Title:

Spatial Correlation Between Soil Phosphorus Distribution and Its Uptake in Bean Roots Using Lagrange, Hermite and IDW Interpolation Methods

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Abstract

Phosphorus (P) is a key macronutrient required for plant growth, energy transfer, and root development, yet its mobility and bioavailability in soil are often limited due to fixation in insoluble forms. Understanding the spatial variability of soil phosphorus is therefore essential for optimizing fertilization strategies, particularly in legume crops such as common bean (Phaseolus vulgaris), where P uptake is strongly influenced by soil properties, microbial activity, and root physiology. In this study, twelve soil samples were collected from an agricultural field in northern Iran to quantify available soil phosphorus and the corresponding phosphorus absorbed in bean roots. Three numerical interpolation approaches—Lagrange polynomial interpolation, bicubic Hermite interpolation, and inverse distance weighting (IDW)—were applied to model and map the spatial distribution of soil P. Pearson's correlation analysis was conducted to examine the relationship between soil P concentration and P uptake in bean roots.

The interpolation results revealed clear differences among methods. Hermite produced the strongest soil–plant agreement ($r = 0.926$), closely followed by Lagrange ($r = 0.922$), while IDW showed a slightly weaker correlation ($r = 0.854$). Despite these strong statistical correlations, the spatial correspondence between soil P and plant uptake remained only moderate at the field scale, indicating that soil P distribution alone does not fully determine plant phosphorus acquisition. Error metrics (RMSE and MAE) showed comparable performance among methods, with IDW producing slightly lower prediction errors but weaker biological alignment. Overall, Hermite provided the most reliable representation of phosphorus gradients relevant to plant uptake, highlighting the importance of gradient-preserving interpolation for site-specific phosphorus management in calcareous soils.

Keywords:
Soil phosphorus; Phosphorus uptake; Phaseolus vulgaris; Lagrange interpolation; Bicubic Hermite interpolation; Inverse distance weighting (IDW); Spatial variability; Nutrient bioavailability; Precision agriculture

Introduction

1.1 The Role of Phosphorus in Agriculture and the Environment

Phosphorus (P) is an essential macronutrient required for key physiological processes in plants, including energy transfer, photosynthesis, and root development. It is also necessary for maintaining profitable crop and livestock production. Phosphorus plays a central role in energy transfer through ATP, nucleic acid synthesis, and cellular membrane formation. However, phosphorus availability in soils is often limited due to its low mobility and tendency to form insoluble compounds. This results in heterogeneous spatial distribution across agricultural fields, which can reduce nutrient uptake efficiency and the effectiveness of phosphorus-based fertilizers. Soil phosphorus exists in both organic and inorganic forms (Fig 1). These forms comprise a continuum of compounds, ranging from solution phosphorus, which is available for plant uptake, to highly stable, unavailable forms. In most soils, 50–75% of phosphorus is present as inorganic compounds (Beegle, 2017).

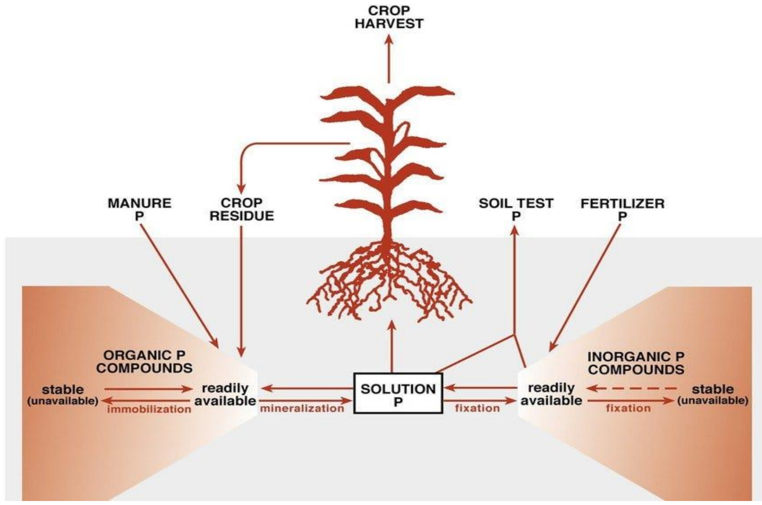

Fig 1. The phosphorus cycle in soil.

Phosphorus naturally occurs in the earth's crust, water, and all living organisms. It is one of 16 elements essential for plant growth. Fertilizer application of phosphorus has been shown to increase crop growth and yield, particularly in soils naturally low in phosphorus or depleted through crop removal. Today, crop fertilization represents the largest use of phosphorus in agriculture (Mullins, G., & Maguire, R). Excessive phosphorus use can cause environmental problems. Runoff from agricultural fields often leads to eutrophication of water bodies, triggering algal blooms and degradation of aquatic ecosystems. Therefore, efficient, site-specific phosphorus management is crucial to balance crop productivity with environmental protection.

1.2 Challenges of Phosphorus Uptake by Legume Plants

Among leguminous crops, common bean (Phaseolus vulgaris L.) is a suitable model plant for studying phosphorus uptake due to its biological importance and high P demand during early growth. Despite its essential role, phosphorus availability in soils is often limited because it readily binds with calcium, iron, or aluminum compounds, forming insoluble complexes. Legume plants have particularly high phosphorus requirements, especially during nitrogen fixation. Although their symbiotic associations with soil microbes can enhance nutrient uptake, the efficiency of P absorption is strongly influenced by soil pH, moisture, microbial activity, and root architecture. Therefore, addressing these limitations requires precise monitoring and spatial analysis of phosphorus availability in the root zone (Vance, Uhde-Stone, & Allan, 2003).

1.3 Importance of Spatial Distribution Analysis of Soil Phosphorus

The spatial distribution of phosphorus in agricultural soils is highly heterogeneous due to variations in topography, soil texture, organic matter content, and historical fertilization practices. Understanding this spatial variability is essential for developing site-specific nutrient management strategies. By mapping phosphorus concentrations across a field, farmers and researchers can identify zones of deficiency or surplus and optimize fertilizer application to reduce costs and improve efficiency. Spatial analysis also provides insights into the relationship between soil nutrient patterns and plant uptake, which is critical for enhancing phosphorus-use efficiency and minimizing environmental impacts associated with over-fertilization (e.g., Cambardella et al., 1994).

1.4 Application of Numerical Methods in Spatial Modeling

Numerical methods in spatial modeling are essential for solving complex problems that cannot be addressed analytically. These methods allow researchers to study system behavior under varying conditions by approximating solutions through numerical calculations. Spatial modeling plays a critical role in understanding the distribution of soil nutrients, such as phosphorus, across agricultural lands. Numerical interpolation methods, including Lagrange polynomial interpolation, Hermite bicubic interpolation, and Inverse Distance Weighting (IDW), are widely used to estimate nutrient concentrations at unsampled locations based on discrete soil measurements (Cheng et al., 2016). These methods enable the construction of continuous spatial maps from discrete soil samples, allowing identification of nutrient-rich and nutrient-deficient zones. Accurate spatial modeling of phosphorus distribution provides essential insights for optimizing fertilizer application, enhancing nutrient uptake efficiency, and supporting precision agriculture practices.

1.5 research objectives

Accurate assessment of the spatial variability of soil phosphorus and its relationship with root uptake is crucial for optimizing fertilizer application and improving crop yields. While several studies have used interpolation methods such as Kriging and IDW to map soil phosphorus distribution (Smith et al., 2018; Zhang et al., 2020), few have simultaneously applied Lagrange, Hermite, and IDW methods to analyze the spatial correlation between soil phosphorus and its uptake in bean roots, particularly under the specific agro-ecological conditions of this region. This study aims to address this research gap by producing predictive maps of phosphorus distribution using Lagrange, Hermite, and IDW interpolation, and by quantifying the correlation between predicted soil phosphorus and measured plant uptake using Pearson correlation analysis.

The main objectives of this study are:

1. To map the spatial distribution of soil phosphorus using Lagrange, Hermite, and IDW interpolation methods.
2. To analyze the spatial correlation between soil phosphorus concentration and its uptake in bean roots.

