified version is an adequate substitute for model 9 is currently unknown.

All of the models described above can be fitted within a Bayesian or likelihood framework (Hadfield & Nakagawa, 2010). For the latter, the `metafor` package (Viechtbauer, 2010) for `R` (R Core Team, 2021) is particularly attractive as it is freely available and was written specifically for the purposes of conducting meta-analyses. Maximum likelihood (ML) or restricted maximum likelihood (REML) estimation can be used for model fitting (the latter usually being the preferred choice; see Patterson & Thompson, 1971), which will provide estimates of the variance components included in a particular model, the estimate of μ (i.e., $\hat{\mu}$), and its corresponding standard error (i.e., SE[$\hat{\mu}$]). Likelihood ratio tests and profile likelihood confidence intervals provide inferences for the variance components. An approximate 95% Wald-type confidence interval for μ can be obtained with $\hat{\mu} \pm 1.96\text{SE}[\hat{\mu}]$. Analogously, H_0: $\mu = 0$ can be tested by comparing $z = \hat{\mu}/\text{SE}[\hat{\mu}]$ against the critical values (i.e., ± 1.96) of a standard normal distribution.

Simulation

We explored the consequences of using the various models described in the previous section based on a simulation study.

Simulation Setup

In our setup, the primary studies considered in a meta-analysis could provide one or multiple effect size estimates for one or multiple species. We set $(N_{studies}, N_{species})$ either to $(20, 40)$ or $(50, 100)$ to examine the difference between a smaller versus larger meta-analysis. Furthermore, we set σ^2_u, σ^2_s, σ^2_n, and σ^2_p to either 0, 0.05, or 0.3 (plus an additional parameter α to be described below to either 0.5, 1, or 2) to define a particular condition within the simulation study. Table 1 provides an overview of the 158 conditions that were studied in

this manner. Note that we used a 'conditional factorization' of the four variance components to keep the number of conditions manageable and to generate scenarios where one of the models described in equations 3, 5, 7, and 9 corresponds to the true data generating mechanism. Within a particular condition, the following steps were repeated 1000 times.

First, the number of effect sizes provided by the studies (i.e., the N_i values) were simulated from a right-skewed distribution, as typically observed in practice. For this, we generated $N_{studies}$ random values from a Beta$(1.5, 3)$ distribution, which were then multiplied by 39, rounded to the closest integer, and increased by 1. Therefore, the number of estimates per study could vary between 1 and 40 (with a mean, median, and mode of approximately 14, 13, and 9, respectively).

In the next step, we simulated the species indices (i.e., the k values) by generating N_{total} random values from a Beta$(2, 2)$ distribution, which were multiplied by $N_{species} - 1$, rounded to the closest integer, and then increased by 1. Accordingly, the number of times that the various species were studied followed a symmetric unimodal distribution (with mean/median/mode equal to $(N_{species} + 1)/2$). In order to guarantee that all species appear at least once in the meta-analysis, a randomly chosen $N_{species}$ random numbers generated this way were replaced with the integers from 1 to $N_{species}$.

Next, we generated a phylogenetic tree for the species using the `rtree()` function from the `R` package `ape` (Paradis & Schliep, 2019), which uses a simple recursive random splitting algorithm to simulate a phylogeny (Paradis, 2012). The branch lengths were then computed using the `compute.brlen()` function from the same package based on the method by Grafen (1989), using the power parameter α to essentially adjusts the 'height' of branch lengths at the tips of the phylogenetic tree, leading to phylogenetic relationships that are generally stronger when branches are shorter at the tips or weaker when branches are longer at the tips. Fig. 1 shows an example of such a simulated tree for 40 species modified by different α values. Finally, the correlation matrix that represents the phylogenetic relationships (denoted by $\mathbf{A}$ in equation 10) was calculated from the tree by using the `vcv()` function based on

a Brownian model of evolution (i.e., $\mathbf{A}_{k,k'} = 1 - b_{k,k'}$, where $b_{k,k'}$ is the branch length for a pair of species to their most recent common ancestor).

We then generated the values for the four random effects, corresponding to the variance components σ_u^2, σ_s^2, σ_n^2, and σ_p^2, either as independent draws from normal distributions for the first three components or from a multivariate normal distribution for the last one. In conditions where a particular variance component is equal to 0, the corresponding random effect values are then just a series of 0s of the appropriate length. To complete the data generating step, the sampling variances (i.e., the v_{ij} values) were simulated from a right-skewed Beta$(2, 20)$ distribution (and hence had a value of .091 on average) which were then used to generate the N_{total} sampling errors from a normal distribution with mean 0 and variance v_{ij}. We then summed up the random effects and sampling errors as shown in equation 9, setting $\mu = 0$ without loss of generality.

