Revisiting Taylor's law, rediscovering Smith's law: A dual framework for scaling heterogeneity

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Abstract: Taylor's (1961) power law (TPL)—variance scales with mean density—originated in agricultural entomology but now spans the natural and social sciences and even the humanities. Cited over 8,000 times and featured in *Nature* for decades, its reach far exceeds its ecological origins. Yet TPL has been dormant in *Nature* for the past two decades, though it remains active in *PNAS*. Smith's (1938) law (SPL)—crop yield variance scales with plot area—tells a different story. Cited approximately 500 times, almost exclusively in the agricultural literature, Smith's law has itself never appeared in *Nature* or *PNAS*. This disparity is not a minor oversight. It is a missing half. SPL captures environmental heterogeneity — the structured properties of the abiotic template. TPL captures ecological heterogeneity — the interacting distribution of organisms. Together, they form a dual framework for scaling heterogeneity across agents and their environment. We propose a hierarchical, testable model: Taylor's exponent depends on Smith's exponent and species traits shaped by evolution. The parallel to AI is striking. OpenAI's scaling law (agent) and DeepMind's refinement (template) mirror TPL and SPL. Both have played critical roles in guiding LLM training — yet neither cited Taylor or Smith. This convergence suggests a general principle: complex systems — from evolving ecosystems to engineered AI — exhibit dual scaling laws for agents and the templates (policies, environments, rules) that shape them. Smith's law was not wrong. It was forgotten. Rediscovering it offers a unified lens to predict heterogeneity across space, time, and disciplines — from ecology to economics to AI governance.

Keywords: Taylor's power law (TPL), Smith's power law (SPL), ecological heterogeneity, environmental heterogeneity, scaling laws, template vs. agents, AI scaling laws, scale invariance, universality

Introduction

More than sixty years ago, Taylor (1961) published in *Nature* a power law relating the variance of population density to its mean — a pattern now known as Taylor's power law (TPL), cited over 8,000 times. Decades earlier, Smith (1938) described a different power law, relating the variance of crop yield to plot area — largely confined to agricultural literature, cited ~500 times. Despite shared mathematical form, these two laws have rarely been discussed together. The only high-profile discussion of both was a comprehensive review of TPL by Eisler, Bartos & Kertész (2008) in *Advances in Physics*, who mistakenly considered TPL a rediscovery of Smith's law — an error later corrected by Xu (2015) in a preprint reviewing 50 years of TPL though Xu did not pursue the matter further.

TPL appeared regularly in *Nature* from 1961 (Taylor 1961) to about 2003 (Kilpatrick & Ives 2003), then vanished — though it became active in *PNAS*. Smith's law has never appeared in *Nature* or *PNAS*. Here we revisit TPL and "rediscover" Smith's law. Following Shavit & Ellison (2021), heterogeneity is not merely variation. It is the property of collectives whose constituent entities interact and integrate into larger, functioning wholes — fundamentally different from diversity, which merely counts variation within non-interacting populations. A zoo is diverse: many species live in separate enclosures, never interacting. An ecosystem is heterogeneous: plants, animals, and microbes constantly interact, compete, cooperate, and reshape their environment. Within this framework, TPL captures ecological heterogeneity — the structured, interacting distribution of organisms — while Smith's law captures environmental heterogeneity — the structured properties of the abiotic template (soil, resources, habitat) that both shape and are shaped by those interactions (Ma & Ellison 2026, Ma et al. 2026).

Importantly, TPL has proven suitable for other complex systems — from insects through the stock market to computational intelligence — where it describes variance scaling in everything from population counts to price fluctuations to fitness distribution in genetic algorithms (Ma, 2012). A concrete contemporary example is the scaling of tokens in AI research: the performance of large

language models follows predictable power laws with respect to model size, data volume, and compute (Kaplan et al., 2020; Hoffmann et al., 2022). This suggests that our discussion of TPL and Smith's law may open a door for general complex science: investigating the heterogeneity of templates versus players, or more broadly, how the scaling of variance differs between the "environment" (the fixed template) and the "agents" (the players operating within it).