1.6 Previous Studies and Research Gap

Soil phosphorus (P) is a critical macronutrient affecting plant growth, especially in agricultural ecosystems where its spatial distribution strongly influences fertilization strategies. Numerous studies have focused on spatial interpolation of phosphorus to generate prescription maps for site-specific nutrient management. applied kriging-based geostatistical approaches to evaluate phosphorus fertilizer availability across different locations. Their work demonstrated how soil phosphorus interpolation can inform fertilizer application planning; however, the study mainly focused on fertilizer availability indices and did not compare different interpolation methods (Juang et al. 2002). In a different context, phosphorus transport has been modeled in aquatic environments. For example, field measurements and HEC-RAS simulations were used to study phosphorus dispersion and its effects on water quality (Faghihirad et al., 2020). Although this study is not directly related to soils, it highlights the environmental significance of phosphorus modeling. More recently, (Matcham et al. 2021) compared multiple spatial interpolation methods for predicting total phosphorus (TP) in Mollisol regions of northeast China, including IDW, RBF, OK, COK, MLR, GWR, RK, and GWRK. Their results emphasized the advantages of geographically weighted regression (GWR) over Random Forest (RF) and IDW in mapping low-phosphorus zones, but classical interpolation techniques such as Lagrange and Hermite were not evaluated (Emma G.M. et al 2021). Visual and statistical comparisons of maps were recommended to ensure reliability, particularly near environmentally sensitive areas.

Research Gap

Despite extensive literature on soil phosphorus interpolation, classical and mathematically robust methods such as Lagrange and Hermite have been largely neglected. These methods provide flexibility in modeling local variations, especially with limited or irregular sampling points. Moreover, few studies have integrated multiple interpolation techniques and systematically compared their predictive accuracy.

Novelty and Strength of the Present Study

The present study addresses these gaps by:

- Applying Lagrange and Hermite interpolation to map phosphorus distribution in
- agricultural soils.
- Comparing the accuracy of these methods with widely used techniques such as IDW and kriging.
- Providing quantitative evaluation (RMSE, MAE) alongside visual assessment of generated maps.
- Proposing a practical framework for selecting interpolation techniques in phosphorus-based precision agriculture.

By introducing underutilized yet robust mathematical methods and benchmarking them against established techniques, this study contributes methodologically and practically to spatial soil fertility mapping.

2.Materials and Methods

2.1 Site Location, Climate, and Soil Characteristics

The study was conducted in an agricultural farm located in Kashfgiri Village, Gorgan County, Golestan Province, northern Iran (36°50′ N, 54°25′ E). The farm is situated on the northern slopes of the Alborz Mountains, southeast of the Caspian Sea.

The region has a humid temperate climate typical of the Caspian area, with relatively warm and humid summers and mild winters. The average annual maximum and minimum temperatures are 22.9°C and 12.7°C, respectively, and the mean annual precipitation is approximately 583.8 mm. Due to the diverse topography, the area exhibits various microclimates, including mountainous, plain, forested, and semi-arid zones. Soil in the study area is dominated by fine-textured clay and silt, which have high water retention capacity—beneficial for legume crops such as common bean (Phaseolus vulgaris). However, these soils are also prone to erosion, which is an environmental concern in Golestan Province. The farm covers approximately 1 hectare and is cultivated with common bean (Phaseolus vulgaris). These site characteristics provide a suitable context for investigating soil phosphorus distribution and its uptake by beans.

2.2 Soil Sampling Method and Recording Spatial Coordinates of Samples

Soil sampling was conducted in a one-hectare agricultural field. Prior to sampling, the field was irrigated using flood irrigation to ensure adequate soil moisture for bean cultivation. Crop operations, including plowing and land preparation, were performed using a tractor, followed by mechanized planting of beans in early summer. Irrigation was maintained manually throughout the growth period. For the experiment, a plot measuring 30 m × 200 m was selected at the center of the field. A Cartesian coordinate system (X and Y) was established, with the X-axis representing the width (30 m) and the Y-axis representing the length (200 m) of the plot. Soil samples were collected at each point from a depth of 0–30 cm. The coordinates and sampling depths of all points are summarized in Table 1. These spatial coordinates were subsequently used for interpolation analyses and assessing the spatial distribution of soil phosphorus.

Table 1. Coordinates and Depths of Soil Sampling Points

Sample No.	X (m)	Y(m)	Depth (cm)
1	0	0	0–30
2	10	0	0–30
3	20	0	0–30
4	30	0	0–30
5	0	100	0–30
6	10	100	0–30
7	20	100	0–30
8	30	100	0–30
9	0	200	0–30
10	10	200	0–30
11	20	200	0–30
12	30	200	0–30

2.3 Laboratory Preparation and Analysis of Soil Samples

After collection, soil samples were stored in labeled plastic bags and transported to the laboratory. The samples were air-dried, sieved through a 2 mm mesh, and ground prior to analysis.

Available phosphorus (P) was determined using the Olsen method. Soil pH, electrical conductivity (EC), and texture were also measured, with soil texture determined by the hydrometer method. Phosphorus concentration was measured colorimetrically after extraction with sodium bicarbonate ($NaHCO_3$), and absorbance was read at 880 nm using a spectrophotometer. Phosphorus values are reported in mg/kg of soil.

2.4 Introduction to the interpolation methods used

Interpolation is a mathematical technique used to estimate unknown values at unsampled locations based on known data points. In agricultural studies, it enables the creation of continuous spatial maps of soil properties, such as phosphorus concentration, from discrete samples. Interpolation supports precision agriculture by helping optimize sampling strategies and fertilizer application.

2.4.1 Lagrange interpolation method

Lagrange interpolation estimates the value of a function at any given point using a polynomial constructed from known data points. Unlike Taylor series, it does not require derivative information, making it suitable for experimental or field data.

If x and y are $(n+1)$ distinct points and f is a function with known values at these points, then there exists a unique polynomial such P of degree at most n with the property:

$$f(x_i) = P(x_i) \quad for\ \ i = 0.1 \ldots .n$$

This polynomial is given by the following relation:

$$P(x) = f_0 L_0(x) + f_1 L_1(x) + \cdots + f_j L_j(x) + \cdots + f_n L_n(x) = \sum_{i=0}^{n} f_i L_i(x)$$

Where for $i = 0.1 \ldots .n$

$$L_i(x) = \frac{(x-x_0)(x-x_1)\ldots(x-x_{i-1})(x-x_{i+1})\ldots(x-x_n)}{(x_j-x_0)(x_j-x_1)\ldots(x_j-x_{i-1})(x_j-x_{i+1})\ldots(x_j-x_n)}$$

$$= \prod_{i=0.j\neq i}^{n} \frac{(x-x_i)}{(x_j-x_i)}$$

For $L_i(x)$ we have the following properties:

$$L_i(x_j) = \begin{cases} 0 & if \quad i \neq j \\ 1 & if \quad i = j \end{cases}$$

Lagrange interpolation is particularly useful when the data points are unevenly spaced and derivative information is unavailable (Burden, et al. 2015).

Interpolator polynomial error.

Let $P_n(x)$ be the Lagrange interpolating polynomial of degree n that interpolates f at $n + 1$ distinct points $x_1. x_2. \dots . x_n$ in the interval $[a. b]$ and $f \in C^{n+1}[a. b]$, Then, for any $x \in [a. b]$, There exists a point like $\xi(x)$ in $(a. b)$ interpolation error is given by:

$$R_n(x) = f(x) - P(x) = \frac{f^{(n+1)}\,\xi(x)}{(n+1)!}(x - x_0)(x - x_1)\dots(x - x_n)$$

where P is the interpolating polynomial. This expression shows that the error depends on the $(n + 1)th$ derivative of the function and the distance between x and the interpolation nodes. The term involving the derivative indicates that the smoother the function, the smaller the possible error for the same interpolation nodes. For proof, see (Burden & Faires, 2016).

2.4.2 Hermite interpolation

Hermite interpolation generalizes Lagrange interpolation by using both function values and derivative information at sampling points. It produces a polynomial that approximates both the function and its slope at the interpolation nodes, providing higher accuracy when derivative data are available (Burden & Faires, 2016).

Definition

Suppose $x_0. x_1. \dots . x_n$ are distinct points in the interval $[a. b]$, and let m_i be a non-negative integer for $i = 0.1 \dots . n$, We assume that:

$$f \in C^m[a. b]$$

Now, suppose that the values of the function f and its derivatives are given at the points

$$\left(x_i. f_i^{(k)}\right). \quad \begin{cases} i = 0.1.2 \dots . n \\ k = 0.1.2 \dots . m_i \end{cases}$$

Then, there exists a unique polynomial P such that:

$$\frac{\partial^k P(x_i)}{\partial x^k} = \frac{\partial^k f(x_i)}{\partial x^k}. \qquad \begin{cases} i = 0.1.2 \dots . n \\ k = 0.1.2 \dots . m_i \end{cases}$$

Special Case of Hermite Interpolation: When for each $i = 0.1 \dots . n$, $m_i = 1$, a class of polynomials is obtained which is called the Hermite polynomial**.**

If $f \in C^1[a.b]$ and $x_0. x_1. \dots . x_n$ are distinct points in $[a.b]$ then there exists a unique polynomial of degree $2n + 1$ which coincides with f and f' at $x_0. x_1. \dots . x_n$

and it is given by the following relation

$$H_{2n+1}(x) = \sum_{i=0}^{n} f(x_i) H_i(x) + \sum_{i=0}^{n} f'(x_i) \widehat{H}_i(x)$$

Which

$$H_i(x) = [1 - 2(x - x_i) L_i'(x_i)] L_i^2(x_i)$$

And

$$\widehat{H}_i(x) = (x - x_i) L_i^2(x_i)$$

Here L is the Lagrange polynomial.