After generating the data, we fitted the four models shown in equations 3, 5, 7, and 9, using REML estimation as implemented in the `rma.mv()` function from the `metafor` package. For model 3, we simply treated each estimate as a separate study (one can also think of this as model 5 without the addition of the study-level random effect). For each model, we then saved the estimate of μ, the variance component estimates, the bounds of the 95% Wald-type confidence interval for μ, and the model fitting time. In case any one of the four models did not converge within a particular iteration (with the default settings of the `rma.mv()` function), the iteration was discarded and a new iteration was run to guarantee that a 1000 successful model fits were available for all four models.

After the 1000 iterations, we computed the mean of the $\hat{\mu}$ values for each model, the mean of the variance component estimates, the proportion of iterations where 0 was included in the confidence interval (i.e., the empirical coverage rate for μ), the mean confidence interval width, the convergence rate, and the mean model fitting time. The simulation was run on a workstation with an Intel Xeon E5-2630v4 processor utilizing 15 cores in parallel. Completion time for the simulation was approximately 7 days (roughly 2520 core hours in total).

We generated two other sets of conditions to investigate specific questions. In the first set, we examined conditions where the phylogenetic relationships could also be weaker than in the main scenarios to test the performance of model 9 under such conditions. These conditions were generated by setting α to $(0.1, 0.2, 0.3, 0.4, 0.5, 1, 2)$ when $(N_{studies}, N_{species}) = (50, 100)$, the estimate- and study-level variance components were both large (0.3), and the levels of the remaining variance components were factorized with values of 0.05 and 0.3 (for a total of 28 different conditions). In the second set, we compared the performance of model 9 and the simplified model 11 (that leaves out the non-phylogenetic species-level random effect). For this, we set $(N_{studies}, N_{species}) = (50, 100)$, $\sigma^2_u = 0.05$, $\sigma^2_s = 0.05$, and $\alpha = 1$, and then generated different conditions by factorizing different values of only σ^2_n and σ^2_p, where the former was set to values from 0 to 0.3 with increments of 0.05, whereas the latter was set to either 0, 0.05, or 0.3 (for a total of 21 different conditions).

Results

Fig. 2a displays boxplots based on the mean $\hat{\mu}$ value for each of the four models across the 158 conditions, separated by which model was the true data generating mechanism. Generally, the means were clustered tightly around 0, indicating little to no bias in $\hat{\mu}$, although in a small set of conditions there was some slight positive bias in the estimates of the overall mean. These conditions were characterized by non-zero values for all four variance components (i.e., when model 9 was the true model), $(N_{studies}, N_{species}) = (20, 40)$, a weak phylogenetic relationship ($\alpha = 0.5$), and a large phylogenetic variance ($\sigma^2_p = 0.3$).

In contrast to the results for the overall mean, the coverage rates of the 95% confidence interval for μ differed markedly across models (Fig. 2b). For conditions where model 3 was the true data generating mechanism, all models achieved coverage rates close to or slightly above the nominal 95% confidence level regardless of the specific conditions. As the other variance components were introduced into the data, however, the coverage rates of models that did not account for these additional sources of variability started to decrease, at times

severely so. Only model 9 was able to achieve rates close to the nominal level across the majority of conditions, although the rates also fell somewhat below the nominal level for certain conditions when all variance components were larger than zero.

Given that estimates of μ were relatively unbiased for all models, the closer to nominal coverage rates of model 9 would be expected to be mainly a consequence of wider confidence intervals (that consequently have a better chance of capturing the true value of μ). Fig. 2c confirms this, showing the mean confidence interval widths for the various models across the various conditions. However, what is particularly noteworthy is that the use of model 9 under conditions where actually a simpler model is the true data generating mechanism only leads to a relatively minor increase in the mean interval width.

Fig. 3 displays the bias in the variance component estimates of model 9 under the 28 different conditions generated by varying α, σ_n^2, and σ_p^2 (while holding σ_u^2 and σ_s^2 constant at 0.3). The results show no bias in the estimates of σ_u^2 and σ_s^2. Furthermore, the model is able to estimate σ_n^2 and σ_p^2 with little to no bias, except when the strength of the phylogenetic relationships decreased. As expected, under such conditions, the model struggles to provide unbiased estimates of the non-phylogenetic and phylogenetic species-level variance components (especially when $\sigma_n^2 \neq \sigma_p^2$).

Fig. 4a shows the coverage rates of the confidence interval for μ for models 9 and 11 as the size of the non-phylogenetic species-level variance component (i.e., σ_n^2) was systematically increased. While model 9 provided rates close to or somewhat below the nominal level, the rates for model 11 were often equal to 100% and hence the confidence interval for the overall mean tended to be too wide (i.e., was overly conservative). Furthermore, Fig. 4b demonstrates that the bias in the phylogenetic variance component of model 11 inflated rapidly as the true value of σ_n^2 increased (the value of σ_p^2 had no noteworthy influence on the size of the bias and hence we averaged these results over the three possible values of σ_p^2). In contrast, model 9 estimated these two variance components essentially without bias under these scenarios.

