

1 Decomposing phenotypic skew and its effects on the
2 predicted response to strong selection

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9 **The major frameworks for predicting evolutionary change assume that a phenotype's**
10 **underlying genetic and environmental components are normally distributed. However,**
11 **the predictions of these frameworks may no longer hold if distributions are skewed.**
12 **Despite this, phenotypic skew has never been decomposed, meaning the fundamental**
13 **assumptions of quantitative genetics remain untested. Here, we demonstrate that**
14 **the substantial phenotypic skew in the body size of juvenile blue tits (*Cyanistes***
15 **caeruleus) is driven by environmental factors. Although skew had little impact on**
16 **our predictions of selection response in this case, our results highlight the impact**
17 **of skew on the estimation of inheritance and selection. Specifically, the non-linear**
18 **parent-offspring regressions induced by skew, alongside selective disappearance, can**
19 **strongly bias estimates of heritability. The ubiquity of skew and strong directional**
20 **selection on juvenile body size implies that heritability is commonly overestimated,**
21 **which may in part explain the discrepancy between predicted and observed trait**
22 **evolution.**

23 Two equations describe how traits respond to selection, the breeders equation¹ and Lande's
24 gradient equation². Both describe evolutionary change in terms of selection and inheritance.
25 Although these frameworks are generally thought to be interchangeable, they only converge
26 when phenotypes (and their genetic and environmental components) are normally distributed
27 or fitness functions are linear³. Given that fitness functions are highly unlikely to be linear in
28 practice^{4;5}, skew can lead to problems with the application of these equations. Consequently,
29 normality is seen as a fundamental assumption in quantitative genetics⁶⁻⁸, yet to our knowledge
30 has not been directly tested, despite the major consequences it has for how traits are predicted
31 to respond to selection⁹⁻¹⁷.

32 The breeders equation gives the predicted response to selection as the heritability (h^2) multiplied
33 by the selection differential (S). The most natural interpretation of heritability in the context
34 of the breeders equation is the slope of a *linear* parent-offspring (PO) regression^{12-14;18},
35 whilst S describes the *linear* relationship between a phenotype and fitness. The accuracy
36 of the breeders equation relies heavily on the linearity of both of these functions - if both
37 are non-linear, the residuals from the linear functions may be correlated, creating a 'spurious
38 response to selection'¹⁴. The linearity of the parent-offspring relationship breaks down when
39 the amount of skew differs between genetic and environmental components^{19;20}, with genetic
40 and environmental skew causing curvature in opposite directions (Figure 1). Lande's gradient
41 equation gives the response to selection as the additive genetic variance (V_A) of the trait
42 multiplied by the selection gradient (β). Whilst the gradient equation is robust to environmental
43 skew, it doesn't correctly describe the response to selection in the presence of genetic skew if
44 the fitness function is non-linear¹¹. Environmental skew can, however, impact the estimation
45 of β when using Lande-Arnold regression^{5;17;21}.

46 Although extensions to these two equations have been derived that allow for the non-linearity
47 of the PO-regression¹² and the non-normality of genetic values¹¹, the majority of the work in
48 this area remains theoretical. Whilst non-linearity in PO-regressions has been demonstrated
49 in the lab^{12;22-26} and ad-hoc methods have been used to test for skew at the genetic level^{27;28},
50 to our knowledge, no study has 1) relaxed the normality assumptions when making statistical
51 inferences to examine the origin and extent of skew at different levels, and 2) explored how
52 observed patterns of natural selection interact with skew to determine how well breeders and

53 gradient equations predict selection response in the wild.

54 Juvenile body size is under strong, persistent, directional selection across taxa²⁹, yet is known
55 to show little response to this selection³⁰. We show that juvenile body size is highly negatively
56 skewed across bird species, but the origin of this skew is unknown. To determine this, we
57 developed statistical methods to decompose the phenotypic distribution into a set of skew-t
58 distributions, and predict the shape of PO-regression based on the estimated skew. We applied
59 these methods to data from a long-term cross-fostering experiment of a wild bird population.
60 By estimating survival selection acting on juvenile body size, we tested the robustness of the
61 predicted response to selection from the breeders and gradient equations.

62 **Results**

63 **Prevalence of Phenotypic Skew**

64 Across 27 species of birds, tarsus length (a common measure of structural size) was substantially
65 negatively skewed in juveniles (-1.054 [-1.394, -0.686], pMCMC<0.001), but not adults
66 (-0.302 [-0.641, 0.052], pMCMC=0.086), with tarsus length being significantly more skewed
67 in juveniles than adults (-0.752 [-1.124, -0.366], pMCMC<0.001; Figure 2).

68 **Decomposing Phenotypic Skew**

69 Using data on four juvenile body size traits (tarsus length, head-bill length, mass and wing
70 length), measured on 15 day old chicks from a long-term cross-fostering experiment on a wild
71 population of blue tits, we decomposed phenotypic skew into genetic, between- and within-nest
72 environmental components. We used a mixed model approach with skew-t distributed random
73 effects which allowed the extent and direction of skew to vary between these levels. There
74 was considerable phenotypic skew in all four traits, with the coefficient of skew ranging from
75 -0.51 to -1.60 (Figure 3). There was little evidence of genetic skew in any trait (Figure 3,
76 Tables S5, S8, S11 and S12 and further discussion in supplementary methods). Phenotypic
77 skew was instead driven by considerable environmental skew at both between- and within-nest
78 levels, with the relative magnitude of this skew varying between traits (Figure 3, Tables S6,
79 S9, S12 and S15).

80 Given the environmental origin of the negative phenotypic skew, we would expect a convex
81 PO-regression for all traits¹⁹ (Figure 1C). Through deriving a method to compute this non-linear
82 PO-regression (Equation 1), we can show that for all traits the slope in the lower tail of the
83 distributions is close to zero, but becomes steeper with increasing body size (Figure 3).

84 **Selection on Juvenile Body Size**

85 To quantify selection acting on body size, we estimated the linear and quadratic effects of
86 body size on survival from both day 15 to fledging (leaving the nest) and fledging to local
87 recruitment in a bivariate probit event-history model. As expected, all traits showed significant
88 positive linear effects of body size on survival at both stages, with survival increasing at larger
89 body sizes (Figure 4, Tables S16-19). Interestingly, all quadratic effects of juvenile size on
90 survival between day 15 and fledging were positive, with these effects being suggestive and

91 significant for mass and wing length, respectively (Figure 4, Tables S16-19), indicating an
92 accelerating effect of size on offspring survival. In contrast, negative quadratic effects were
93 typical for survival from fledging to recruitment although this effect was only suggestive in the
94 case of tarsus length (Figure 4, Tables S16-19). The fitness functions over both events were
95 generally concave (Figure 4), which would indicate stabilising selection, but the hypothesis
96 that the optimal trait value lay outside of the observed phenotypic range for any trait could
97 not be rejected (proportion of iterations with an internal optimum: tarsus 0.853; head-bill
98 0.543; mass 0.757; wing 0.017).

99 Using these fitness functions, we were able to estimate selection gradients (β) for each trait by
100 taking the partial derivative of the individual relative fitness function with respect to the trait
101 and averaging it over the trait's distribution. However, β is more frequently approximated
102 using a Lande-Arnold regression of fitness on a trait²¹ and phenotypic skew can bias this
103 approximation when the fitness function is not linear or quadratic (as is the case for survival
104 functions)²¹. To test this, we calculated the expected estimates of β that would be obtained
105 from the Lande-Arnold approach without (β_1) and with (β_2) a quadratic term fitted^{21,31,32},
106 over the posterior distribution of the survival models (Equations 10 and 11). Figure 4 shows
107 that generally there is little meaningful difference between estimates, with the exception of
108 wing length, where there is suggestive evidence that β_1 would underestimate β by nearly 30%
109 (β_1/β : 0.711 [0.532, 0.915], pMCMC=0.012).