Unifying TPL and Smith's law addresses how heterogeneity propagates across coupled ecological and environmental systems. This would allow researchers to predict population variance from environmental variability, design sampling protocols that account for both biotic and abiotic heterogeneity, and build a common scaling language across organisms and their habitats. We set aside "universality" — powerful in physics, but often ambiguous and a potentially conceptual bag in ecology, economics, medicine, and other complexity science fields where precise experiments are difficult due to ethics and the uncontrollability of Mother Nature. Instead, we focus on what these two laws share, where they differ, and how they might be connected. Our goal is not to declare a grand unified theory, but to reopen a conversation dormant in *Nature* for two decades: how power laws describe heterogeneity across ecological and environmental domains, and what that tells us about scaling in complex systems. By extending this conversation beyond entomology and ecology, we invite researchers from economics, computer science, and complex systems theory to explore a general framework for heterogeneity scaling — one that distinguishes between the statistical properties (such as AI infrastructure) of the template (the environment, shaped by regulatory strategies, policies, or chip architectures) and the players (the agents within it, such as LLMs).

Taylor's power law (TPL)

Taylor (1961) related the variance of population density to a power of the mean density:

$$V = am^b \quad (b>0) \tag{1a}$$

To see the scaling property, change the scale by a factor k (e.g., double the sampled area). If the mean scales proportionally ($m_k = km_0$), then substituting into Taylor's power law (TPL) Eqn. (1a) gives:

$$V_k = a(m_k)^b = a(km_0)^b = k^b\left(am_0^b\right) = k^b V_0 \quad (1b)$$

Thus, TPL implies scale-invariance with respect to the mean: variance scales as k^b times the original variance (V_0). No correction is needed because the mean *m* is itself the natural scaling variable — there is no external, independent quantity that biases the exponent. TPL captures ecological heterogeneity (the structured, interacting distribution of organisms).

Smith's power law (SPL)

Smith (1938) related the variance of crop yield to plot area:

$$V_x = ax^{-b_s'} \quad (2a)$$

However, the observed exponent b' depends on the total field size *n*. Unlike TPL, where the mean defines the scale, here plot size *x* and field size *n* are not independent. To remove this dependence, Smith corrected to an **infinite field** ($n \to \infty$) yielding a true exponent *b* that is independent of field size. The corrected law retains the same mathematical form:

$$V_x = ax^{-b_s} \text{ (after infinite field correction)} \quad (2b)$$

Thus, SPL implies **scale-invariance with respect to plot size**, but only after applying the infinite-field correction. The negative exponent ($-b_s$) indicates that variance decreases as plot size increases — a signature of environmental heterogeneity.

The Nature Series and the Modern Synthesis

The publication of Taylor's power law (TPL) in *Nature* framed the central debate about the exponent bb (ranging from 1 to 3) from its empirical discovery (Taylor, 1961) through mechanistic links to migration and behaviour (Taylor & Taylor, 1977; Taylor, 1980; Taylor et al., 1983), culminating in the direct question of whether the exponent's specificity was taxonomic or functional (Taylor et al., 1988; Ma, 1991). This series of *Nature* papers initiated extraordinary cross-disciplinary research that expanded far beyond ecology — into statistics, physics, finance, computer science, and even the humanities — and continued actively in *PNAS* and other leading journals after disappearing from *Nature* around 2003. Over the six decades since, TPL research has evolved across three distinct periods: from ecological mechanisms and statistical distributions,

through mathematical foundations and sampling theory, to complex networks and early-warning signals for tipping points (Ma & Taylor, 2025).

Smith's Law: The Missing Environmental Quantification

Taylor et al.'s (1988) *Nature* paper identified TPL's context-dependence and robustness but lacked a metric to account for its differences between dissimilar habitats. That metric was published 50 years earlier. Smith's (1938) exponent b_s (range: 0.05 to 0.85) explicitly quantifies the scale-dependence of environmental variance — the spatial autocorrelation of the environmental template — the stage upon which ecological actors perform and which is quantified by TPL. Unlike TPL, Smith's law remains largely unknown outside crop science and experimental agriculture, having never appeared in *Nature* or *PNAS*. Does this disproportionate attention to TPL and neglect of Smith's law suggest a missed opportunity for unifying ecological and environmental heterogeneity?

Cross-Disciplinary Parallels: Scaling Laws in Deep Learning

Strikingly parallel power-law relationships have emerged in deep learning. Kaplan et al. (2020) of OpenAI first demonstrated that test loss scales as a power law with model size, dataset size, and compute — spanning seven orders of magnitude, cited over 8,000 times. This paper served as the blueprint for the modern scaling era of AI, inspiring GPT-3, GPT-4, and beyond.