Model fitting times differed between the various models (Table 2), with model 9 requiring the most amount of time on average, regardless of the true data generating mechanism. The most challenging conditions for the more complex models were those scenarios where model 3 corresponded to the true data generating mechanism. In this case, a single fit of model 9 took around 26 seconds on average when $(N_{studies}, N_{species}) = (50, 100)$. In these conditions, convergence rates were also the lowest, although even model 9 then converged in approximately 96% of the iterations.

Illustrative Example

To illustrate an application of the models examined in the previous section, we used the data from the meta-analysis by Rios Moura *et al.* (2021) on size-assortative mating (SAM). Each study included in the meta-analysis provided one or multiple correlation coefficients describing the similarity in some measure of body size in mating couples. For the analysis, the correlation coefficients were transformed with Fisher's r-to-z transformation. We focus here on the estimate of the overall mean (transformed) correlation coefficient, leaving aside the issue of differences between studies where correlations were computed with or without pooling of data across different timepoints or areas (i.e., temporal/spatial pooling). Also, using the method by Grafen (1989), we turned the phylogenetic tree used by Rios Moura *et al.* (2021) into an ultrametric tree before fitting models 9 and 11, to bring these analyses more in line with how our simulation study was conducted. The dataset includes 1828 effect size estimates (i.e., transformed correlations) collected from 457 studies and 341 species.

Table 3 presents the results obtained from fitting each model to the dataset. Interestingly, the estimate of the overall mean tended to be somewhat larger in the more complex models, although differences between models 7, 9, and 11 were relatively small. More importantly, we see a substantial increase in the standard error of the estimated overall mean for the more complex models. As a result, the confidence intervals become wider, the values of the test

statistics smaller, while the respective p-values increase. Although each model suggests that the overall mean significantly differs from 0 (at the conventional 0.05 level of significance), the p-value for model 9 and especially for model 11 was approaching the rejection threshold.

Further, examining the estimates of the variance components shows some interesting patterns. While the simple random-effects model 3 cannot distinguish between different sources of variability and essentially attributes all of the heterogeneity to differences between the individual effect size estimates, model 5 suggests that the variance in the effects is more related to differences between studies than particular estimates within studies. However, once species-level variability is considered, it becomes apparent that this is actually the dominant source of heterogeneity. Moreover, model 9 shows that this variability is to approximately equal parts attributable to non-phylogenetic and phylogenetic species-level differences. In contrast, when ignoring the non-phylogenetic variance component in the simplified model 11, part of the variance from that component is forced back into the study-level variance component. Furthermore, $\hat{\sigma}_p^2$ in the simplified model is substantially inflated compared to model 9 which may be an example of the inflation in this component when σ_n^2 is excluded (see Fig. 4b). Based on these findings and the Akaike Information Criteria (AIC) values of the various models, we would strongly favor model 9 in this comparison, illustrating that both non-phylogenetic and phylogenetic variance components should be considered in the analysis.

Discussion

Meta-analyses in the fields of ecology and evolution typically need to address the fact that multiple effect size estimates can be extracted from at least some of the studies and that the estimates are based on various species that are related to each other due to their shared evolutionary history. In this paper, we investigated the performance of the phylogenetic multilevel meta-analytic model proposed by Hadfield & Nakagawa (2010) and Nakagawa &

Santos (2012) that captures these intricacies along with some simpler models. Despite the concerns we raised in the introduction, the model can successfully estimate the overall mean and its uncertainty. It also provides approximately unbiased estimates of all variance components, including the non-phylogenetic and phylogenetic species-level variances, as long as there are at least moderately strong phylogenetic relationships among the species. In addition, despite its complexity, the model does not appear to suffer from convergence problems and model fitting does not require excessive computational times.

Estimating the Overall Mean and its Uncertainty

Not only the phylogenetic multilevel meta-analytic model, but also the simpler models that leave out certain variance components provide essentially unbiased estimates of the overall mean, regardless of the nature of the true model that underlies the data (Fig. 2a). However, the uncertainty in the overall mean will only be estimated accurately when the fitted model includes the variance components that do contribute to the heterogeneity and dependencies among the underlying true effects. Fitting underspecified models typically led to severe undercoverage of the confidence interval for the overall mean and hence anticonservative inferences. In fact, subtracting the coverage rates shown in Fig. 2b from 1 yields the Type I error rates for the test of the overall mean, which could go as high as 91% when using a simple random-effects model that ignores the multilevel structure and the species-level variance components.