110 Predicted Response to Selection

111 In the absence of genetic skew, the correct response to selection is given by Lande's gradient
112 equation ($V_A\beta$), which for these traits gives: tarsus: 0.085mm [0.034, 0.127]; head-bill:
113 0.069mm [0.037, 0.102]; mass: 0.094g [0.052, 0.139]; wing: 0.175mm [0.077, 0.280]. The
114 breeders equation is equal to the gradient equation when the Lande-Arnold regression without
115 the quadratic term gives good estimates of the selection gradient, irrespective of whether the
116 PO-regression is linear or not (i.e if $\beta_1 = \beta$ then $h^2S = V_A\beta$). Given the similarity between
117 β and β_1 for tarsus, head-bill and mass, the breeders equation will therefore give accurate
118 predictions of the selection response for these traits. However, it underestimates the response
119 to selection in wing length by nearly 30%, as the proportional change in the predicted response
120 to selection is equal to β_1/β (shown above).

121 Selection Bias and Heritability Estimation

122 The heritability in the breeders equation is the heritability *before* selection (h_b^2) which can
123 be interpreted as the slope of the PO-regression averaged over all individuals irrespective
124 of their fitness. However, direct estimates of the PO-regression can only be obtained from
125 individuals that survive to become parents and so to some extent measure the heritability
126 *after* selection (h_a^2 ; note these terms are used differently from¹⁴). Since larger individuals are
127 more likely to survive, and the PO-regression is steeper for these individuals, direct estimates
128 of the PO-regression are likely to upwardly bias estimates of heritability. To demonstrate this,
129 we obtained direct estimates of the PO-regression from the 182 individuals (118 male and
130 64 female) that were measured as chicks and survived to produce offspring that were also
131 measured. Although the estimated linear regression (blue line in Figure 5) is similar to the
132 predicted non-linear PO-regression (red line in Figure 5) for the large surviving individuals

133 (the direct estimate and the predicted regression fit the data equally well for all traits; tarsus
134 $p = 0.195$, head-bill $p = 0.087$, mass $p = 0.060$ and wing $p = 0.052$), the two diverge
135 substantially at small body sizes (Figure 5). In order to directly compare h_a^2 and h_b^2 , we used
136 the parameters of the quantitative genetic and survival models described above calculate h_a^2
137 as the linear PO-regression weighted by the fitness of the parents (Equation 16) and h_b^2 as
138 V_A/V_P . For tarsus, head-bill and mass, h_a^2 was substantially and significantly higher than h_b^2 ,
139 with a proportional increase in h_a^2 of over 60% for head-bill and mass (h_a^2/h_b^2 : tarsus 1.223
140 [1.137, 1.333], pMCMC=0.002; head-bill 1.664 [1.421, 1.951], pMCMC<0.001; mass 1.645
141 [1.325, 2.046], pMCMC<0.001; wing 1.584 [0.373, 2.551], pMCMC=0.372).

142 Estimates of h_b^2 will only be accurate if they do not suffer from the same selection bias
143 present in PO-regression. Our experimental cross-fostering design means that the majority
144 of information in our pedigree comes from the comparison of siblings, rather than parents
145 and offspring, suggesting the bias should be small. However, many wild bird pedigrees rely
146 largely on information from parent-offspring relationships to estimate genetic effects - without
147 cross-fostering and using social pedigrees (no within-nest variation in relatedness), sibling
148 comparisons provide little information on genetic effects because they are confounded with
149 common environment (nest) effects. To test this, we simulated data using the parameters
150 from our quantitative genetic and selection models for mass, assuming social and genetic
151 monogamy, with and without skew and with and without partial cross-fostering. As expected,
152 environmental skew caused PO-regressions to be consistently and substantially upwardly biased
153 by a similar amount as we observed in our data, regardless of cross-fostering (estimated/simulated:
154 no cross-fostering 1.609; cross-fostering 1.616). Without cross-fostering, estimates of V_A ,
155 and so heritability, from animal models were upwardly biased, although less than in the
156 PO-regressions (V_A 1.226, h^2 1.228), whereas cross-fostering led to the correct estimation of
157 V_A (1.012) and h^2 (1.015; Figure 6).

158 Discussion

159 A common assumption in quantitative genetics is that phenotypes, and their underlying
160 genetic and environmental components, are normally distributed. Here we demonstrate
161 that this assumption is commonly violated, and in four morphological traits the observed
162 negative phenotypic skew is driven by environmental, rather than genetic, skew. There
163 was strong directional viability selection acting on all four traits, with non-linear fitness
164 functions. Under these conditions the breeders equation may give inaccurate predictions
165 for the response to selection, but Lande's gradient equation - which only assumes genetic
166 values are normally distributed - is expected to be accurate¹¹. However, this assumes that
167 the methods used to obtain estimates of β and V_A are robust to deviations from normality.
168 Here we empirically demonstrate that common methods used to estimate both metrics can
169 produce biased estimates in the presence of environmental skew.

170 Perhaps the most striking result is the apparent absence of genetic skew. Theory shows that
171 directional selection can generate genetic skew, but the direction of the skew differs between
172 models. Under the infinitesimal (Gaussian descendants³³) model (assumed in our analyses),
173 directional selection can drive a Gaussian distribution of breeding values to be skewed in
174 the direction of selection through the build up of linkage disequilibrium^{11;34;35}. However,

175 stabilising selection may mitigate this (¹¹ Eq 46) and the breeding value distribution quickly
176 returns to normality if selection ceases (the skew quarters each generation for unlinked loci;³⁴).
177 Finite allele models also generate genetic skew through changes in allele frequency. Under the
178 rare-alleles model, directional selection after a long period of stabilising selection generates
179 skew in the direction of selection^{10;11} but sustained long term directional selection (with
180 directional mutation) is expected to drive skew in the opposite direction to selection^{36;37}.
181 Given juvenile body size appears to be under sustained positive directional selection²⁹ and
182 gene knockout studies in mice provide evidence for directional mutation, with loss-of-function
183 mutations reducing size more often than increasing it³⁸, we would predict negative genetic
184 skew in our system. However, these models predict that the amount of skew generated
185 through selection should be small, consistent with our finding of no or negligible genetic
186 skew. Other processes, such as few loci, alleles of large effect, extreme allele frequencies
187 or substantial non-additive gene action, particularly directional dominance, could generate
188 greater levels of skew^{20;36;39;40}. This seems unlikely for body size, which appears to be highly
189 polygenic^{41;42}, although the finding that inbred individuals are on average smaller does suggest
190 some directional dominance^{43–46} which would also generate skew in the opposite direction to
191 selection. Two other studies have looked at the distribution of breeding values (indirectly
192 through estimating the skew of breeding values estimated in a Gaussian model) and while
193 one also found little evidence of skew²⁸, the other found skew in the opposite direction to
194 selection²⁷. More widespread assessments of the prevalence of genetic skew are needed to
195 assess the generality of these results.

196 Environmental skew has received little attention from theoreticians, with most studies assuming
197 that environmental effects are normally distributed^{11;12;14}. There are, however, several biological
198 processes that are known to induce environmental skew. For example, asymmetric competition,
199 when larger individuals have a disproportionate negative competitive effect on others, can drive
200 negative skew^{47–50}. Blue tits have moderate levels of hatching asynchrony (hatching spread is
201 approximately 2 days; see⁵¹ for distribution across bird species) which is expected to generate
202 asymmetries in competitive ability⁵² and therefore skew at the within-nest level. However, the
203 dominant source of phenotypic skew is at the between-nest level (contribution to phenotypic
204 skew relates to standardised skew and variance) and so if asymmetric competition was the main
205 driver of phenotypic skew, it would require parental ability to be driven by asymmetric adult
206 competition, perhaps through differences in condition and/or territory quality. An alternative
207 explanation is that (some) chicks have yet to reach their asymptotic size by the time of
208 measurement and so variation in their size at this time is driven by variation in growth rate
209 and asymptotic size. If variation in growth rate is largely at the between-nest level and variation
210 in the asymptote is largely genetic, as has been suggested in great tits⁵³, then the non-linearity
211 of growth functions could result in skew that is primarily environmental in origin (see⁵⁴ for
212 a related result). This skew would be expected to disappear further into development as all
213 chicks reach their asymptotic size, but due to the strong selective disappearance of small
214 chicks this may not necessarily manifest itself (see below).