Hoffmann et al. (2022) of DeepMind later refined this, showing that for compute-optimal training, model size and training data must be scaled equally — a correction (cited over 2,500 times) that led to the Chinchilla model (70B parameters), which outperformed GPT-3 (175B) by being trained on proportionally more data. This shifted industry focus from "bigger is always better" to the economic reality that models had been systematically undertrained.

Though discovered independently, without citation to Taylor or Smith, these deep learning laws share the same mathematical form as TPL and SPL: an aggregate property scales as a power law with a fundamental resource. The parallel extends further: just as TPL dominates while SPL remains neglected, OpenAI's law is far more cited than DeepMind's refinement — despite the latter being a crucial correction. This convergence across ecology, agriculture, and computer science suggests that power-law scaling of aggregate properties may be a general feature of complex systems, whether composed of insects, soil patches, or neural parameters.

Reciprocal Relationships between Ecological and Environmental Heterogeneity

In ecology, heterogeneity branches into two complementary prongs. Environmental heterogeneity influences biodiversity through heterogeneity–diversity relationships (HDR). Ecological heterogeneity influences ecosystem stability through heterogeneity–stability relationships (HSR). These two forms engage in recursive feedback across scales: environmental heterogeneity shapes ecological heterogeneity through niche filtering and resource gradients, while ecological heterogeneity modifies the environmental template through ecosystem engineering and habitat modification (Ma & Ellison, 2026).

This reciprocity extends far beyond ecology. Across statistics, economics, medicine, physics, and computer science — wherever heterogeneity matters — the same dichotomy holds: the template (environment, market conditions, tissue microenvironments, network architecture) and the agents (organisms, traders, cells, nodes) are locked in recursive feedback (Ma & Ellison 2026; Ma et al. 2026). TPL captures the scaling of agent heterogeneity; Smith's law captures the scaling of template heterogeneity. If reciprocal heterogeneity is general across complex systems, then Smith's law deserves the same cross-disciplinary attention as TPL.

Toward a Unified Ecological and Environmental Heterogeneity

The similarity of TPL and SPL leads us to propose that b_T is not a fixed trait but a hierarchical consequence of its dependence on the heterogeneity of habitats:

$$b_T \approx f(b_S, \phi) \tag{3}$$

where ϕ represents species or community traits shaped primarily by evolutionary forces (e.g., migration, dispersal, diapause). Thus, ϕ captures the spatial behaviour of a species. Although the power-law principle has proven universal, extending from metazoan species populations to microbial communities (Ma, 2015), and b_T exhibits invariance within a single, stable habitat (e.g., insects in greenhouses (Taylor et al., 1988) and human microbiomes at specific sites (Ma & Taylor, 2020), it can show significant variance across different habitats and environmental templates (e.g., gut vs. reproductive system microbiomes (Ma & Taylor, 2020). Furthermore, b_T can be an evolutionary property, stable over evolutionary time within a defined phylogenetic clade, whereas ecological communities operate on a much shorter timescale (Ma & Ellison, 2024, 2026). This pattern is precisely the expected outcome if b_T is contingent on a relatively constant b_S within a

single habitat but varies when the underlying environmental scaling (b_S) changes or differs between geographically distinct habitats.

Taylor's power law has been extended to measuring the heterogeneity of ecological networks (Ma, 2025). However, a complete theory for landscape or ecosystem heterogeneity must integrate both environmental and ecological heterogeneities. Combining the frameworks of Smith and Taylor achieves this. Sophisticated heterogeneity analysis methodologies then become necessary. For example, a recent three-layer analysis — base metrics, network, and application layers — offers standardized estimators, captures how local heterogeneity propagates through system architecture, and allows customization without altering core protocols. This hierarchical, modular design ensures scalability, reproducibility, and cross-disciplinary portability (Ma & Ellison, 2025). Its architecture draws inspiration from the ISO 7-layer OSI model (Zimmermann, 1980), which has served as the blueprint for modern digital communications from the early Internet to today's 5G wireless networks.