These findings are in line with those by Chamberlain *et al.* (2012), who demonstrated, based on 30 published meta-analyses, that the inclusion of phylogeny into a random-effects model usually only led to minor changes in the pooled effect size, but had a more substantial impact on the statistical significance of the finding (turning significant findings into non-significant ones in the majority of cases where changes occurred).

Our findings can also be used to alleviate concerns with using the phylogenetic multilevel meta-analytic model when it is actually an overspecified model (i.e., when the actual data

generating mechanism is simpler). In those cases, the mean confidence interval width of the model was just barely wider than that of the simpler models, indicating little to no loss in efficiency by fitting an overly complex model (Fig. 2c). The superfluous variance components then converge towards 0 (or close to it), which appears to be slightly more challenging for the optimization algorithm, leading to longer model fitting times and occasional convergence problems, but not to any worrisome degree (Table 2). Moreover, in practice, for any particular dataset, convergence problems can typically be resolved by selecting a different optimizer or making changes to the settings for the optimization routine, so the convergence rates as given only apply to the default settings.

At the same time, we should point out that the coverage rate of the model did fall slightly below the nominal 95% level in the majority of conditions when all variance components were in fact non-zero (see Fig. 2b, rightmost panel). This undercoverage stems from using an overly simple Wald-type confidence interval using critical values based on a standard normal distribution that ignores the uncertainty in the estimates of the variance components (especially in the study and the two species-level components when $N_{studies}$ and $N_{species}$ are low). A similar issue, but for a simpler model with only between- and within-study variance components (i.e., model 5 in our simulation) was also recently pointed out by Song *et al.* (2020). Improved methods based on the t-distribution, with various approximations for the degrees of freedom, have been proposed and studied extensively in the context of the standard random-effects model (e.g., Sanchez-Meca & Marin-Martinez, 2008) and mixed-effects models in general (e.g., Luke, 2017), but these methods have not been generalized to the present context. As a simple approximation, using the smaller of $N_{studies} - 1$ and $N_{species} - 1$ as the degrees of freedom for a confidence interval based on a t-distribution is likely to bring the coverage rate quite close to the nominal rate in the majority of conditions.

Including and Testing the Phylogenetic Effect

Phylogenies play a central role in the context of phylogenetic comparative studies (Freckleton *et al.*, 2002; Blomberg *et al.*, 2003; Ives *et al.*, 2007). An important step in such studies is testing the significance of the 'phylogenetic signal' in some trait of interest. This test is often performed through a statistic such as λ (Pagel, 1999) or K (Blomberg *et al.*, 2003). Although model 9 does not parameterize the phylogenetic effect in this manner, one can derive information from its output that shows its relationship to the λ statistic. In particular, Pagel's λ is a multiplicative factor that is applied to the off-diagonal values of the correlation matrix that represents the phylogenetic relationships (i.e., the $\mathbf{A}$ matrix). For example, the variance-covariance matrix for three species would be given by

$$\sigma^2 \begin{bmatrix} 1 & \lambda a_{12} & \lambda a_{13} \\ & 1 & \lambda a_{23} \\ & & 1 \end{bmatrix}$$

while the decomposition of the species-level heterogeneity in model 9 implies the variance-covariance matrix

$$\sigma_n^2 \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} + \sigma_p^2 \begin{bmatrix} 1 & a_{12} & a_{13} \\ & 1 & a_{23} \\ & & 1 \end{bmatrix} = (\sigma_n^2 + \sigma_p^2) \begin{bmatrix} 1 & \left(\frac{\sigma_p^2}{\sigma_n^2+\sigma_p^2}\right) a_{12} & \left(\frac{\sigma_p^2}{\sigma_n^2+\sigma_p^2}\right) a_{13} \\ & 1 & \left(\frac{\sigma_p^2}{\sigma_n^2+\sigma_p^2}\right) a_{23} \\ & & 1 \end{bmatrix}$$

and hence $\sigma^2 = \sigma_n^2 + \sigma_p^2$ and $\lambda = \sigma_p^2/(\sigma_n^2 + \sigma_p^2)$ (see also Lynch, 1991; Freckleton *et al.*, 2002). Hence, $\sigma_p^2/(\sigma_n^2 + \sigma_p^2)$ indicates the degree of the phylogenetic signal in the overall variance sourced from the species. A likelihood ratio test of H_0: $\sigma_p^2 = 0$ can be easily performed by comparing $X^2 = -2(ll_7 - ll_9)$ against a chi-squared distribution with one degree of freedom, where ll_7 and ll_9 are the (restricted) log likelihoods of models 7 and 9, respectively.