215 The strong, negative environmental skew led the PO-regression in all traits to be convex. This
216 occurs because the long tail of small individuals are primarily small because of environmental
217 factors and so resemble their offspring less than larger individuals. Most discussions of
218 the linearity of the PO-regression focus on how, in combination with a non-linear fitness
219 function, a non-linear PO-regression leads the breeders equation to be inaccurate, through

220 generating a covariance between the residuals from a linear fitness function and the linear
221 PO-regression^{3;14} (see also Figure S18). This 'spurious response to selection'¹⁴ will be largest
222 when the non-linear fitness function and the PO-regression have the same (e.g. both concave)
223 or opposite (e.g. one concave and one convex) shape, causing a positive or negative covariance
224 between residuals, and so leading the breeders equation to under or over-estimate the response
225 to selection, respectively. Skew generates quite predictable and simple non-linearity in the
226 PO-regression (Figure 1), and so generally accelerating or decelerating fitness functions will
227 be more likely to generate a spurious response to selection, as is seen with wing length (Figure
228 S18).

229 We additionally show that the selective disappearance of small individuals alongside a non-linear
230 PO-regression leads to h^2 estimates that are biased towards the slope of the surviving large
231 individuals. This selection bias is particularly striking in estimates from PO-regression but
232 interestingly also occurs in animal models applied to pedigrees where information about
233 the genetic variance comes primarily from parent-offspring comparisons (e.g. typical bird
234 pedigrees). This occurs because both PO-regression and the animal model assume that
235 the relationship between offspring and parental phenotypes is linear, and so the missing
236 parent-offspring comparisons would follow the same slope. Previous work in this system
237 has shown that selection differentially eliminates negative environmental, but not genetic,
238 deviations for mass over the course of development⁵⁵. This was interpreted as mass being
239 an environmentally correlated target of selection rather than the true target⁵⁶. However,
240 incorporating skew into our models challenges this interpretation as, under our model, size is
241 the true target of selection. As the long tail of small individuals are small for environmental
242 reasons, the selective disappearance of these individuals drives the observed decrease in
243 environmental variance and skew through ontogeny. Given the selective disappearance previously
244 observed was prior to the measurements analysed here⁵⁵ it seems likely that the environmental
245 skew we observe is an underestimation of the true skew, meaning we are likely underestimating
246 the true non-linearity of the PO-regression. Multivariate methods would account for this
247 selective disappearance⁵⁷, however, these proved to be complex to implement in this instance.

248 Given the consistent negative environmental skew we see across the four traits, and the
249 conserved nature of negative phenotypic skew in juvenile (but not adult) size across bird
250 species, we believe a concave PO-regression for juvenile size traits might be a general finding.
251 As found here, juvenile body size is also generally under strong viability selection across
252 taxa²⁹. Together, this suggests that previous heritability estimates of juvenile size are likely
253 to have been systematically over-estimated, especially as a large proportion are based on
254 PO-regressions⁵⁸. Indeed, tarsus length heritability estimates from PO-regressions have been
255 shown to be consistently larger than those from animal models⁵⁸. Juvenile size is a hallmark
256 trait of evolutionary stasis, whereby traits that should respond to selection in the wild appear
257 not to. Although these results do not fully explain this stasis, they do show that the predicted
258 response to selection may be being substantially overestimated in traits with non-Gaussian
259 phenotypic distributions.

260 Lande-Arnold regression is by far the most common method for estimating β ^{5;31;59} and is
261 known to be unbiased in the presence of phenotypic skew only if the fitness function is
262 linear or quadratic *and* this quadratic term is modelled²¹. Although the estimated survival
263 functions deviated from a quadratic for all traits, estimates of β were close to those that

264 would have been obtained under Lande-Arnold regression including the quadratic term (β_2)
265 for all traits, and without the quadratic term (β_1) for three traits. The near equivalence
266 of these different estimates seems at odds with the conclusions of Bonamour *et al.*¹⁷, who
267 demonstrate that selection gradients approximated with Lande-Arnold regression are biased
268 in the presence of phenotypic skew. However, Bonamour *et al.* only modelled the linear term
269 in the Lande-Arnold regression (β_1) whilst assuming a quadratic fitness function - had the
270 quadratic term also been included, the linear term in the Lande-Arnold regression (β_2) would
271 have been unbiased^{3;21}, in correspondence with our wing length results (β_1 underestimated
272 β , but β_2 did not). However, there is no reason to believe including a quadratic term in a
273 Lande-Arnold regression will generally result in a good approximation of β . Indeed, Morrissey
274 & Sakrejda⁵ compared β with that approximated from a quadratic Lande-Arnold regression
275 and found quite large proportional differences (30%), although small differences in absolute
276 terms. We therefore urge caution in assuming that our results are a general statement about
277 the accuracy of Lande-Arnold regression under non-normality.

278 Quantitative genetics uses two frameworks to predict how traits will respond to selection.
279 Here we demonstrate how both of these frameworks are affected by skew at the environmental
280 and genetic levels. Genetic skew can lead both the breeders equation and Lande's gradient
281 equation to be inaccurate. Although little or no genetic skew has been found in the few
282 studies that have tried to quantify it, it remains unknown to what extent this is a generality,
283 and will be highly dependent on the genetic architecture of specific traits. In the absence of
284 genetic skew, the gradient equation presents an accurate prediction of selection response¹¹,
285 although environmental skew provides challenges to the accurate estimation of both β and V_A .
286 Whilst the breeders equation may provide a more intuitive way of thinking about selection
287 response, the extensions to this framework that allow for non-linearity¹² are complex and
288 computationally expensive. We therefore recommend a focus on the gradient equation (and
289 its extensions¹¹) in wild systems, where fitness functions are highly likely to be non-linear and
290 trait distributions are commonly skewed.

291 Methods

292 This study was preregistered (see <https://osf.io/7qyp4/>). We have highlighted in the following
293 sections where our methods deviate from those planned.

294 Meta-analysis of Skew

295 We collected raw data on juvenile and adult tarsus length from several sources: we used a
296 mailing list to request data, we searched the dryad repository for 'tarsus', we emailed groups
297 with known long-term avian datasets that were not represented in these sources and included
298 any tarsus length data that we otherwise encountered. When datasets from different studies of
299 the same population overlapped in time, we use the largest single dataset available. Datasets
300 were taken from ^{42;60-97}.

Sample standardised skew was estimated from raw data z as

$$\frac{\frac{1}{n} \sum_{i=1}^n (z_i - \hat{\mu})^3}{\left[\frac{1}{n} \sum_{i=1}^n (z_i - \hat{\mu})^2\right]^{3/2}} \frac{\sqrt{n(n-1)}}{n-2}$$

with sampling variance as

$$\frac{6n(n-1)}{(n-2)(n+1)(n+3)}$$

301 where n is sample size and $\hat{\mu}$ the estimate of the trait mean.

302 Using this data, we ran a random-effect meta-analytic model in MCMCglmm with age (juvenile
303 or adult) as a fixed factor and random effects of species and study. Models were run for
304 65000 iterations, with a burnin of 15000 and a thinning intervals of 50. The priors for the
305 random-effect variances were scaled (by 100) $F_{1,1}$ and the prior for the residual variance was
306 inverse-gamma with a shape and scale of 0.001. The fixed effects had a diffuse normal prior
307 (mean=0, variance= 10^{10}).

308 Study population

309 We used data from a nest-box population of blue tits (*Cyanistes caeruleus*), on the Dalmeny
310 estate, Edinburgh, United Kingdom, collected from 2011 to 2018, with 253 nest-boxes over
311 two sites. Detailed methods are described in ^{55;98}. Briefly, all nests were visited regularly until
312 the discovery of the first egg, and then daily for egg cross-fostering, when eggs were weighed.
313 From 2011-2013 and 2016-2018 a partial egg cross-fostering design was used to enable additive
314 genetic and nest-of-rearing effects on offspring size to be separated ⁵⁵. In 2014-2015 a mixture
315 of full and partial cross-fostering was used as part of a separate experiment. Full details of
316 cross-fostering can be found in ⁹⁹. After egg laying was complete, nests were left undisturbed
317 for 11 days and then checked daily for hatching. At hatching (day 0), all chicks were uniquely
318 marked (within a nest). The chicks had blood samples taken at day 3 and were given a unique
319 metal ring at day 9. At day 15, chick's tarsus, wing and head-bill lengths were measured and
320 they were weighed. For the morphometric measurements, one chick from each nest was
321 measured twice in order to account for measurement error ⁵⁵. From day 10, adults were

322 caught at the nest in order to identify them; blood samples and morphometric measurements
323 were taken and the birds were uniquely ringed. At the end of the season we checked all
324 nests and recorded any dead chicks left in the nest. From this we could infer which chicks
325 fledged. Chicks were considered recruited if they were recaptured as breeders in subsequent
326 years.