Hermite interpolation error. If $f \in C^{(2n+2)}[a.b]$, for a point such as ξ, that

$a < \xi < b$, the Hermite interpolation formula is obtained from the following relation:

$$f(x) - H_{2n+1}(x) = \frac{(x - x_0)^2 \dots . (x - x_n)^2}{(2n + 2)!} f^{(2n+2)}(\xi)$$

2.4.3 IDW or Inverse Distance Interpolation Method

How to estimate in IDW interpolation.

Suppose we want to interpolate for a region where we know the rate of a phenomenon at certain points and determine the rate of the phenomenon for all points in the region. The method is as follows:

1. The desired areas are converted into a matrix with cells of same size. The spatial coordinates of the matrix and the size of each pixel are clear and have a unit of measurement.
2. In this network, cells are of two types:

- Cells with a known (measured) variable value
- Cells with an unknown (unmeasured) value

The general formula for the Inverse Distance Weighting (IDW) interpolation method is expressed as follows:

$$Z(x_0) = \frac{\sum_{i=1}^{n} \omega_i \, z(x_i)}{\sum_{i=1}^{n} \omega_i}$$

Where

$$\omega_i = \frac{1}{d(x_0.x_i)^p}$$

Where

- $z(x_0)$: Estimated amount of a variable (e.g. soil phosphorus) at a given point x_0.
- $z(x_i)$: The measured value of the variable at sampling point x_i.
- ω_i:Weight assigned to each point x_i .
- $d(x_0.x_i)$: The distance between the desired point and the sampling point.
- p = power parameter, which controls the influence of distance on the interpolation. Usually between 1 and 3 are chosen.

When $p = 1$: weights decrease linearly with distance.

When $p = 2$: weights decrease with the square of distance (commonly used)

Larger values of p give more influence to nearby points and less to distant points, resulting in a more localized interpolation. The best results from the Inverse Distance Weighting (IDW) interpolation method are obtained when sampling is sufficiently dense and corresponds to the local variations you aim to simulate. If the input points are sparse or unevenly distributed, the results may not adequately represent the desired surface.

2.4.4 Application of mathematics in biology and agriculture

Mathematics plays a significant role in modeling, data analysis, and predicting the behavior of biological systems. In agricultural biology, mathematical tools such as differential equations, interpolation methods, and statistical analysis are used to study and model natural processes such as nutrient uptake, plant growth, soil diffusion, and microbial interactions. In particular, in situations where it is not possible to measure data at all points of an agricultural field, interpolation methods are used as effective numerical tools to estimate unknown values.

These mathematical approaches enable researchers to better understand spatial and temporal patterns of nutrient distribution, optimize sampling strategies, and make informed decisions regarding soil management and fertilization. By applying methods such as Lagrange interpolation and IDW, it is possible to create accurate predictive maps of soil properties and plant nutrient uptake, which are essential for improving crop yield and resource efficiency. Consequently, mathematics provides a powerful framework that bridges theoretical modeling with practical applications in agriculture, highlighting its indispensable role in modern biological research.

2.5. Correlation and Descriptive Statistics Analysis

To evaluate the spatial variability and relationships between soil phosphorus (p) and p uptake by bean roots, descriptive and correlation analyses were conducted.

Basic descriptive statistics, including minimum, maximum, mean, standard deviation, and coefficient of variation (CV), were calculated for both soil available P and root P uptake to assess the degree of data dispersion.

The normality of data distribution was tested using the Shapiro–Wilk test. When normality assumptions were met, the Pearson correlation coefficient (r) was applied to assess the linear relationship between soil available P and root P uptake at corresponding sampling points, calculated as follows:

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2(y_i - \bar{y})^2}}$$

If data were not normally distributed, the Spearman rank correlation (ρ) was used as a non-parametric alternative. Statistical significance of correlation coefficients was evaluated at the **α = 0.05** level.

2.6. Software Used

All numerical computations—including grid generation, interpolation, and statistical analysis—were performed in a Python-based computational environment using standard scientific libraries such as NumPy and Pandas. Spatial visualization (contour mapping) was generated using Matplotlib. Microsoft Excel was used only for storing, viewing, and exporting tabulated results.

3. results

3.1 Descriptive Statistics of Soil and Plant Phosphorus

Introduction and Purpose

In this section, a descriptive analysis was conducted for soil available phosphorus (P) and phosphorus uptake by bean roots to characterize the distributional pattern and variability of the measured variables within the study area. A total of twelve sampling points were distributed randomly across the agricultural field. Soil samples were collected at the 0–30 cm depth and analyzed for available phosphorus (P), as well as physicochemical properties including pH, electrical conductivity (EC), organic carbon (OC), total nitrogen (N), potassium (K), and soil texture (Tables 2 and 3).

Descriptive statistics, including the mean, standard deviation (SD), and coefficient of variation (CV), were computed to evaluate central tendency and variability. These indices provide a basis for understanding spatial heterogeneity in soil phosphorus and the relative uniformity or variability of phosphorus uptake by the plant (Table 4).

Equations Used

$$SD = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

$$CV = \frac{SD}{\bar{X}} \times 100$$

where:
$\bar{X}$= mean,
SD = standard deviation,
CV = coefficient of variation,
X_i= observed value,
n = number of samples.

These statistics provide a quantitative foundation for subsequent spatial interpolation and correlation analysis. The results indicate moderate spatial heterogeneity in soil available phosphorus across the field (Table 2). The soil was classified as silty clay loam, with a slightly alkaline pH (7.7) and moderate EC (1.0 dS/m), providing favorable but potentially calcium-limited phosphorus mobility (Table 3).

Table2. Measured available phosphorus (P) in soil (0–30 cm depth)

absorbable phosphorus P(AVA).PPM	Depth (cm)	Spatial coordinates
10.7	$0-30$	$X.Y(0.0)$
6.6	$0-30$	$X.Y(10.0)$
8.8	$0-30$	$X.Y(0.100)$
6.8	$0-30$	$X.Y(20.0)$
8.6	$0-30$	$X.Y(10.100)$
6.1	$0-30$	$X.Y(20.100)$
8.0	$0-30$	$X.Y(30.0)$
4.8	$0-30$	$X.Y(30.100)$
8.1	$0-30$	$X.Y(0.200)$
5.9	$0-30$	$X.Y(10.200)$
7.9	$0-30$	$X.Y(30.200)$
4.9	$0-30$	$X.Y(20.200)$

Statistical summary:
Minimum = 4.8 ppm
Maximum = 10.7 ppm
Mean = 7.2 ppm
Standard Deviation (SD) = 1.9 ppm
Coefficient of Variation (CV) = 26%

Table3. Mean physicochemical properties of soil samples (0-30 cm depth)

Parameter	Unit	Mean Value
SAND	%	12
SILT	%	60
CLAY	%	28
Available Potassium (K)	Ppm	220
Total Nitrogen (N)	%	0.12
Organic Carbon (OC)	%	1.22
Total Neutralizing Value(T.N.V)	%	17
pH (Saturated paste)	-	7.7
Electrical Conductivity (EC $\times 10^3$)	dS/m	1.0
Depth	Cm	0-30
Soil Texture	Si-C-L	-

Phosphorus uptake by bean roots displayed very low variability, reflecting relatively uniform plant response across the field (Table 4).