Estimating the Non-Phylogenetic and Phylogenetic Variance Components

Given the informative nature of these two variance components, it is essential to estimate their true values accurately. We found that model 9 was usually able to estimate these variance components unbiasedly, but should note that the model struggles to separate the non-phylogenetic and phylogenetic species effects when phylogenetic relationships are weak. In essence, the two sources of variability then start to collapse into one, with a total variance of $\sigma_n^2 + \sigma_p^2$. This total variance is then distributed in approximately equal parts into the two estimates, which explains the apparent low bias when (coincidentally) $\sigma_n^2 = \sigma_p^2$ (Fig. 3a and d). However, when $\sigma_n^2 \neq \sigma_p^2$, the bias in the two estimates becomes quite apparent (Fig. 3b and c). Therefore, we would caution against the use of model 9 when phylogenetic relationships are weak. As a rough guideline, for $\alpha = 0.5$, the mean correlation in the $\mathbf{A}$ matrix (excluding the diagonal) is around 0.2 and hence a lower mean correlation would call into question the trustworthiness of the estimates of σ_n^2 and σ_p^2.

Some meta-analyses in ecology and evolution have used model 11 to reduce model complexity (e.g., Garamszegi *et al.*, 2012; Moore *et al.*, 2015). Our results indicate that this approach cannot be recommended. As we increased the value of σ_n^2 in the actual data, the bias in the phylogenetic variance component inflated massively as a result of leaving the non-phylogenetic variance component out of the model (Fig. 4b). As a result, the relevance of the phylogeny could be greatly overestimated. In addition, the confidence interval for the overall mean then becomes extremely conservative with coverage rates at or very close to 100%. This in turn implies a loss of efficiency for estimating the size of the overall mean and a loss of power for testing H_0: $\mu = 0$. The illustrative example also shows this phenomenon.

Caveats and Conclusions

For the simulation study, we used a 'generic' effect size measure, that is, we directly simulated the sampling errors from a normal distribution and treated the sampling variances (i.e., the v_{ij} values) as exactly known. These conditions only apply asymptotically to measures typically used in practice (e.g., standardized mean differences, response ratios, r-to-z transformed correlation coefficients, log-transformed risk/odds ratios). The present results therefore reflect the performance of the various models under idealized conditions (i.e., assuming that the sample sizes of the individual studies are sufficiently large, such that the sampling distributions of the estimates are indeed approximately normal and that any inaccuracies in the estimated sampling variances are negligible). The advantage of using a generic measure is that we were able to identify problems that are inherent to certain models and not (potentially) a consequence of violations to the model assumptions. On the other hand, it remains to be determined how well the phylogenetic multilevel model performs when the effect sizes are generated based on the exact distributional assumptions underlying specific measures.

Therefore, at least for the moment, the present results suggest that model 9 is the most appropriate tool for conducting a multi-species meta-analysis in ecology and evolution. For the vast majority of conditions examined, it provides approximately unbiased estimates of the variance components and the overall mean and a confidence interval for the latter that has a close to nominal coverage rate. Therefore, we recommend that meta-analysts in ecology and evolution use the phylogenetic multilevel model as the de facto standard when analyzing multi-species datasets.

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$N_{studies}$	$N_{species}$	σ^2_u	σ^2_s	σ^2_n	σ^2_p	α	Conditions	True model
20	40	0, 0.05, 0.30	0	0	0	1	3	Model 3
20	40	0.05, 0.30	0.05, 0.30	0	0	1	4	Model 5
20	40	0.05, 0.30	0.05, 0.30	0.05, 0.30	0	0.5, 1, 2	24	Model 7
20	40	0.05, 0.30	0.05, 0.30	0.05, 0.30	0.05, 0.30	0.5, 1, 2	48	Model 9
50	100	0, 0.05, 0.30	0	0	0	1	3	Model 3
50	100	0.05, 0.30	0.05, 0.30	0	0	1	4	Model 5
50	100	0.05, 0.30	0.05, 0.30	0.05, 0.30	0	0.5, 1, 2	24	Model 7
50	100	0.05, 0.30	0.05, 0.30	0.05, 0.30	0.05, 0.30	0.5, 1, 2	48	Model 9

Table 1: Overview of the conditions examined in the simulation study. The first two columns show the number of studies and species, respectively. The next four columns indicate the true values of the variance components. The α column represent the power parameter. All values were crossed within a particular row of the table. The last two columns respectively represent the number of conditions generated in each row and the model that corresponds to the true data generating mechanism for the conditions in a particular row.