327 Social parentage was assigned through catching parents at the nest. When no female was
328 caught, the social female was assigned a dummy mother identity. When no male was caught,
329 the social father was assigned as the genetic sire with the largest proportion of paternity in
330 a nest, either a male caught at a different nest that year, or an unsampled male assigned a
331 dummy identity.

332 For the assignment of genetic parentage and chick sex, genotypes were obtained using blood
333 and tissue samples from adults and chicks. Genotyping and pedigree reconstruction largely
334 followed protocols outlined in⁵⁵ and⁹⁸. However, adults not caught in the focal year but that
335 were known to be alive (because they were caught in subsequent years and were aged 2 years
336 or over) were allowed to be parents of chicks in the focal year. The distance between the
337 nest-of-origin of the chicks and the nest at which the candidate parents were caught in the
338 subsequent year was fitted as a covariate. Mothers were allowed to be polygamous when
339 (half) sib-ships were assigned to chicks with unknown fathers (see Supplementary Methods).
340 When assigning chick sex, we used morphological sexing of recruits over molecular sexing from
341 chicks (sexing didn't match for 5 chicks).

342 For our analysis we included data on chick size measured at day 15 post-hatching, collected on
343 this project from 2011-2018, and additionally chick recruitment data from 2019 and 2020. We
344 included all nests for which hatching date was known. Although similar morphological data
345 was collected in 2010, we excluded all records from this year as egg size was not measured.
346 Egg size was used to account for nest-of-origin effects in our models (see below). We also
347 excluded data from an additional two nests where egg size was not measured, from chicks for
348 which molecular sexing was not successful (n=20 chicks) and where we did not have one of
349 the day 15 measurements (n=11 chicks). In total, we had records of 5123 day 15 chicks in
350 715 nests, with 642 chicks repeatedly measured.

351 **Statistical analysis**

352 All models were run in a Bayesian framework. From all models posterior means and 95%
353 credible intervals are presented. A p-value for the fixed effects and covariances in these
354 models was approximated (pMCMC) as two times the smaller number of iterations where the
355 parameter value is either less than zero or greater than zero¹⁰⁰. We use a threshold of 0.005
356 to refer to results as significant and those between 0.05 and 0.005 as suggestive¹⁰¹.

357 **Decomposing phenotypic skew using hierarchical models**

358 We modelled the four traits (tarsus length, head-bill length, mass and wing length) measured
359 at day 15 using linear mixed effects models with sex (2 level factor), year (8 level factor), time
360 of day (continuous - hours from midnight) and egg size (continuous) as fixed. Additive genetic
361 and nest-of-rearing effects were modelled as random. Because we have repeated measurements
362 of tarsus, wing and head-bill lengths, we additionally modelled measurement error effects in

363 these traits, by including bird identity effects, which are equivalent to the residuals in a model
364 without repeat measures, and the residuals are measurement error effects⁵⁵. In contrast to
365 past analyses^{55,98}, we do not model nest-of-origin effects but rather include egg size as a
366 covariate to account for these effects (see⁵⁵ and Supplementary materials). As estimating
367 skew-t distributed random effects (see below) is parameter heavy, including a covariate rather
368 than a random effect is preferable, especially as nest-of-origin effects are very small for these
369 traits^{55;98}.

370 Skew due to the fixed effects was obtained by multiplying the fixed effect design matrix by the
371 fixed effects and either estimating the parameters for the skew-t distribution of the resulting
372 variable to use for the calculation of non-linear parent offspring regression or obtaining the
373 sample skew for plotting. This assumes that the joint distribution of the covariates is equal to
374 the empirical distribution we observe. In combination with a diffuse prior on the fixed effects,
375 this assumption probably leads to a small inflation in the estimated (absolute) skew. Time
376 of day was excluded from this estimate as any skew induced by this is due to our sampling
377 design rather than being biologically relevant.

378 In order to estimate skew in the random effects, we fitted random effects with skew-t
379 distributions. The residuals for the repeat measured traits were treated as Gaussian as these
380 represent measurement errors. As with the normal distribution, the skew-t distribution¹⁰²⁻¹⁰⁵
381 has a location ξ and scale ω parameter, but also parameters δ and ν which modify the skew
382 and tailness of the data, respectively. The distribution converges on a normal distribution when
383 $\delta = 0$ and ν approaches infinity. As δ moves away from 0 and ν decreases the (absolute) skew
384 in a variable increases, with the sign of δ signifying the direction of the skew. The skew-t
385 distribution is unbounded and readily allows for considerable amounts of positive and negative
386 skew. The reasons for the use of this distribution are further discussed in the supplementary
387 materials. Our approach to modelling the additive genetic effects is to extend standard
388 quantitative genetic models by allowing the base population breeding values to have a skew-t
389 distribution, with normally distributed Mendelian sampling deviations in the descendants (with
390 variance $\omega^2(1-\bar{F})/2$ where \bar{F} is the average inbreeding coefficient of the individual's parents).
391 This assumes that inheritance occurs under the Gaussian descendants infinitesimal model^{33,106},
392 i.e. the Mendelian sampling deviations are normally distributed within families, and any genetic
393 skew results from selection. In practice, however, the Mendelian sampling deviations are largely
394 confounded with residual effects in our data because there are few parent-offspring comparisons
395 (due to high migration and low recruitment) and so inferences are probably quite robust to
396 any violation of the Gaussian descendants assumption. Initially we tried to fit this model in an
397 animal model framework, but due to poor mixing we chose to approximate the model using
398 a dam-sire model. This model discards information about the Mendelian-sampling deviations
399 and subsumes them in the residual effects which then come from a mixture distribution¹⁰⁷.
400 Given there is little information in our data about the Mendelian-sampling deviations the
401 dam-sire and animal models are expected to give almost identical answers (see Supplementary
402 Materials). Although this method allows us to directly estimate skew in breeding values, when
403 the environmental residuals are skew-t, as assumed here, the mixture distribution does not
404 have standard form. Here, we approximate the mixture distribution as skew-t and although
405 we cannot derive the full distribution of the environmental residuals we are able to obtain their
406 variance and skew. These models provided little evidence for genetic skew in any trait and so
407 we reverted to an animal model with normally distributed breeding values - the animal model

408 approach having the advantage that the environmental residual skew can then be directly
409 estimated. The dam and sire effects were modelled in a multi-membership model where the
410 two sets of effects were constrained to having the same skew-t distribution.

411 Initially we intended to model chick mass over ontogeny in a multivariate framework (see
412 preregistration), as in previous studies of this population^{55;98}. However, implementing the
413 required multivariate skew-t models proved too challenging. Since there is strong directional
414 selection on chick body mass throughout ontogeny^{55;98}, our estimates of skew at day 15
415 are likely underestimates as the univariate analysis used will fail to account for selective
416 disappearance prior to day 15^{55;98}. We also planned to have a global box-cox parameter
417 in case there was a single transformation that would make everything linear and additive.
418 However, given the problems we had with implementing more complex models, we chose not
419 to include this additional complexity.

420 It should also be noted that estimates from these skew-t models seem to be more sensitive
421 to unmodelled heteroskedasticity than standard Gaussian mixed effects models, even when
422 skew exists, and this can lead to biased fixed effect and variance estimates. This led us
423 to fit a reduced set of fixed effects compared with previous analyses^{55;98} and outlined in
424 our pre-registration (see Supplementary materials). To partly address this issue we also ran
425 equivalent Gaussian models for all skew-t models, and present the results in the Supplementary
426 materials. There were small differences between models but the results remain qualitatively
427 the same (see SM; Figure S17, Tables S4-15).