Implications and Future Directions

From Scaling to Prediction

As suggested by Eqns. (1) and (2), the scaling of variance is a remarkably general empirical phenomenon, shown for fluctuation scaling in physical systems (Eisler et al., 2008). TPL and SPL are different manifestations of the same self-similar scaling of variance. Their integration opens surprising new directions for a unified scaling theory. For example, a recent synthesis argues that TPL bridges ecological and statistical realms, connecting to prime number distributions, while providing a framework for detecting tipping points and analyzing heterogeneity–stability in complex networks (Ma & Taylor, 2025). We do not invoke the strong physical meaning of "universality" (same exponent, same class, theoretical derivation). Instead, we note the empirical generality: the same mathematical form recurs across domains. Our combination of b_S and b_T provides the groundwork for such advances by anchoring network and stability analyses in measurable interactions between environmental structures and biological traits.

The hierarchical relationship (Eqn. 3) transforms the dual framework from a descriptive analogy into a testable model. Three implications follow.

First, it resolves a persistent confusion. b_T is not a universal constant but contingent on environmental heterogeneity (b_S) and species traits (ϕ). This prevents the common error of treating TPL as standalone and Smith's law as a footnote.

Second, it enables cross-scale prediction. Changes in land use, habitat fragmentation, or resource patchiness (b_S) should predictably shift b_T. Cross-habitat comparisons already show that microbiomes from different body sites yield different b_T values, consistent with differences in their environmental templates (Ma & Taylor, 2020).

Third, it generates falsifiable experiments. Eqn. (3) is testable. In microcosms — e.g., flour beetle populations or microbial metacommunities — manipulating environmental grain (b_S) should measurably alter population aggregation (b_T). If no change occurs, the framework fails.

Crucially, this hierarchical model is falsifiable through such experiments. For complex systems beyond model ecosystems, three-layer heterogeneity analysis (Ma & Ellison, 2026), traditional simulation modeling, and AI/ML technologies can be applied.

These implications are not abstract. They turn a historical synthesis into an active research program.

Conclusion

Taylor's power law and Smith's power law were discovered independently, decades apart, in different journals, for different communities. Yet they share the same mathematical core: variance scales as a power law. Their separation was an accident of disciplinary history, not a necessity of nature.

We have argued that TPL captures ecological heterogeneity — the structured, interacting distribution of organisms — while SPL captures environmental heterogeneity — the structured properties of the abiotic template (soil, resources, habitat) that both shape and are shaped by those interactions (Ma & Ellison, 2026). Together, they form a dual framework that captures the recursive feedback between agents and their environment. The hierarchical model [i.e., $b_T \approx f(b_S, \phi)$] makes this framework testable, predictive, and open to extension beyond ecology.

Notably, TPL applies not only to spatial heterogeneity but also to temporal heterogeneity (stability) scaling, which is why it has been used to detect early warning signals for tipping points and to study phase transitions in complex systems (Ma & Taylor, 2025). This temporal dimension further reinforces the generality of the dual framework. Smith's law, though originally formulated for the spatial scaling of environmental variance, should have temporal analogues as well — a direction we leave for future inquiry.

The neglect of Smith's law is not a minor oversight. It is the missing half of a complete theory of spatial heterogeneity and temporal stability. 'Rediscovering' it — and joining it with Taylor's law — offers a unified lens to see, measure, and predict the complex fabric of ecosystems, where the scaling of life is inextricably linked to the scaling of its environment, across both space and time.

Beyond ecology, this unification carries profound implications for cross-disciplinary fields. Consider AI: the scaling laws of deep learning (Kaplan et al., 2020; Hoffmann et al., 2022) describe how test loss improves with model size, data, and compute — the "agent" side. But the most compelling challenge facing AI today is not scaling models further. It is alignment, safety, and security — how to ensure that powerful agents behave reliably. These challenges are fundamentally about the template: the regulatory frameworks, governance policies, and technical guardrails that shape the environment in which AI agents operate. Different administrations implement different rules — some permissive, some restrictive — creating heterogeneous templates whose scaling properties remain unquantified. Where is the analogue of Smith's law for AI governance? Does the variance of safety outcomes scale with the complexity of regulatory templates? Unifying TPL and SPL in ecology suggests that analogous dual laws may govern other complex systems: in economics (market volatility vs. regulatory architectures), in medicine (tumor heterogeneity vs. treatment protocols), and in social sciences (behavioral heterogeneity vs. institutional structures). Recognizing this pattern transforms how we ask questions across disciplines: not "what is the scaling law for agents?" but "what are the dual scaling laws for agents and the templates — policies, rules, and environments — that constrain and shape them?"

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