(a) $(N_{studies}, N_{Species}) = (20, 40)$

	True Model			
Model Fit	Model 3	Model 5	Model 7	Model 9
Model 3	0.939 (100.00%)	0.668 (100.00%)	0.700 (100.00%)	0.700 (100.00%)
Model 5	2.653 (99.81%)	1.104 (100.00%)	1.151 (100.00%)	1.162 (100.00%)
Model 7	2.484 (97.53%)	1.876 (100.00%)	0.868 (100.00%)	0.858 (100.00%)
Model 9	3.316 (96.56%)	3.053 (99.73%)	2.288 (99.99%)	1.463 (99.99%)

(b) $(N_{studies}, N_{Species}) = (50, 100)$

	True Model			
Model Fit	Model 3	Model 5	Model 7	Model 9
Model 3	1.589 (100.00%)	1.313 (100.00%)	1.307 (100.00%)	1.294 (100.00%)
Model 5	3.934 (99.78%)	1.986 (100.00%)	1.999 (100.00%)	1.959 (100.00%)
Model 7	19.823 (96.86%)	14.752 (100.00%)	7.364 (100.00%)	7.393 (100.00%)
Model 9	25.980 (95.63%)	23.540 (99.60%)	18.641 (100.00%)	11.005 (100.00%)

Table 2: Average model fitting times in seconds and convergence rates (in parentheses) of all models under the different data generating mechanisms.

	$\hat{\mu}$	$SE[\hat{\mu}]$	95% CI	Z	p	$\hat{\sigma}^2_u$	$\hat{\sigma}^2_s$	$\hat{\sigma}^2_n$	$\hat{\sigma}^2_p$	AIC
Model 3	0.24	0.007	0.23, 0.25	34.15	<0.0001	0.0641	–	–	–	1082.8
Model 5	0.30	0.015	0.27, 0.33	20.42	<0.0001	0.0149	0.0806	–	–	429.0
Model 7	0.34	0.020	0.30, 0.38	17.37	<0.0001	0.0143	0.0195	0.0815	–	386.3
Model 9	0.37	0.130	0.11, 0.62	2.83	0.0046	0.0145	0.0192	0.0555	0.0512	344.7
Model 11	0.36	0.172	0.02, 0.70	2.07	0.0383	0.0149	0.0557	–	0.0914	367.2

Table 3: Results derived from fitting the various models to the example dataset. The first five columns show the estimated overall mean, its standard error, the 95% confidence interval, the test statistic, and the p-value for testing H_0: $\mu = 0$, respectively. The next four columns show the estimates of the variance components in the respective models. The last column shows the Akaike Information Criteria (AIC) values.

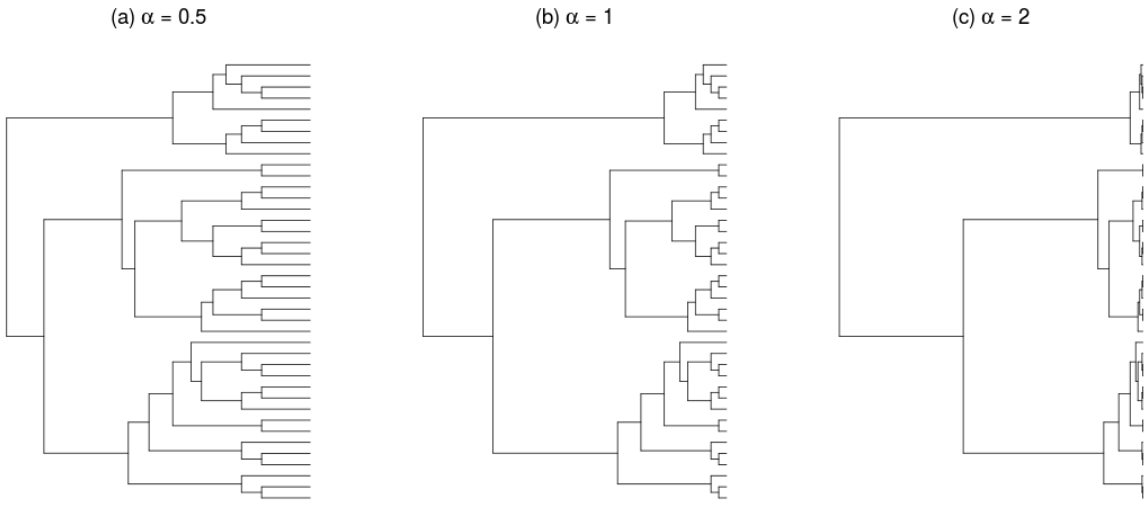

Figure 1: An example of a simulated phylogenetic tree for 40 species modified with different values of the power parameter α (i.e., 0.5, 1, and 2).