428 These models were run using Stan (version 2.21.0)¹⁰⁸ using the cmdstanr package (Stan
429 Development Team, 2019) in R (version 4). Four chains were run for each model with
430 a warmup of 4000 iterations and 6000 iterations post-warmup, with the exception of the
431 dam-sire wing length model which was run with a warmup of 5000 iterations and 10000
432 iterations post-warmup. Convergence of individual chains was visually assessed, as well as
433 ensuring that the Gelman–Rubin diagnostic (R-hat) across chains was less than 1.1¹⁰⁹. We
434 used diffuse normal priors for fixed effects (mean=0 and standard deviation=100), half-Cauchy
435 priors (mean=0 and standard deviation=10) for standard deviations and uniform priors from
436 -1 to 1 for δ and 4 to 40 on ν . The choice of priors is discussed further in the Supplementary
437 materials.

438 **Non-Linear Parent-Offspring Regression**

439 The PO-regression function is defined as $E[z_o|z]$ where z_o is the phenotype of offspring from a
440 parent with phenotype z . Assuming random mating and environmental values in the offspring
441 (e_o) are independent of parental phenotypes this becomes $\frac{1}{2}E[g|z] + \frac{1}{2}E[g] + E[e_o]$ under
442 the Gaussian descendants assumption, where g is breeding value. Have θ_g be the parameters
443 of the breeding value distribution and θ_e the parameters of the environmental distribution.
444 Then,

$$E[g|z] = \frac{\int (z-e)p(z-e|\theta_g)p(e|\theta_e)de}{\int p(z-e|\theta_g)p(e|\theta_e)de} \quad (1)$$

445 The integrals have to be evaluated numerically, which is time consuming, and so the regression
446 function was evaluated at the posterior mean of the parameters from the skew-t animal models

447 to give $E[z_o|z]$ for each trait (Figure 5). Also note that in the presence of pre-breeding survival
 448 selection the term $\frac{1}{2}E[g]$ in the intercept of the regression function should be replaced by
 449 $\frac{1}{2}(E[g] + \Delta g)$ where Δg is the change in mean breeding value due to selection such that
 450 $E[g] + \Delta g$ is the expected breeding value of the other parent.

451 Selection on chick body mass

452 Given that we were not able to model chick body mass in a multivariate framework, we did
 453 not model survival throughout ontogeny as originally planned (see preregistration), but rather
 454 modelled survival from day 15 to fledging and fledging to recruitment. We modelled this as
 455 an event history in a probit regression (binomial error distribution and probit link function)
 456 including a quadratic effect of chick size at day 15 on both events, allowing us to model the
 457 stabilising component of selection. These models accounted for measurement error in tarsus,
 458 head-bill and wing lengths, using the repeated measurements of these traits. Originally we
 459 planned to correct our measurements for time of day effects (see preregistration). However,
 460 these effects proved to be very small and for most traits non-significant (see Supplementary
 461 Results). We therefore decided not to add this extra complexity into our models.

462 Sex, day of hatching within the nest, year, clutch size, male presence, nest hatch date were
 463 also included as fixed effects. All fixed effects were allowed to differ between the two events.
 464 Finally we modelled the 2x2 covariance matrix of nest-of-rearing effects. This model was
 465 run using Stan. Four chains were run for each model with 5000 iterations and a warmup of
 466 2500 iterations with a thinning interval of 10. Convergence of chains was assessed as above.
 467 Diffuse priors for fixed effects (mean=0 and standard deviation=100), half-Cauchy priors for
 468 all standard deviations (mean=0 and standard deviation=10) and LKJ priors on correlations
 469 with shape=2¹¹⁰.

470 The Individual Relative Fitness Function

471 Partitioning the linear predictors for each survival event (1: day 15 to fledging, 2: fledging to
 472 recruitment) into a part due to the trait and a part due to remaining terms (denoted η), and
 473 assuming that the distribution of $\eta^{(1)}$ and $\eta^{(2)}$ are bivariate normal conditional on the trait z ,
 474 then the absolute fitness function has the form:

$$W(z) = F_{MVN}(s|\Sigma) \quad (2)$$

475 where F_{MVN} is the multivariate normal cumulative density function in which the first argument
 476 is the quantile to be evaluated and the second argument is the (co)variance of the variates
 477 (the means are zero and are therefore not given). For event i

$$s^{(i)} = E[\eta^{(i)}] + \frac{COV(\eta^{(i)}, z)}{\mu_2}(z - \mu) + \beta^{(i)}z + \frac{1}{2}\gamma^{(i)}z^2 \quad (3)$$

478 where $\beta^{(i)}$ and $\frac{1}{2}\gamma^{(i)}$ are the linear and quadratic effect of the trait on event i , μ is the trait

479 mean and μ_i the i^{th} central moment of the phenotypic distribution.

$$\Sigma^{(i,j)} = COV(\eta^{(i)}, \eta^{(j)}) - \frac{COV(\eta^{(i)}, z)COV(\eta^{(j)}, z)}{\mu_2} + COV(u^{(i)}, u^{(j)}) + \delta^{(i,j)} \quad (4)$$

480 where $u^{(i)}$ are the nest effects for event i and $\delta^{(i,j)} = 1$ when $i = j$ and represents the residual
481 variance.

482 The partial derivative of $W(z)$ with respect to z is given by

$$\begin{aligned} \frac{\partial W(z)}{\partial z} = & f_N(s^{(1|2)}|\Sigma^{(1|2)}) \left(\frac{COV(\eta^{(1)}, z)^2}{\mu_2} + \beta^{(1)} + \gamma^{(1)}z - \frac{\Sigma^{(1,2)}}{\Sigma^{(2)}} \left(\frac{COV(\eta^{(2)}, z)^2}{\mu_2} + \beta^{(2)} + \gamma^{(2)}z \right) \right) \\ & F_N(s^{(2)}|\Sigma^{(2)}) + f_N(s^{(2)}|\Sigma^{(2)}) \left(\frac{COV(\eta^{(2)}, z)^2}{\mu_2} + \beta^{(2)} + \gamma^{(2)}z \right) F_N(s^{(1|2)}|\Sigma^{(1|2)}) \end{aligned} \quad (5)$$

483 where f_N and F_N are the density and cumulative density functions for a centred normal
484 distribution, and

$$s^{(1|2)} = s^{(1)} - \frac{\Sigma^{(1,2)}}{\Sigma^{(2)}}s^{(2)} \quad \Sigma^{(1|2)} = \Sigma^{(1)} - \frac{(\Sigma^{(1,2)})^2}{\Sigma^{(2)}} \quad (6)$$

485 Solving Equation 5 to find the stationary point(s), and therefore the optimal trait value, is
486 difficult. Instead we evaluated the derivative of Equation 5 at the minimum and maximum
487 observed trait value and assessed whether the derivative at the minimum is positive and
488 negative at the maximum. This condition implies an optimal trait value within the range of
489 observed trait values.

490 Selection Gradients

491 The Lande-Arnold method²¹ for estimating the selection gradient is only robust to phenotypic
492 skew if the fitness function is quadratic and both the mean-centered trait value and its square
493 are fitted in the regression^{3,21}. We therefore computed three selection gradients. Using the
494 notation in³², we calculated our best estimate of it¹¹¹,

$$\beta = E \left[\frac{\partial w(z)}{\partial z} \right] = \int \frac{\partial w(z)}{\partial z} p(z) dz \approx \frac{1}{n} \sum_{i=1}^n \frac{\partial w(z)}{\partial z} \Big|_{z_i} \quad (7)$$

495 where $p(z)$ is the probability density function for z , $w(z)$ is the relative fitness function
496 obtained by dividing $W(z)$ by mean fitness ($E[W] = \int W(z)p(z)dz$) and z_i are the observed
497 trait values. Put simply, we calculated the mean partial derivative of individual fitness function
498 (from Equation 5) across our observed phenotypic distributions, divided by mean fitness.