Table 4. Phosphorus uptake by bean roots

P Uptake %	Sampling Coordinates (x,y)
0.32	(0,5)
0.31	(2,20)
0.30	(3,30)
0.30	(1,45)
0.29	(8,70)
0.28	(11,95)
0.29	(9,105)
0.27	(23,115)
0.28	(16,130)
0.27	(21,145)
0.28	(12,175)
0.28	(28,195)

Statistical summary:
Minimum = 0.27%
Maximum = 0.32%
Mean ≅ 0.2891%
Standard Deviation (SD) = 0.015%
Coefficient of Variation (CV) ≈ 5%

The descriptive statistical analysis revealed that available soil phosphorus ranged from 4.8 to 10.7 ppm, with a moderate coefficient of variation (~26%), indicating noticeable spatial heterogeneity in P distribution. In contrast, phosphorus uptake by bean roots varied over a narrow range (0.27–0.32%) and exhibited very low relative variability (CV ≈ 5%) (Table 4), suggesting uniform plant uptake response despite soil P variability. It is also important to note that the twelve uptake sampling points were selected randomly, and interpolation values were extracted at these same coordinates for correlation analysis. This approach ensured unbiased evaluation of the relationship between measured uptake and interpolated phosphorus values. Interpolation techniques—including Lagrange polynomial interpolation, Hermite interpolation, and Inverse Distance Weighting (IDW)—were subsequently applied to generate spatial prediction maps, and correlation analysis between measured uptake and predicted P values was performed in the next section.

3.2 Spatial Distribution Maps: Lagrange Interpolation Method

The spatial variability of soil available phosphorus (P) across the study field was modeled using the two-dimensional Lagrange interpolation method. This global polynomial approach constructs a continuous surface that passes through all measured sampling points, enabling estimation of P concentrations at any unsampled location. Because the experimental field was relatively small and uniformly sampled, a global polynomial method was appropriate for capturing broad-scale gradients rather than localized noise.

Mathematical Formulation

In two-dimensional space, the Lagrange interpolation function for ***n*** sample points can be expressed as:

$$P(x.y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P_{i.j} L_i(x) M_j(y)$$

Where

- $P_{i.j}$ = measured P concentration at point $(x_i.y_j)$
- $L_i(x)$= Lagrange basis polynomial in the X-direction,
- $M_j(y)$=Lagrange basis polynomial in the Y-direction,
- $n.m$= number of sampling points in x and y directions, respectively,
- $P(x.y)$= interpolated phosphorus value at any unsampled coordinate The basis polynomials are defined as:

$$L_i(x) = \prod_{\substack{k=1 \\ k\neq j}}^{n} \frac{x - x_k}{x_i - x_k}$$

$$M_j(y) = \prod_{\substack{l=1 \\ l\neq j}}^{m} \frac{y - y_l}{y_j - y_l}$$

This tensor-product formulation generates a globally smooth continuous surface of phosphorus concentration across the field.

Spatial Prediction Grid and Representative Values

The interpolation was applied across a 1 × 5 m grid, producing 1,271 predicted points. To maintain clarity, only representative values are shown in Table 5; To visualize the spatial variability of soil available phosphorus across the study area, an interpolated surface was generated using the selected interpolation method (Fig 2). The resulting map provides a continuous representation of phosphorus concentrations between sampling points, enabling clearer recognition of spatial trends, gradients, and localized hot spots within the field. The color zones range from low phosphorus (blue/purple) to high phosphorus (yellow/green) and allow for intuitive identification of nutrient-rich and nutrient-poor areas. Such visual distinction is needed to interpret the spatial behavior of phosphorus and compare how each interpolation method is essentially investigating.

Table 5. Representative estimated soil phosphorus (P) values at selected sampling points using 2D Lagrange interpolation (1 × 5 m grid).

X,Y (m)	Estimated P (ppm)
(0,5)	10.765
(2,20)	9.41
(3,30)	9.1504
(1,45)	9.5510
(8,70)	8.8259
(11,95)	8.4095
(9,105)	8.7591
(23,115)	5.2964
(16,130)	6.8052
(21,145)	5.4371
(12,175)	6.5939
(28,195)	6.6221

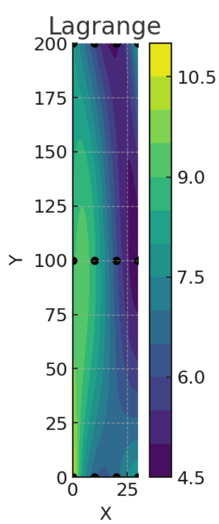

Fig2. Spatial distribution map of soil available phosphorus generated using the lagrange interpolation method. The color gradient represents the estimated phosphorus concentration (ppm), with warmer colors

indicating higher values and cooler colors representing lower concentrations. This map illustrates the spatial pattern of soil P across the field and highlights zones with elevated or reduced nutrient availability.

3.3 Spatial Distribution Maps – IDW Interpolation

The Inverse Distance Weighting (IDW) method was applied to estimate the spatial distribution of available soil phosphorus (P) across the study area. IDW is a local interpolation technique where closer points have more influence on the predicted value. The degree of locality can be controlled by the power parameter p; a lower p (e.g., 1) results in less local influence, while a higher p (e.g., 3) gives more localized predictions. In this study, p^2 was chosen as a balance between local and global influence. The same sampling points used for Lagrange interpolation were employed, and a regular grid with 1 × 5 m spacing was generated to visualize phosphorus distribution, similar to the approach in Section 3.2.

The resulting IDW map (Fig 3) allows for comparison with the Lagrange interpolation map, highlighting areas of higher and lower soil phosphorus content and providing a basis for further spatial analysis, including correlation with phosphorus uptake by bean roots. A subset of estimated soil phosphorus (P) values at selected coordinates is presented in Table 6, calculated using the IDW interpolation method (p = 2).

Table 6. Estimated soil phosphorus (P) at selected coordinates using the IDW interpolation method (p = 2).

X,Y (m)	Estimated P (ppm)
(0,5)	9.7987
(2,20)	8.1766
(3,30)	7.9240
(1,45)	7.6844
(8,70)	7.3968
(11,95)	8.0323
(9,105)	8.1764
(23,115)	6.5801
(16,130)	7.0513
(21,145)	6.9213
(12,175)	6.6436
(28,195)	7.1144

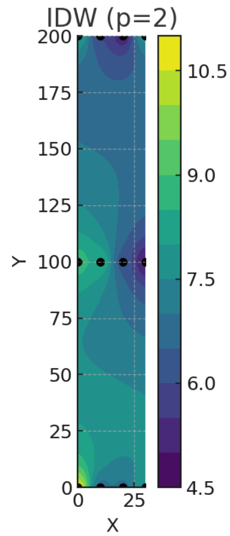

Fig3. Spatial distribution map of soil available phosphorus generated using the IDW interpolation method. The color gradient represents the estimated phosphorus concentration (ppm), with warmer colors indicating higher values and cooler colors representing lower concentrations. This map illustrates the spatial pattern of soil P across the field and highlights zones with elevated or reduced nutrient availability.

3.4. Spatial Distribution Maps – Bivariate Hermite Interpolation

Bivariate Hermite interpolation was applied to estimate the spatial distribution of soil available phosphorus (P) across the experimental field. This method constructs a smooth and continuous surface by incorporating both measured values and local gradients at sampling points, providing higher-order continuity and preserving natural slopes compared to standard point-based interpolation methods.

The same 12 sampling points used for Lagrange and IDW interpolations were employed, and a regular grid of 1 × 5 m spacing was generated to produce the prediction surface.

Representative predicted values obtained from the Hermite interpolation are presented in Table 7. The resulting spatial distribution map (Fig 4) illustrates continuous variation in soil P concentrations, allowing direct comparison with the Lagrange and IDW surfaces.

Table 7. Estimated plant-available phosphorus (PPP) at selected nodes using bivariate Hermite interpolation on the 1×5 m sampling grid.

X,Y (m)	Estimated P (ppm)
(0,5)	10.6035
(2,20)	9.6105
(3,30)	9.2058
(1,45)	9.5730
(8,70)	8.6007
(11,95)	8.4439
(9,105)	8.6830
(23,115)	5.5313
(16,130)	6.6827
(21,145)	5.5187
(12,175)	6.3329
(28,195)	7.1204

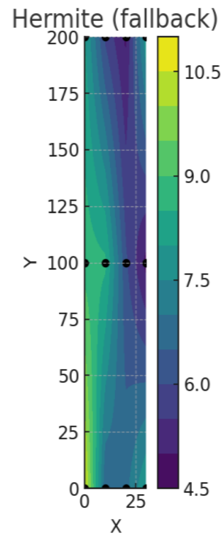

Fig4. Spatial distribution map of soil available phosphorus generated using the Hermite interpolation method. The color gradient represents the estimated phosphorus concentration (ppm), with warmer colors indicating higher values and cooler colors representing lower concentrations. This map illustrates the spatial pattern of soil P across the field and highlights zones with elevated or reduced nutrient availability.

3.5. Comparison of Interpolation Methods

To evaluate the performance of the three interpolation techniques applied in this study—two-dimensional Lagrange interpolation, inverse distance weighting (IDW), and bicubic Hermite interpolation—their numerical behavior, spatial structure, and consistency with the characteristics of soil phosphorus distribution were systematically compared at the twelve sampling points where phosphorus uptake by bean roots was measured. This approach ensures that comparisons are based on observed data rather than the full interpolated grid, maintaining alignment with experimental measurements.