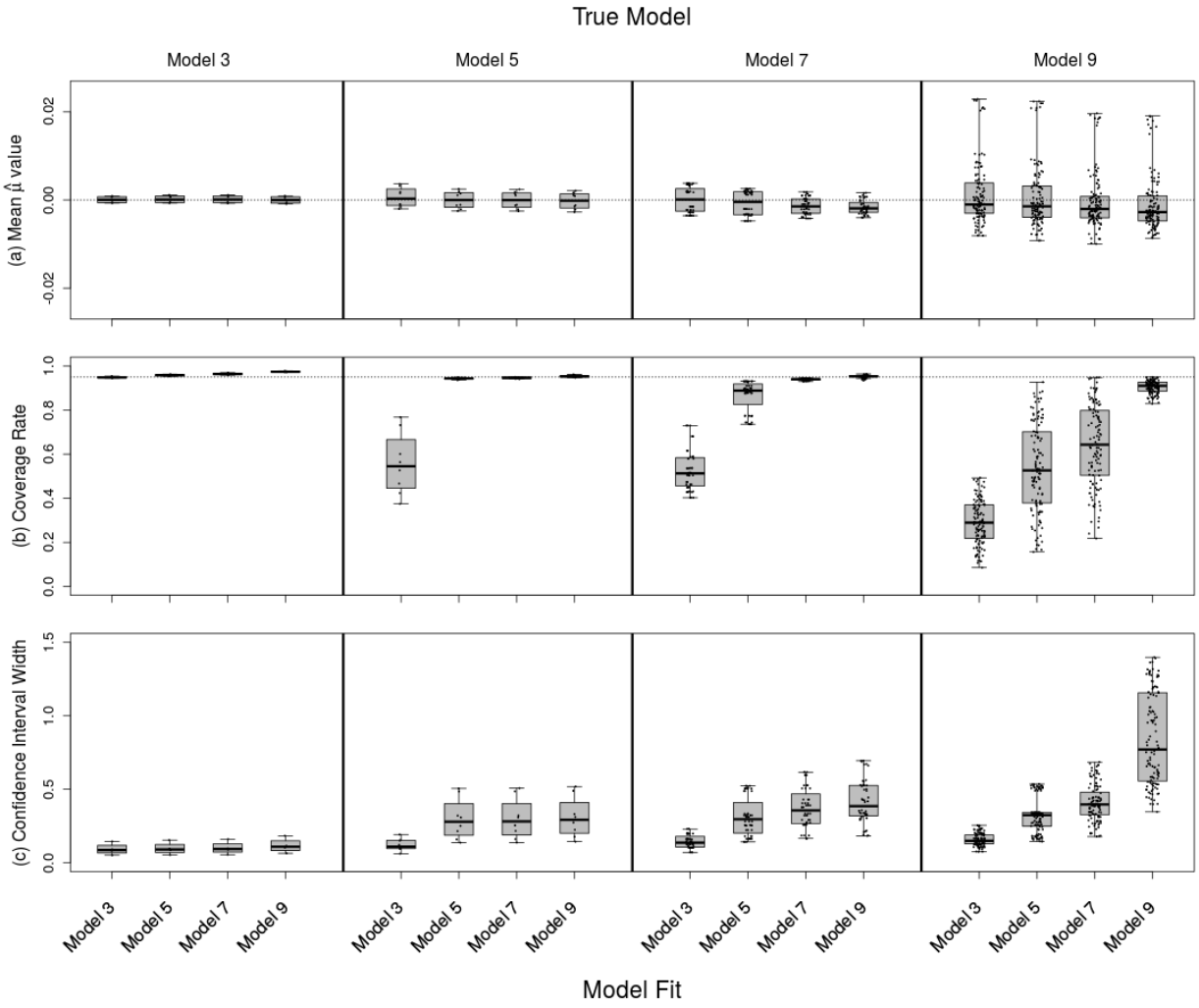

Figure 2: Boxplots based on the (a) mean $\hat{\mu}$ values, (b) coverage rates of the 95% confidence interval for μ, and (c) mean confidence interval widths for each of the four models across the 158 conditions, separated by which model was the true data generating mechanism.