499 The linear selection differential is defined as

$$S = \int zw(z)p(z)dz - \mu \approx \frac{1}{n} \sum_{i=1}^n z_i w(z_i) - \hat{\mu} \quad (8)$$

500 and the quadratic selection differential as

$$C = \int (z - \mu)^2 p(z) w(z) dz - \mu_2 \approx \frac{1}{n} \sum_{i=1}^n (z_i - \hat{\mu})^2 w(z_i) - \hat{\mu}_2 \quad (9)$$

501 From these we can calculate the expected linear regression coefficient from the Lande-Arnold
502 method when only the linear term was fitted:

$$\beta_1 = \frac{S}{\hat{\mu}_2} \quad (10)$$

503 and the linear regression coefficient from the Lande-Arnold method when both the linear and
504 quadratic term are fitted (Eq. 29.28a from³):

$$\beta_2 = \frac{(\hat{\mu}_4 - \hat{\mu}_2^2)S - \hat{\mu}_3 C}{\hat{\mu}_2(\hat{\mu}_4 - \hat{\mu}_2^2) - \hat{\mu}_3^2} \quad (11)$$

505 Selection cannot operate on between-sex differences in trait values (the average fitness of
506 the two sexes is constrained to be equal) and we assume that selection does not operate on
507 between-year differences in trait values (which might occur if juvenile size impacts on adult
508 survival). We therefore estimated each β as the average of each sex by year combination
509 (Figure 4 e-h), calculated across the posterior distribution of the survival model.

510 Response to Selection

511 The extension of Lande's gradient equation to a non-normal distribution of genetic effects is
512 (combining Equations 26 and 42 from¹¹):

$$\Delta z = \sum_{j=1}^{\infty} K^{j+1}(g) \frac{1}{j!} \int \frac{\partial^j w(z)}{\partial z^j} p(z) dz \quad (12)$$

513 where $K^j(x)$ denotes the j^{th} cumulant of x , which up to the third cumulant (skew) is

$$\Delta z = V_A E \left[\frac{\partial w(z)}{\partial z} \right] + \frac{S_A}{2} E \left[\frac{\partial^2 w(z)}{\partial z^2} \right] \quad (13)$$

514 where S_A is the skew in the additive genetic effects. When the distribution of additive genetic
515 values is normal and/or the fitness function is linear, Equation 12 reduces to Lande's gradient
516 equation

$$\Delta z = V_A E \left[\frac{\partial w(z)}{\partial z} \right] = V_A \beta \quad (14)$$

517 since all cumulants > 2 of the genetic distribution are zero.

518 Heritability

519 We compared how well our inferred non-linear PO-regression (Equation 1) performed at
520 predicting offspring phenotype compared to linear single-parent mid-offspring regression. Using
521 the 182 individuals (118 male and 64 female) that were measured as chicks at day 15 and
522 survived to produce offspring that were also measured at day 15, we fitted a weighted (by
523 family size) regression with our inferred non-linear PO-regression fitted as an offset. We then
524 compared the fit of this model to an identical model but where the raw parental phenotype
525 was also fitted as a covariate with a free parameter.

526 We then compared estimates of the heritability before and after selection (h_b^2 and h_a^2 , respectively).
527 The heritability can be defined as the regression coefficient of a linear mid-PO-regression, and
528 can be calculated before selection

$$h_b^2 = 2 \frac{COV(z_o, z)}{\mu_2} = \frac{V_A}{V_P} \quad (15)$$

529 or after selection

$$h_a^2 = 2 \frac{E[w(z)z_o z] - E[w(z)z_o]E[w(z)z]}{E[w(z)z^2] - E[w(z)z]^2} \quad (16)$$

530 The posterior distribution of h_b^2 was evaluated directly, but the i^{th} posterior sample of h_a^2
531 was obtained by simulating 10^4 values of z and z_o using the parameters sampled at the
532 i^{th} iteration of the trait model, calculating expected fitness for each sampled z using the
533 parameters sampled at the i^{th} iteration of the fitness model, and then evaluating the relevant
534 expectations.

535 Simulations

536 To test how different sampling designs and standard estimation procedures (PO-regression
537 and Gaussian animal model) impact estimates of heritability in the presence of skew and
538 selection, we simulated data according to the posterior mean of the parameters from our
539 skew-t quantitative genetic and selection models for mass. A closed population with 1000
540 breeding pairs was simulated over three generations, with 10 measured full-sib offspring per
541 pair. Simulations were set up with either no cross-fostering or with nests paired and five
542 offspring reciprocally crossed and with either skew t-distributed random effects (with ω , δ
543 and ν parameters set to their posterior means) or normally distributed random effects with
544 matching variance. The probability of a chick recruiting to be a parent was obtained by
545 applying the estimated survival model for chick mass to the simulated phenotype. Each of the
546 four scenarios were simulated 2000 times and for each data set the heritability was estimated
547 directly using PO-regression and as the estimate of the additive genetic variance over the sum
548 of all variance estimates from a Gaussian animal model fitted in ASRemI-R¹¹².

549 Data availability

550 All data and code can be found at <https://doi.org/10.5281/zenodo.5342526>.

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557 **Author contributions**

558 JLP and JDH conceived and designed the project. JLP, HEL, CET and JDH generated the
559 data. JLP and JDH analysed the data and wrote the paper. All authors have read and
560 approved the paper.

561 **Competing interests**

562 The authors declare no competing interests

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842 **Figures**

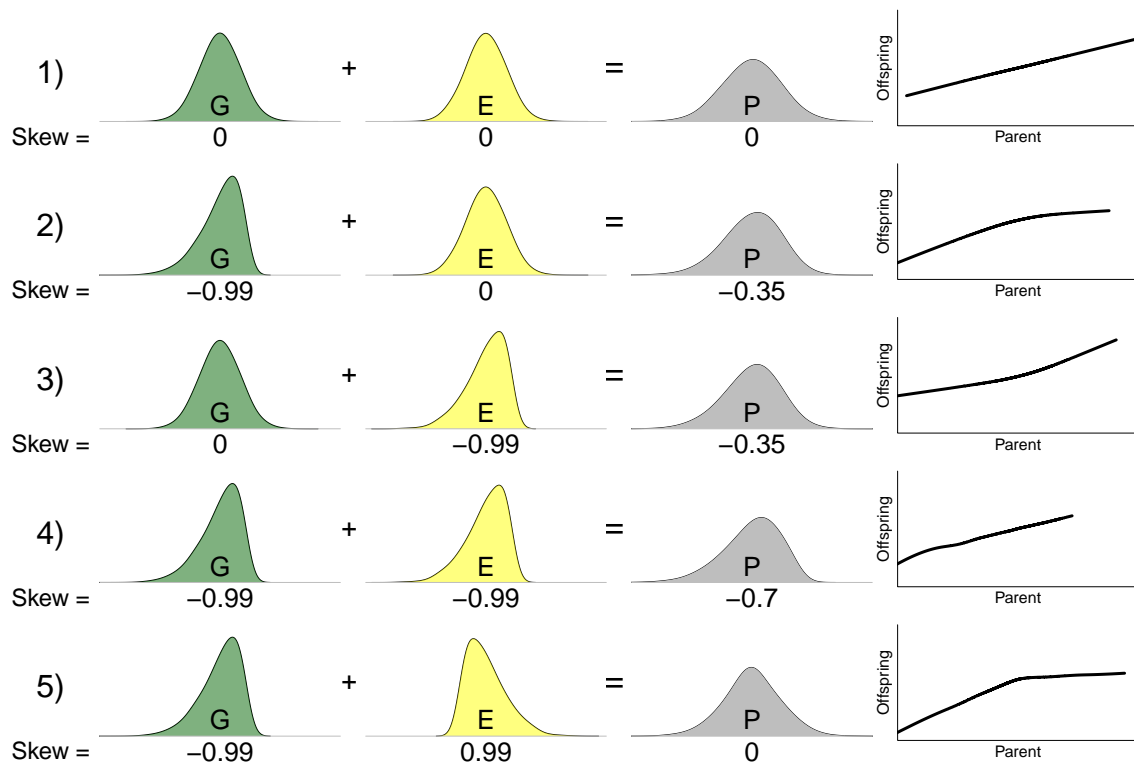


Figure 1: The effects of different distributions of breeding values (G) and environmental values (E) on the phenotypic distribution (P) and the shape of the PO-regression. When both genetic and environmental values are normally distributed (1), as typically assumed, there is a linear PO-regression. Negative genetic (2) and environmental (3) skew affect the shape of the parent-offspring relationship in opposite directions, whilst inducing the same phenotypic skew. If genetic and environmental distributions are skewed in the same direction (4) their effects on the parent-offspring relationship can cancel each other out, giving a linear parent-offspring relationship, despite considerable phenotypic skew. If genetic and environmental are skewed in opposite directions (5), although they may cancel each other out at the phenotypic level, they induce a highly non-linear parent-offspring relationship. 1-5) are all simulated with a heritability (V_A/V_P) of 0.5.