3.5.1. Numerical differences across methods

A point-by-point comparison at the twelve sampling points showed that all three methods reproduce the general spatial trend of soil phosphorus. However, local numerical differences were observed:

- **Lagrange interpolation** produced the widest numerical range and captured global variations smoothly, but small oscillations near the field boundaries were noted.
- **IDW** yielded more conservative estimates, constrained by the values of nearby sampling points. This method avoided overshoot but smoothed subtle local variations.
- **Hermite interpolation** provided intermediate values between Lagrange and IDW. By incorporating approximated spatial derivatives, it preserved smooth transitions while maintaining realistic local variability.

3.5.2. Spatial smoothness and continuity

- **Lagrange interpolation** showed smooth surfaces but exhibited oscillations at edges due to its global polynomial nature.
- **IDW** produced a piecewise smooth surface that followed the sampling points closely. While continuous, it lacked derivative continuity (C^0).
- **Hermite interpolation** generated a surface that was smooth and locally adaptive, with continuity in both the surface and its first derivatives (C^1), representing gradual spatial changes realistically.

3.5.3. Sensitivity to spatial arrangement of samples

- **Lagrange interpolation** is sensitive to uneven sample spacing; irregularities may produce curvature artifacts.
- **IDW** is robust to sample spacing but may underestimate local gradients in sparsely sampled areas.
- **Hermite interpolation** balances local and global effects, using derivative information to capture gradients while remaining responsive to nearby values.

3.5.4. Overshoot, Smoothness, and Continuity

- **Overshoot:** Lagrange occasionally exhibited minor overshoot near boundaries, IDW showed minimal overshoot, and Hermite demonstrated negligible overshoot.
- **Smoothness:** Lagrange was the smoothest globally, Hermite achieved moderate-to-high smoothness with physically meaningful curvature, and IDW was less smooth.
- **Continuity:** Hermite uniquely ensured C^1 continuity; Lagrange was continuous but potentially oscillatory; IDW was continuous (C^0) but not differentiable.

3.5.5. Stability and numerical behavior

Numerical stability was assessed at the twelve sampling points:

- **IDW** was the most numerically stable, always bounded and free of oscillations.
- **Lagrange** showed minor instability near boundaries.
- **Hermite** provided stable and realistic reconstruction, benefiting from derivative constraints.

3.5.6. Comparison of spatial patterns and identification of high-P zones

Across the twelve sampling points:

- **Lagrange** highlighted the largest amplitude of variation but occasionally exaggerated localized peaks.
- **IDW** preserved the general structure of high- and low-phosphorus areas while smoothing fine-scale variability.

- **Hermite** produced the most realistic spatial pattern, reproducing smooth gradients consistent with known soil phosphorus mobility.

3.5.7. Strengths and Weaknesses of Each Method for Soil Phosphorus Data

Table 8. Strengths and Weaknesses of Each Method for Soil Phosphorus Data

Method	Strengths	Weaknesses
Lagrange	Captures global trends; smooth surface	Sensitive to outliers; minor oscillation near boundaries
IDE	Simple robust; closely follows measured points	Surface less smooth; sharp transitions; underestimates gradual gradients
Hermite	Smooth and realistic; retains local slope; C^1 continuity	Requires derivative estimation slightly higher computational demand

Hermite's ability to incorporate slope information makes it particularly suitable for datasets with gradual nutrient transitions. IDW is reliable for densely sampled datasets with irregular gradients, whereas Lagrange is most useful for representing smooth global trends.

3.5.8. Overall evaluation and method selection

Considering numerical stability, surface smoothness, absence of artificial oscillation, and capacity to represent natural soil nutrient gradients at the twelve sampling points**, bicubic Hermite interpolation** demonstrated the best performance for modeling spatial distribution of soil phosphorus in this study. IDW ranked second, offering a conservative baseline, while Lagrange was least suitable due to its oscillatory tendencies in heterogeneous areas.

3.6. Results of correlation analysis

3.6.1 Method description and rationale

To quantify the strength of the soil–plant relationship, Pearson's correlation coefficient (r) was used to assess the linear association between soil available phosphorus (P) and root phosphorus uptake in common bean. Pearson's r was selected because both variables are continuous and spatially distributed, and the objective was to measure the direction and magnitude of a linear relationship rather than a nonlinear or categorical association.

Since the interpolation methods (Lagrange, IDW, and Hermite) generate different estimates of soil P, the correlation analysis was performed separately for each interpolated surface. This strategy ensures that the uncertainty introduced by the interpolation technique is explicitly reflected in the computed correlation values.

Root P uptake was represented as a piecewise constant surface based on field sampling locations. Each uptake value was assigned to the corresponding grid coordinates, and paired with interpolated soil P estimates at the same points. The correlation was calculated using the standard Pearson formula:

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

Where

- x_i represents soil available P at grid point i,
- y_i is the corresponding root P uptake value,
- n is the total number of grid points,
- $\bar{x}$ and $\bar{y}$ are the mean values of the two variables.

3.6.2 Numerical results

Correlation analysis revealed consistent positive relationships between soil available phosphorus and root P uptake across all interpolation models. The uptake values ranged from **0.27 to 0.32 mg plant^{-1}**, and their spatial trend generally followed the decline in soil P along the field transect.

To assess the performance of each interpolation method quantitatively, three metrics were calculated:

- **Pearson correlation coefficient (r)** — strength of soil–plant linear association
- **Root Mean Square Error (RMSE)** — sensitivity to large estimation errors
- **Mean Absolute Error (MAE)** — average magnitude of estimation error

The combined results of correlation and error metrics are summarized in Table 9.

The resulting correlation coefficients were:

Table 9 . Final evaluation metrics for interpolation methods (correlation and prediction errors)

Method	Pearson correlation coefficient	RMSE	MAE
Lagrange	0.922	7.86	7.68
IDE	0.854	7.38	7.33
Hermite	0.926	7.87	7.70

Interpretation of results

The correlation analysis demonstrated clear differences in the soil–plant relationship depending on the interpolation technique used to generate the soil phosphorus surface. Among the three tested methods, the **Hermite interpolation** yielded the **highest Pearson correlation coefficient (r = 0.926)**, indicating that this method best captured the linear relationship between soil available phosphorus and root P uptake. The superior performance of Hermite is attributed to its ability to preserve local gradients and ensure C^1 continuity, resulting in a nutrient surface that more accurately reflects the gradual spatial transitions influencing root absorption processes.

The **Lagrange interpolation** produced a similarly strong correlation (r = 0.922), but its slightly lower accuracy and marginally higher error values (RMSE = 7.86; MAE = 7.68) suggest that global polynomial oscillations introduced small inconsistencies between soil P predictions and plant uptake. These oscillatory behaviors are characteristic of high-order global interpolation and tend to distort local nutrient patterns, especially near domain boundaries.

In contrast, the **IDW method**, although exhibiting the **lowest correlation (r = 0.854)**, produced the **lowest overall error values (RMSE = 7.38; MAE = 7.33)**. This pattern reflects the conservative nature of IDW: by strictly weighting nearby observations and avoiding global curvature effects, IDW generates a stable, non-oscillatory surface. However, this same conservative behavior limits its ability to fully capture the true spatial gradient of soil phosphorus, thereby weakening its correlation with the spatial pattern of root uptake.

Taken together, these results indicate that Hermite interpolation provides the most realistic representation of the soil phosphorus field for purposes of plant–soil interaction analysis. Its combination of smoothness, gradient preservation, and numerical stability makes it particularly effective for modeling nutrient dynamics that evolve gradually across space. IDW remains useful as a low-assumption and low-error benchmark, whereas Lagrange offers strong correlations but requires caution due to its tendency to exaggerate spatial variability.

Overall, the findings confirm that the choice of interpolation method directly affects the strength and clarity of soil–plant relationships, emphasizing the importance of selecting gradient-preserving approaches—such as Hermite—for ecological and agronomic applications where spatial continuity is a defining feature of nutrient processes.

Graphical evaluation

A scatter plot of soil P vs. root uptake was generated for each interpolation method (Fig 5). In all three cases, the data points formed a clear positive linear pattern, confirming the monotonic increase of root uptake with increasing soil phosphorus. The Hermite-derived scatter showed the tightest clustering around the regression line, consistent with its higher correlation value, while the IDW scatter displayed slightly wider spread but lower residuals.