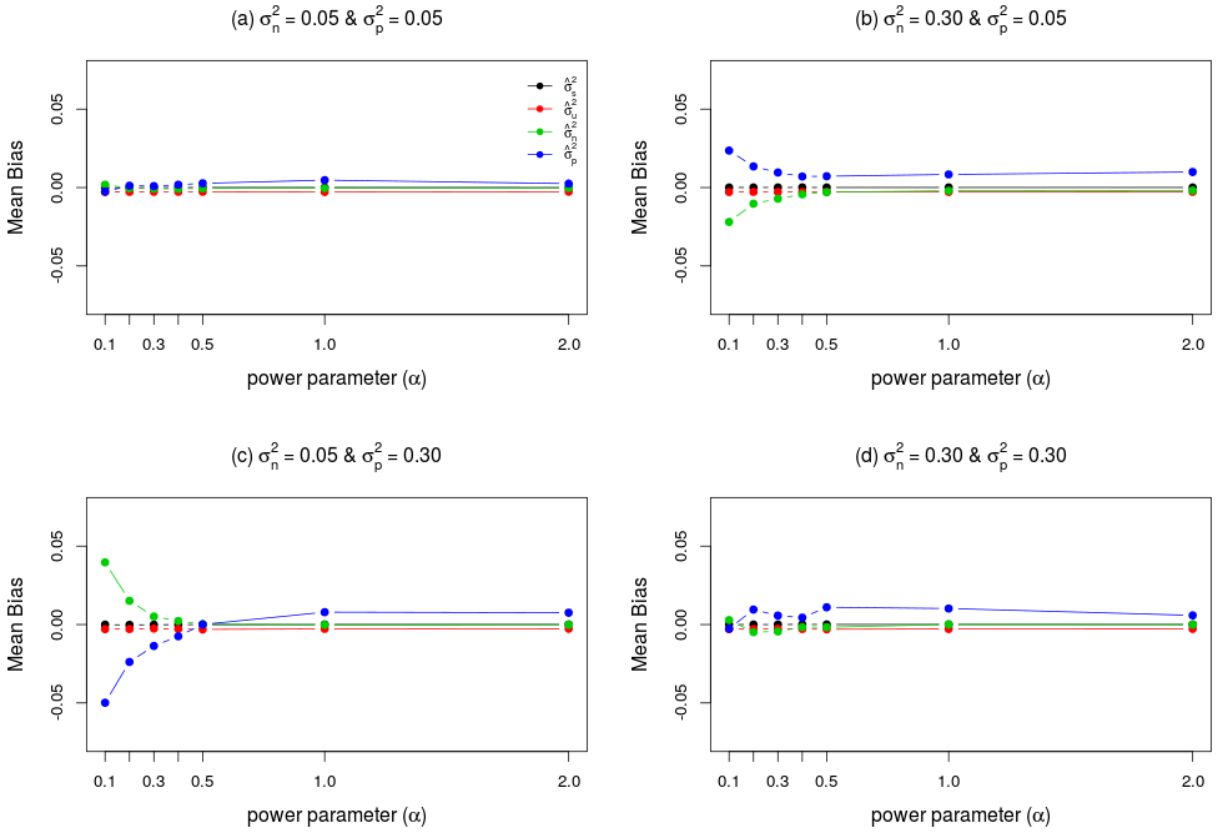

Figure 3: Mean bias of the variance component estimates of model 9 under different combinations of the power parameter (α) and the non-phylogenetic and phylogenetic variance components (σ_n^2 and σ_p^2, respectively).

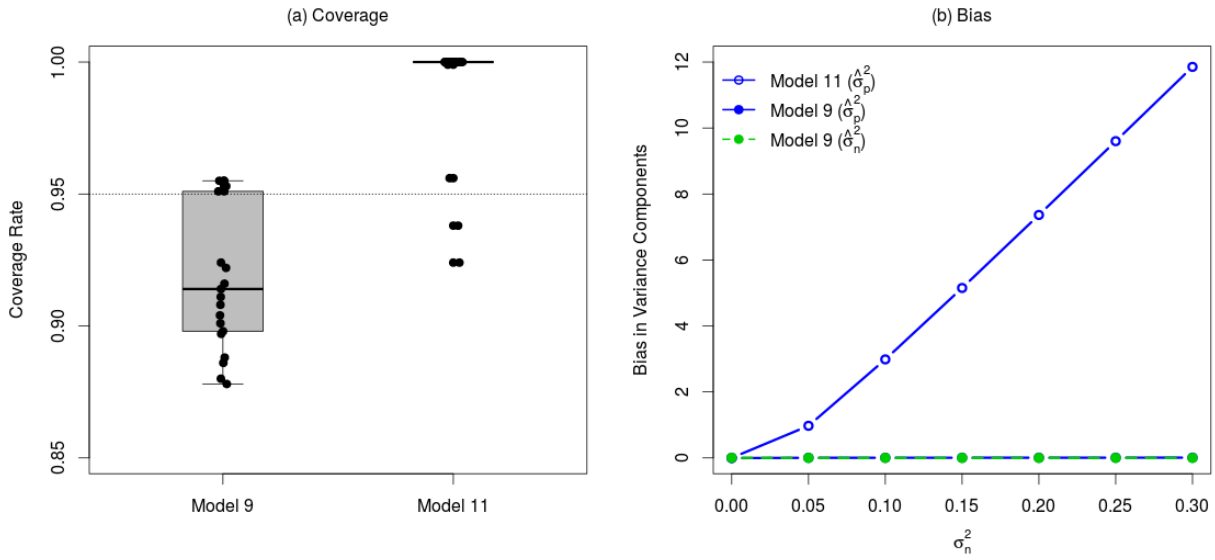

Figure 4: Comparison of models 9 and 11 as the size of the non-phylogenetic species-level variance component (i.e., σ^2_n) was systematically increased. (a) Coverage rates of the 95% confidence intervals for μ, (b) bias in the non-phylogenetic and phylogenetic variance components.