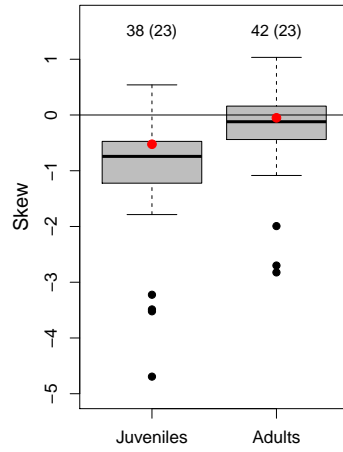


Figure 2: Skew in the distribution of avian tarsus lengths across different species. In the boxplots, the center line shows the median; box limits show upper and lower quartiles; whiskers show 1.5x interquartile range; points show outliers. Numbers above the plots show the number of estimates, and species in parenthesis. The red points show the skew in our blue tit data.

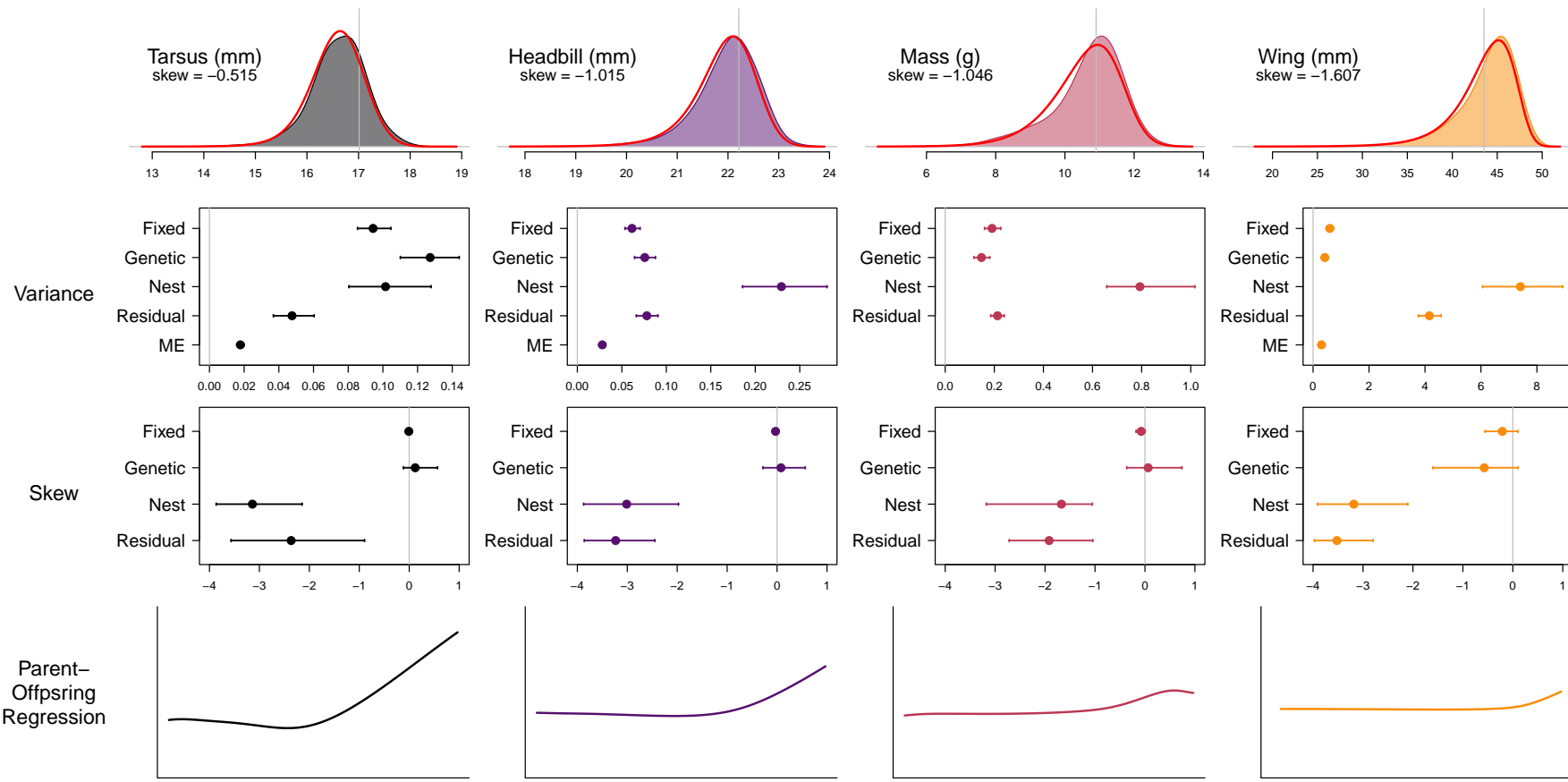


Figure 3: Decomposition of variance and skew in juvenile body size traits in blue tits. Top plots shows the phenotypic distribution of the traits, with the red line showing the distribution predicted from the skew models. The middle rows show the variance and skew (top and bottom, respectively) for each component for all four traits, with all model estimates coming from the skew-t animal model, except the genetic skew which was estimated in the skew-t dam-sire model (see methods). ME stands for measurement error. The bottom row shows the predicted shape of the PO-regression based on the model estimates.