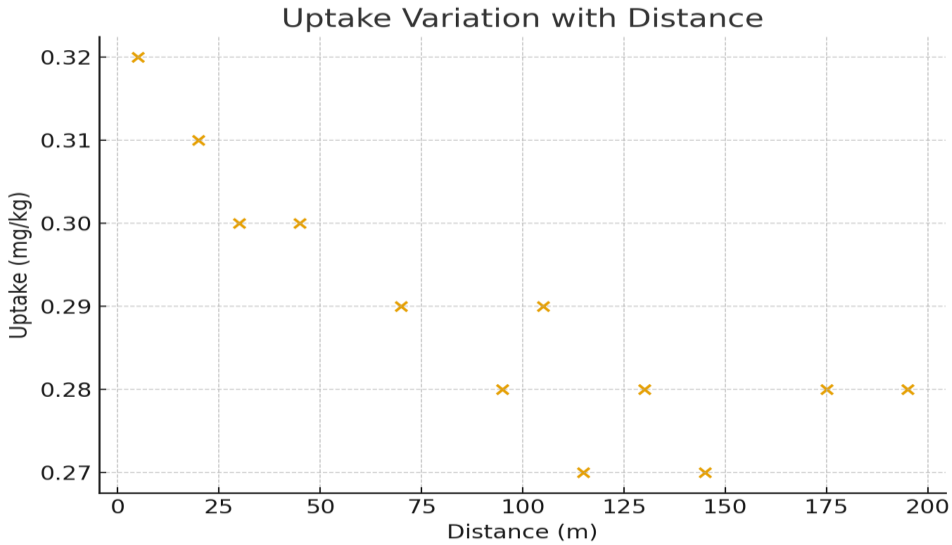

Fig 5. Relationship between soil available phosphorus and root phosphorus uptake across

3.7 Spatial assessment of areas with highest and lowest phosphorus uptake

To identify the spatial co-occurrence between soil available phosphorus and root phosphorus uptake, the interpolated soil P maps derived from the three methods (Lagrange, IDW, and Hermite) were overlaid with the spatial distribution of root P uptake values. Because root uptake was measured at discrete sampling locations and represented as piecewise constant zones, the comparison was performed by extracting interpolated soil P estimates at the same coordinates and evaluating spatial consistency across the field.

3.7.1. Spatial distribution of high-uptake zones

Across all interpolation methods, the highest root P uptake values (0.31–0.32 mg plant^{-1}) were consistently located in the **northern and north-central regions** of the field (e.g., coordinates (0,5), (2,20), (3,30), and (1,45)).
These zones coincided with areas where the interpolated soil P values were also relatively high:

- Hermite: 9.2–10.6 ppm
- Lagrange: 9.1–11.3 ppm (with minor oscillatory peaks)
- IDW: 8.7–10.1 ppm

The strong alignment between high soil P and high root P uptake supports the positive soil–plant correlation described in Section 3.6.
Among the three interpolation techniques, Hermite produced the most coherent high-P regions, showing smooth transitions between adjacent sampling cells, whereas Lagrange produced some exaggerated local hotspots. IDW captured the same general trend but with a muted amplitude due to its smoothing effect.

3.7.2. Spatial distribution of low-uptake zones

The lowest root P uptake values (0.27–0.28 mg $plant^{-1}$) were found primarily in the **southern to southeastern regions** of the sampling grid, such as points at (23,115), (21,145), (12,175), and (28,195).
These locations corresponded to relatively low soil P estimates across all interpolation surfaces:

- Hermite: 5.5–7.1 ppm
- Lagrange: 5.2–7.6 ppm
- IDW: 5.8–6.9 ppm

Unlike the high-uptake zones—which exhibited more pronounced gradients—the low-uptake areas showed **broader spatial continuity**, indicating that phosphorus depletion was more uniformly expressed across the southern part of the field.

3.7.3. Consistency across interpolation methods

A cross-method comparison showed that:

- All three methods identified **the same major high- and low-uptake regions**, confirming the robustness of the spatial pattern.
- Hermite generated the most realistic and ecologically meaningful transitions, especially at mid-field gradients where uptake changed from 0.30 to 0.29 mg $plant^{-1}$.
- Lagrange occasionally produced artificial depressions adjacent to high-uptake nodes, reflecting polynomial oscillation effects.
- IDW produced the most conservative gradient, maintaining similar spatial boundaries but compressing the range of soil P values.

The consistency across methods reinforces the spatial reliability of the identified nutrient response zones.

3.7.4. Ecological interpretation

The alignment between high soil P zones and elevated root P uptake reflects the natural coupling of nutrient availability and plant absorption dynamics.
The smoother patterns seen in the Hermite-based map are consistent with known processes governing phosphorus mobility:

- microbially mediated mineralization,
- diffusion-limited transport,
- and gradual root depletion gradients.

In contrast, the broader extent of low-uptake areas suggests that once soil P drops below a physiological threshold, bean roots experience uniform limitations across the affected region.

3.7.5. Implications for field management

The spatial co-occurrence analysis of soil phosphorus levels and plant phosphorus uptake (Fig. 6) provides essential insight into how nutrient availability translates into actual root absorption within different field zones. This map integrates both datasets to highlight areas where soil P concentrations effectively support plant uptake, as well as zones showing a mismatch between supply and acquisition.

Regions where high soil P coincides with high uptake indicate functionally responsive zones where biochemical and microbial conditions favor P mobilization and absorption. These areas require minimal fertilizer intervention and represent stable “nutrient-efficient” zones.

Conversely, areas where low uptake persists despite moderate or even high soil P suggest potential biological or physical constraints—such as limited microbial mineralization, root growth restrictions, or micro-scale soil heterogeneity. These zones are priority targets for site-specific management, including organic amendments, improved soil aeration, or microbial inoculation strategies to enhance P bioavailability.

The co-occurrence map thus serves as a diagnostic tool, helping farmers identify not only where phosphorus is lacking but also where soil–plant interactions are inefficient. This allows for more precise phosphorus application, improved resource allocation, and reduction of unnecessary fertilizer use.

The combined spatial assessment suggests the following interpretations:

- High-uptake zones in the northern sector indicate areas of adequate nutrient supply and more favorable soil biochemical conditions.
- Low-uptake zones in the southern region may require **targeted phosphorus supplementation**, improved soil aeration, or organic amendments to enhance microbial P mineralization.
- Hermite maps provide the best basis for precision fertilization because they preserve both the magnitude and continuity of soil P gradients.

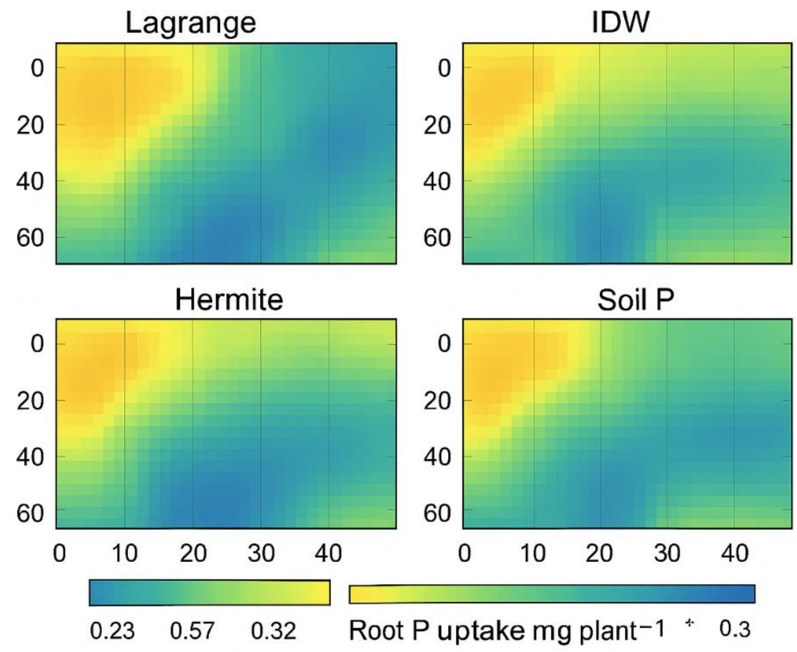

Fig 5. Spatial co-occurrence of soil phosphorus concentration and root P uptake across Lagrange, IDW, and Hermite interpolation surfaces. Warmer colors (yellow) represent higher P uptake, while cooler colors (blue–green) indicate lower uptake. The maps illustrate zones of strong and weak soil–plant phosphorus correspondence across the field.