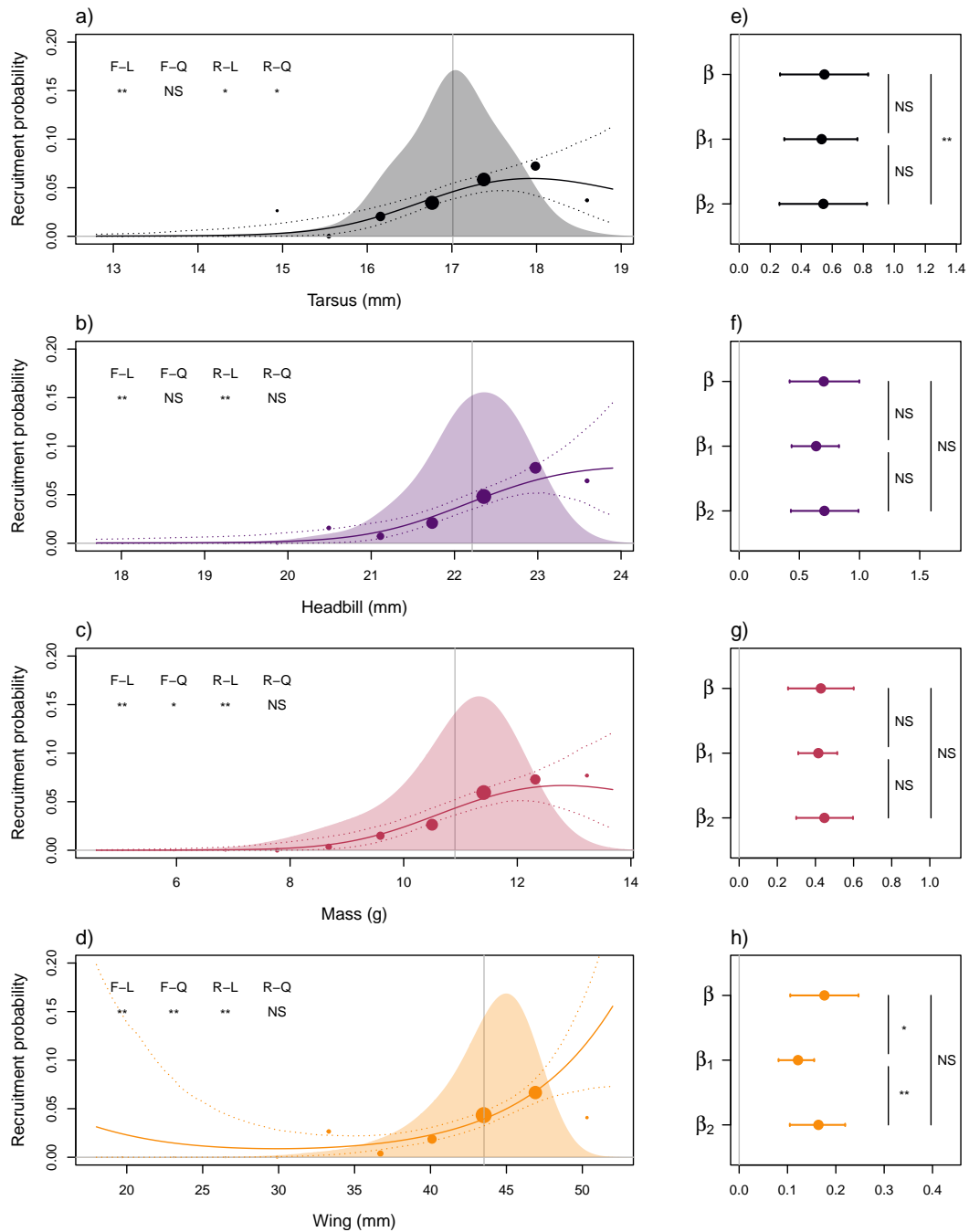


Figure 4: Average fitness functions (a-d) and selection gradients (e-h) for tarsus length, head-bill length, mass and wing length, respectively. In plots a-d, solid lines show the posterior mean fitness functions, dotted lines show the 95% credible intervals, and points show the average survival of individuals in equally spaced intervals. The size of the points is proportional to the square root of the sample size. The phenotypic distribution of the traits is shown, with the grey vertical line showing the phenotypic mean. The significance of the effect of the trait on fitness is also shown, 'F' and 'R' are survival to fledging and recruitment respectively, and 'L' and 'Q' and linear and quadratic effects. In plots e-h, β refers to the selection gradient, β_1 and β_2 refer to the approximations from the Lande-Arnold regression excluding and including a quadratic term, respectively. In all plots 'NS' indicates $p > 0.05$, '*' indicates $0.05 > p > 0.005$ and '**' $p < 0.005$.

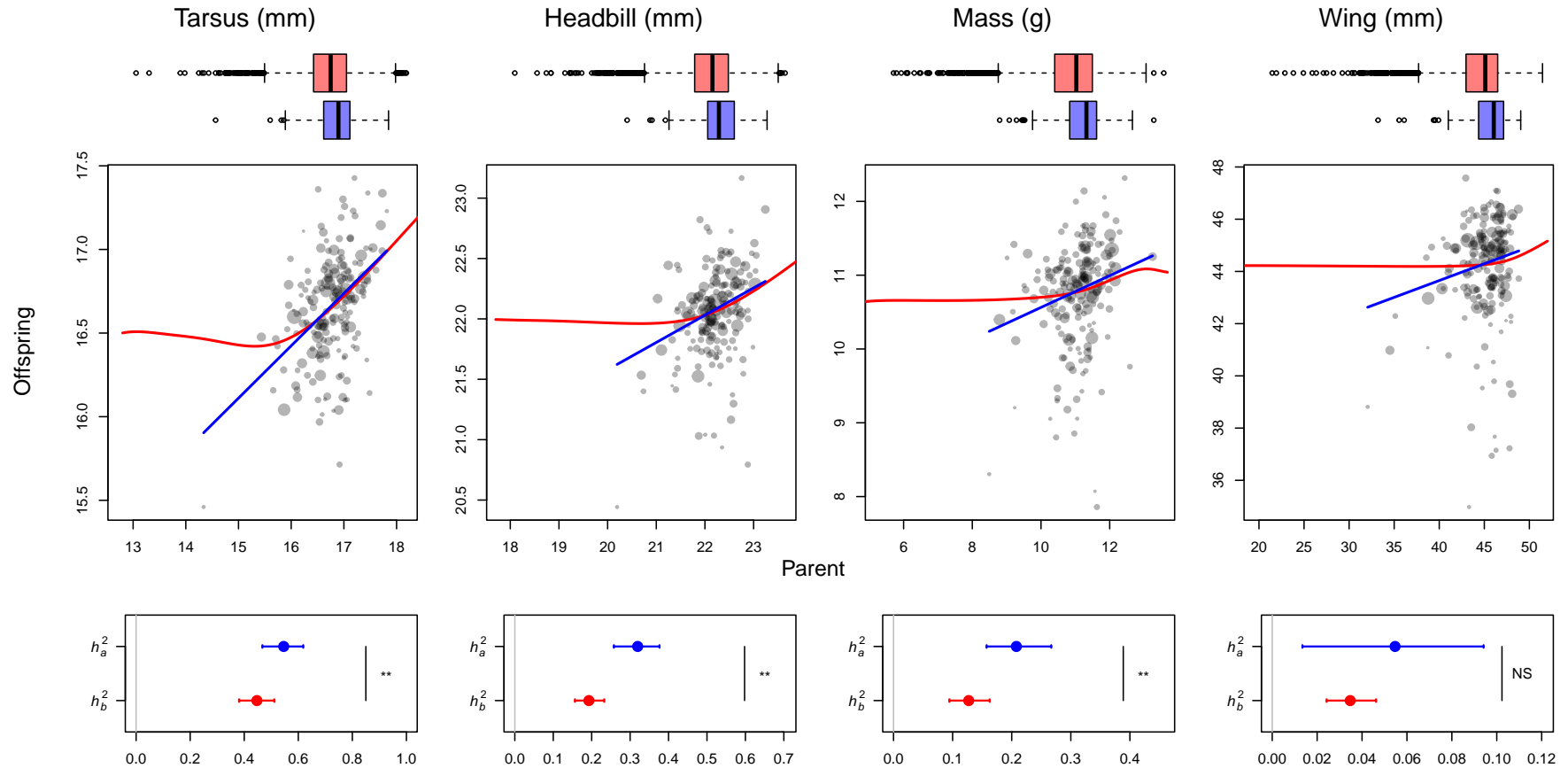


Figure 5: PO-regressions for four body size traits. Top panels show distribution of all chicks (red) and those that survived to recruit (blue). Scatter plots show mid-offspring versus single parental traits. Values are corrected for year, sex and time of day at which they were measured, and the size of the points is proportional to the square root of the family size. The red line is the predicted non-linear PO-regression based on the posterior means of the parameters from the skew-t quantitative genetic model and the blue line is the fit of a weighted (by family size) linear regression to the actual data. These are not corrected for measurement error. Lower panels show the comparison between heritabilities calculated before (h_b^2) and after (h_a^2) selection, calculated across the posterior distribution of the skew-t animal model trait models. In these lower plots all heritabilities account for measurement error. In all plots 'NS' indicates $p > 0.05$, '*' indicates $0.05 > p > 0.005$ and '**' $p < 0.005$.

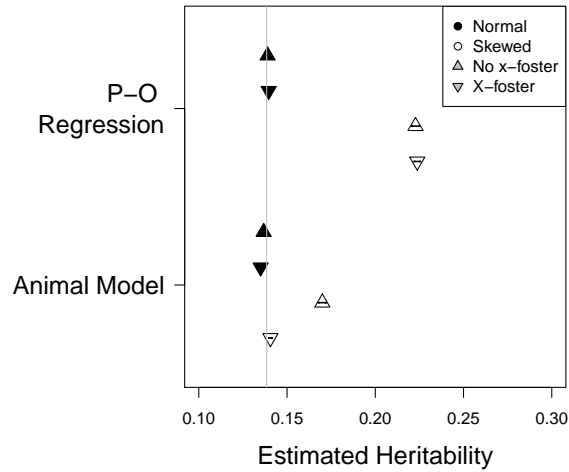


Figure 6: Average estimates of heritability from PO-regression and Gaussian animal models across 2000 simulated data sets. Three-generation simulations were set up with either no cross-fostering or with nests paired and half of each nest's offspring reciprocally crossed. Phenotypes were simulated according to the model estimated for chick mass exactly (skewed) or as Gaussian with matching variance. The probability of a chick recruiting to be a parent was obtained by applying the estimated survival model for chick mass.