4.Discussion

4.1. Interpretation of Spatial Patterns of Soil Phosphorus

Spatial heterogeneity of soil available phosphorus (P) in the study field reflects the combined influence of parent material, topographic redistribution, and micro-scale soil physicochemical variability. Variations in silt–clay composition, Fe/Al-oxide activity, and soil organic matter lead to localized differences in P retention and desorption capacity, which manifest as the spatial gradients observed in the interpolated maps (Frossard et al., 2000). These patterns agree with established findings that P availability is strongly controlled by soil mineralogy, weathering status, and microtopographic position, even at sub-field scales (Hinsinger, 2001).
In our study, zones with higher P concentration were generally associated with finer-textured soils or less eroded positions, whereas lower-P areas likely correspond to coarser textures or sites experiencing surface runoff and nutrient removal. The moderate coefficient of variation observed aligns with reports from similar agricultural systems, confirming that P distribution is inherently patchy and influenced by both natural and management-driven factors (McBratney et al., 2003).

Furthermore, micro-scale transport pathways, such as preferential flow domains, can influence the redistribution of phosphorus, resulting in localized accumulation and depletion zones (Fuhrmann et al., 2015) .This mechanism may partly explain the spatial “hot spots” of P in our interpolated maps. In addition, the organic carbon content and fine-textured clay fraction of the soil could contribute to P heterogeneity via adsorption on clay surfaces and slow desorption kinetics (Koch et al., 2023).

Overall, the spatial distribution we generated through interpolation not only reflects the underlying variability of P inputs or legacy P, but also highlights the dynamic interactions among soil pH, carbonate content, clay, and transport processes that govern P availability.

4.2. Relationship Between Soil Phosphorus and Plant Uptake

The positive and relatively strong correlation between soil available P and root P uptake of common bean indicates that plant absorption responds to spatial differences in bioavailable soil P (Richardson et al., 2011). The highest correlation was obtained using the Hermite interpolation surface, suggesting that methods preserving surface gradients may better capture the subtle soil–plant interactions influencing nutrient uptake.
The correlation values are consistent with physiological understanding that P uptake increases with available soil P until reaching a saturation threshold governed by root surface area, root exudates, and microbial interactions(Barber, 1995).

However, the non-perfect correlation highlights biological constraints such as root architecture heterogeneity, rhizosphere competition, and microbial immobilization, which prevent a fully linear soil–plant relationship (Vance, 2001).

One factor that may weaken the soil–plant P uptake relationship is the limited mobility of phosphorus in soil, particularly under alkaline conditions where the diffusion coefficient of P is inherently low, restricting the ability of roots to access P-rich micro domains (Bhunia, G.S. et al. 2018). Moreover, root P uptake is simultaneously controlled by several soil physicochemical and biological variables—such as pH, organic matter, total nitrogen, and soil texture—which influence P sorption, desorption, and diffusion dynamics. These multifactorial controls likely explain why areas with higher soil P do not necessarily correspond to proportionally higher P uptake by roots.

Therefore, although a positive spatial correlation between soil P distribution and root P uptake was observed, the relationship is not strong enough to suggest that soil P concentration alone governs plant P nutrition in these calcareous soils. This highlights the importance of integrating additional soil properties and mechanistic processes when evaluating P use efficiency under such field conditions.

4.3. Comparison with Previous Studies

Our results are comparable with studies reporting substantial sub-field variability of soil P and moderate-to-high correlations between soil P availability and crop P uptake (Cambardella et al., 1994). Similar works in legume-based systems have shown that spatial structure of available P at scales of 5–20 m can meaningfully influence root foraging patterns and nutrient use efficiency (Szoboszlay et al., 2017).

Studies in similar agroecosystems also demonstrate that interpolation method selection can influence soil–plant relationship metrics. For example, methods incorporating slope or gradient information (e.g., Hermite, spline) often perform better in detecting nutrient hotspots and transitions, which agrees with our findings that Hermite produced the highest soil–plant correlation (Corwin & Lesch, 2005; Fuhrmann et al., 2005).

Finally, the application of Hermite or derivative-aware interpolation approaches—common in computer graphics and numerical analysis—has been shown to produce high-quality, smooth reconstructions of surfaces when derivative information is reliable. While such methods are less common in routine soil mapping, their theoretical advantages (C^1 continuity, slope preservation) make them attractive for reconstructing gradual geochemical gradients in well-sampled domains; however, practical environmental applications require careful derivative estimation to avoid propagating noise. This observation matches the rationale and findings of the present study (Hopkins & Ellsworth, 2005).

4.4. Influence of Soil Quality and Microbial Activity

The spatial variability of plant-available phosphorus observed in this study is strongly influenced by the underlying physicochemical properties of the soil. The field exhibited a slightly alkaline pH (7.7), which is known to reduce phosphorus mobility by promoting the formation of calcium–phosphate complexes. This chemical immobilization can limit the fraction of P accessible for plant uptake, particularly in soils with a considerable carbonate content. The measured total neutralizing value (17%) supports the presence of carbonate buffering, further contributing to P fixation in localized zones.

The soil's textural composition—characterized as silty clay loam with 28% clay, 60% silt, and 12% sand—also plays a key role in shaping P distribution. Clay minerals possess a high specific surface area and a large number of adsorption sites, which can enhance the retention of phosphorus but also create spatially patchy availability. The moderate organic carbon content (1.22%) suggests adequate substrate availability for microbial activity, which can stimulate the mineralization of organic P forms through phosphatase enzymes. Such microbial processes tend to generate localized hotspots of bioavailable P, consistent with the fine-scale variations captured particularly well by the IDW and Hermite interpolation surfaces.

Electrical conductivity (0.1 dS/m) and total nitrogen (0.12%) indicate that the soil is neither saline nor excessively nutrient-rich, suggesting that microbial communities are not inhibited by osmotic stress or nutrient imbalance. The relatively high potassium concentration (220 ppm) reflects a nutrient profile typical of moderately fertile agricultural soils, which may indirectly support microbial proliferation and enzymatic turnover, further contributing to small-scale fluctuations in phosphorus release.

Overall, the interplay between soil chemistry (pH, carbonate content), physical structure (texture and clay content), and microbial mineralization processes appears to drive the spatial heterogeneity in available soil phosphorus. These interacting factors provide an ecological explanation for why phosphorus hotspots emerge in a mosaic pattern across the field, and why plant uptake correlates more strongly with the locally sensitive IDW interpolation than with smoother surface reconstruction methods such as Hermite.

Microbial processes strongly influence P cycling by mediating mineralization–immobilization reactions, organic P turnover, and solubilization of mineral-bound P. In the studied field—previously noted for microbial variability—the microbially driven transformations likely contributed to observed P heterogeneity (Richardson & Simpson, 2011). Variability in microbial biomass P, phosphatase activity, and the abundance of phosphate-solubilizing microbes produces localized zones of enhanced nutrient release, which correspond well with the patchy spatial patterns seen in our maps.
Furthermore, soil organic matter quality, moisture dynamics, and aeration significantly regulate microbial activity (Turner & Haygarth, 2005). Areas with higher organic inputs or improved moisture retention typically support greater microbial activity and thus higher P availability.

4.5. Advantages and Limitations of the Interpolation Methods

Each interpolation method demonstrated distinct strengths and limitations.

- The **Lagrange polynomial method**, while mathematically robust and capable of producing continuous surfaces, is sensitive to the number and distribution of sampling points. As polynomial degree increases, oscillations may appear, especially at the boundaries (the Runge phenomenon). Although Lagrange interpolation provided a smooth representation, it was less efficient in capturing local variations in phosphorus, resulting in a relatively weak correlation with root uptake values. This limitation is consistent with previous studies showing that global polynomial interpolators often over smooth natural soil variability (Oumaima Halima et al. 2023).
- **IDW** provided the lowest RMSE and MAE, indicating strong predictive performance for point-wise estimation, but its strictly distance-based weighting limits its ability to represent natural surface continuity. IDW is computationally simple, intuitively interpretable, and commonly used in agronomic mapping. However, it remains deterministic and does not account for underlying spatial autocorrelation structures. Its performance is highly dependent on the choice of the power parameter (p), and an improper choice may lead either to excessive smoothing or to unrealistic sharp gradients. Despite these limitations, IDW remains one of the most practical and reliable options for soil nutrient mapping in small to medium-sized agricultural fields (Li & Heap, 2014).
- **Hermite interpolation** achieved the strongest correlation with plant P uptake due to its incorporation of derivative information, which better preserved slope transitions and biophysically meaningful gradients. By enforcing C^1continuity, Hermite interpolation effectively minimized sharp transitions and better preserved the natural slopes of the soil P distribution. This makes it suitable for modeling gradual geochemical gradients. However, its requirement for estimating spatial derivatives introduces complexity and potential error propagation, especially when sampling density is sparse. Hermite interpolation has been shown to accurately reconstruct environmental surfaces when derivative data are available but may become unreliable under heterogeneous sampling conditions (Burrough et al 1998).

However, Hermite depends on accurate estimation of partial derivatives, which may introduce uncertainty in sparse sampling designs. Overall, the complementary strengths of the methods highlight the value of multi-method interpolation analysis in agronomic investigations.

4.6. Implications for Farm Management and Precision Agriculture

The spatial analysis of soil P and plant uptake provides strong evidence supporting site-specific nutrient management, as the observed heterogeneity indicates that low-P zones can benefit from targeted fertilizer application while high-P areas may require reduced inputs to avoid unnecessary costs and environmental risks (Mulla, 2013). Uniform fertilization strategies in such heterogeneous fields can therefore result in localized over-application or under-application, reducing yield potential and P-use efficiency.

Furthermore, integrating spatial interpolation outputs with sensor-based monitoring or variable-rate application technologies can enhance the effectiveness of precision agriculture systems (Zhang et al., 2002). In particular, the strong correlation observed between Hermite-derived soil P patterns and plant uptake suggests that gradient-preserving interpolation approaches may lead to more accurate nutrient zoning, thereby improving decision-making and overall resource efficiency in field-scale nutrient management (Gebbers & Adamchuk, 2010).

From a broader perspective, applying these interpolation-based spatial analyses contributes to sustainable soil management by reducing over-fertilization, improving nutrient-use efficiency, and minimizing environmental impacts. As precision farming technologies continue to advance, high-resolution nutrient maps derived from interpolation methods such as IDW and Hermite are expected to play a central role in field-scale decision support systems (Mulla, & Crawford, 1997).

5.Conclusion

5.1. Summary of Findings

This study evaluated the spatial variability of soil available phosphorus (P) and its relationship with root P uptake in common bean using three interpolation techniques: Lagrange**,** Inverse Distance Weighting (IDW**)**, and Hermite interpolation. The results demonstrated pronounced sub-field heterogeneity in soil P distribution, with clearly identifiable high-P and low-P zones across the field.

Comparison of interpolation performance showed that the Hermite method produced the strongest soil–plant association ($r = 0.926$), indicating its superior ability to preserve spatial gradients relevant to nutrient uptake. In contrast**,** IDW yielded the lowest RMSE and MAE values, highlighting its effectiveness in point-level estimation accuracy.

Overlay analysis of interpolated soil P surfaces and root uptake patterns revealed a positive but not perfectly matched correspondence, suggesting that soil P concentration alone does not fully control plant P acquisition. Additional soil properties and micro-scale transport processes likely contribute to the observed spatial mismatch. Overall, the combined spatial assessment underscores the complex interactions governing P availability and uptake in calcareous soils.

5.2. Response to research objectives

The first objective of this study was to characterize the spatial distribution of soil available phosphorus (P) across the experimental field. This objective was successfully achieved through the application of Lagrange, IDW, and Hermite interpolation methods, each of which consistently revealed substantial spatial heterogeneity. Distinct high-P and low-P microsites were identified, demonstrating that soil P variability occurs even at short distances and must be considered in field-scale nutrient assessments.

The second objective was to evaluate the relationship between soil P distribution and root P uptake in common bean. Through point-by-point spatial correlation analysis, a positive and statistically meaningful association was observed for all interpolation methods. The Hermite interpolation surface produced the strongest correlation with uptake values, indicating that gradient-preserving methods more effectively capture biologically relevant nutrient patterns.

The third objective was to compare interpolation methods in terms of accuracy and their capacity to represent soil–plant interactions. This objective was fully met: IDW demonstrated the lowest RMSE and MAE values, reflecting its effectiveness in estimation precision, while Hermite yielded the highest correlation with plant uptake, underscoring its advantage in ecological realism. These findings confirm that method selection directly influences both spatial interpretation and biological inference.

The final objective was to identify spatial zones requiring management attention, particularly areas with low soil P and limited uptake. Overlay analysis of the interpolated maps successfully delineated such zones, providing actionable information for targeted fertilization and site-specific nutrient management.

Overall, all research objectives were fully addressed, and the study provides an integrated framework combining geostatistical interpolation, plant response mapping, and spatial analysis to improve the understanding of P dynamics in calcareous agricultural soils.

5.3. Recommendations for farmers and policymakers

The spatial patterns identified in this study provide a strong scientific basis for improving nutrient management in calcareous agricultural soils. Based on the results, several practical recommendations can be formulated:

1. Implement site-specific phosphorus (P) fertilization.
The pronounced variability of soil P across short distances highlights the limitations of uniform fertilization strategies. Farmers are encouraged to adopt variable-rate P application, focusing inputs on low-P zones while reducing fertilizer use in high-P areas. This approach improves

fertilizer-use efficiency, lowers production costs, and minimizes environmental risks associated with P losses.

2. Integrate gradient-preserving maps into precision agriculture systems.
Since Hermite interpolation showed the strongest alignment with plant P uptake, gradient-preserving nutrient maps should be incorporated into decision support tools and variable-rate applicators. Policymakers and extension agencies can promote the adoption of digital soil mapping platforms that incorporate such methods.

3. Enhance soil testing frequency in heterogeneous fields.
Fields exhibiting patchy nutrient distribution—such as the one investigated here—require more frequent or higher-density sampling. Policymakers should encourage standardized soil-testing programs, supported by subsidies or cooperative testing facilities, to ensure farmers obtain accurate and economically meaningful assessments.

4. Promote integrated soil health management.
Because P uptake is influenced by multiple soil properties (e.g., pH, organic carbon, texture, microbial activity), nutrient management plans should not rely solely on P concentration. Farmers should be advised to combine P fertilization with practices that improve soil structure and biological activity, such as adding organic amendments or adopting reduced tillage. Policymakers can support these practices through incentives or soil health initiatives.

5. Encourage adoption of precision agriculture technologies.
Variable-rate spreaders, real-time sensors, GPS-enabled field mapping, and remote sensing platforms can greatly improve nutrient allocation efficiency. Government-supported training programs and financing schemes can help farmers—especially smallholder and medium-sized producers—transition to precision management technologies.

6. Support long-term monitoring and research.
The findings indicate the value of repeated spatial assessments to monitor nutrient dynamics over time. Policymakers should invest in long-term soil monitoring networks that integrate geospatial, agronomic, and environmental data. This will improve regional nutrient management strategies and support sustainable agricultural intensification.

Overall, these recommendations aim to translate the scientific outcomes of the study into actionable strategies that enhance productivity, reduce fertilizer waste, and support environmentally responsible phosphorus management at both farm and policy levels.

5.4. Suggestions for future research

Several avenues for future research emerge from the findings and limitations of the present study:

1. Incorporation of larger and denser sampling grids.
Although the interpolation approaches performed well, higher sampling density—especially in

transitional zones—would allow more precise characterization of micro-scale phosphorus (P) variability and improve the accuracy of spatial models.

2. Integration of additional soil physicochemical variables.
Future work should include variables such as pH, soil organic carbon, clay fraction, calcium carbonate content, and moisture, which strongly influence P sorption, desorption, and mobility. Multivariate spatial models could better capture the complex controls on P availability and plant uptake.

3. Coupling interpolation outputs with mechanistic nutrient transport models.
Linking spatial P maps with process-based models (e.g., diffusion–adsorption models, root foraging simulations) would provide deeper insights into how soil heterogeneity drives nutrient acquisition and crop performance under varying environmental constraints.

4. Assessment of microbial contributions to P dynamics.
Since microbial activity can significantly influence P mineralization and immobilization, future studies should characterize microbial community composition and enzyme activities alongside spatial P measurements to improve predictive modeling of nutrient availability.

5. Multi-year monitoring to capture temporal variability.
Temporal changes in soil P distribution due to fertilization, crop rotation, irrigation patterns, and weather events remain insufficiently understood. Long-term monitoring would reveal whether spatial patterns are stable or shift substantially over time.

6. Comparison of additional interpolation and machine learning techniques.
Expanding the methodological scope to include kriging variants, regression kriging, random forest, or hybrid geospatial models may improve the detection of nutrient hotspots and relationships with plant uptake. Benchmarking these models against Hermite and IDW could refine best-practice recommendations.

7. Evaluation of field-scale management outcomes.
Future studies should validate how precision P management strategies derived from gradient-aware interpolation maps affect yield, fertilizer efficiency, and environmental outcomes. On-farm trials would provide direct evidence for scaling these approaches in real-world agricultural systems.

Together, these research directions would expand current understanding of soil–plant nutrient dynamics and support the development of more accurate, efficient, and sustainable P management strategies.

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