Relativity for the Realm of the Living: A Proposal for an Extended General Theory

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Abstract:

Biological systems are here reinterpreted through a geometric lens that extends the insight of general relativity: organisms and their interactions are modelled as deformations of a multidimensional biological hyperspace. In this view, each entity acts as a source that locally bends a relational field defined by molecular, morphofunctional and ecological axes, thereby altering the trajectories of others through both attractive (for example, trophic or mutualistic) and repulsive (for example competitive or avoidance) effects. The framework is explicitly multiscale, intending to accommodate hierarchical organisation from genes to ecosystems, with a data-fixed ecological metric g estimated from observations (e.g. Mahalanobis/Fisher–Rao) and updated through time, and it incorporates evolutionary directionality, with state-dependent geometry that changes through time. A minimal mathematical programme is outlined that links curvature, interaction tensors and geodesic motion, together with falsifiable predictions, including curvature-dependent convergence or divergence of trajectories, finite-speed propagation of ecological influence with a system-dependent upper bound c_{eco} and local reduction to classical models. Field equations are derived from a variational principle in which the geometric–matter coupling is carried by a scalar field ϕ, so that $\kappa=\kappa(\phi)$; this yields covariant conservation of the total ecological–geometric current (Noether/Bianchi). The aim is methodological rather than doctrinal: to provide a coherent language that unifies ecology, evolution and complexity science and to invite collaboration on a dynamic geometry of life. If successful, this approach could inform both the interpretation of large-scale historical events and practical ecosystem management by making explicit constraints, couplings (via $\kappa(\phi)$) and distributed, porous sources.

1. Introduction

On the Name

The name Extended General Theory of Relativity (if in German, *erweiterte allgemeine Relativitätstheorie*) had been chosen in deliberate reference to Einstein’s (1916) *allgemeine Relativitätstheorie*. In that foundational paper, the general theory is described as the most comprehensive and natural generalisation of gravitation compatible with physical experience. If, as he suggested, this formulation exhausted what was conceivable within that discipline, then any further generalisation must necessarily move beyond that scientific realm: for example, towards the domain of life, but also in all other disciplines that imply the dynamics of complex systems. The point of departure proposed here is biology (hence, it could for short be referred to as Biorelativity) and more specifically evolutionary ecology. The theory is thus grounded in the long-term dynamics of life on Earth, but its underlying logic may be extensible to other domains in which agents co-construct environments, generate relational fields and alter one another’s trajectories in ways that defy simple causal reduction.

Why Biology Needs New Geometry

The most formal scientific disciplines, mathematics and physics, have long influenced biology, from the eccentric work of D’Arcy Thompson (1917) and the energetic models of metabolism (Bauer 1935), through the statistical mechanics of population genetics (Fisher, 1930; Haldane, 1924-34; Wright, 1931) to biomechanics (Alexander, 1968) and catastrophe theory in biological morphogenesis (Thom 1972 – but see Sussmann, 1978). In this sense, it could be conceived that, in biology, the presence and activity of living agents modify the very conditions under which they persist, compete, cooperate and evolve. The environment is not a static background, but a historically contingent, dynamically restructured field co-created by its inhabitants.

Just as Einstein's general theory of relativity emerged from embedding special relativity within the geometric structure of Minkowski spacetime (a four-dimensional pseudo-Riemannian manifold where the invariant interval fuses space and time; see Einstein & Infeld 1938), this conceptual shift in biology proposes that ecological and evolutionary processes can be envisioned as trajectories across a higher-dimensional, dynamically deformable biological hyperspace. Unlike physical systems, however, biological entities are organised hierarchically, from genes to cells to organisms to populations to ecosystems and the whole biosphere, each level capable of influencing and being influenced by the others (Levin, 1992). This nested structure implies that the ecological manifold is not simply continuous in spacetime, but also stratified across organisational scales, leading to multilevel interactions whose effects are neither additive nor strictly local. Unlike the compact, sharply bounded sources of gravitation, biological entities are open and porous, partly continuous with the medium that contains and integrates them, so identity is better represented as a distributed field with moving fuzzy interfaces rather than a discrete mass. This multiscalar, co-constructive nature of life, where organisms and their environments are mutually influential, requires an extended framework that accommodates emergence, plasticity and context-dependence. While in Minkowski space all observers agree on the form of physical laws, in the biological analogue, the very structure of the space may vary depending on the ecological and evolutionary history of its occupants.

Hence, I explore here the possibility of conceptualising biological systems through the lens of geometric deformation, further generalising Einstein's (1916) insight that gravity is the curvature of spacetime. The core proposal is that biological entities deform an abstract, multidimensional space that reflects their genetic, morphological, behavioural and ecological characteristics as well as all the abiotic factors (salinity, rainfall, substrate): the ecomorphological hyperspace (as in niche theory; Hutchinson, 1957; cf. Grinnell's, 1917, habitat-centred niche grounding abiotic axes; Soberón, 2007). In this analogy, each biological entity acts as an ecological mass-energy that locally deforms the space. Other entities respond to this deformation by moving closer or further away, giving rise to ecological and evolutionary dynamics.

In this theory, the analogue of inertial frames are eco-inertial frames: local charts of eco-space, co-moving with a community's dynamical equilibrium, in which free dynamics without net external forcing follow geodesics and the equations take their simplest form, that is, normal coordinates with Christoffel symbol comes to zero ($\Gamma=0$). This is an ecological reformulation of the local equivalence principle in which, over sufficiently small space-time domains, curvature is negligible and dynamics are approximated by a flat metric, exactly as in general relativity when coordinates are constructed in such a way that nullify the Christoffel symbols at a point (Carroll 2004). Operationally, the frame is identified by removing the mean drift of the system and minimising the covariant acceleration of trajectories; this occurs, for example, in the neighbourhood of Lotka–Volterra limit cycles (to be discussed elsewhere, Vallejo & Fariña 2025) and near equilibria of consumer–resource models, and in co-moving frames with environmental fronts for animal movement where step-selection models show no residual trend (Lotka 1925; Volterra 1926; MacArthur 1970; Tilman 1982; Fortin et al. 2005; Avgar et al. 2016). The locality of such frames is constrained by the characteristic speed at which ecological reconfigurations propagate, which can be bounded using reaction–diffusion wave speeds and observed climate velocity (Fisher 1937; Kolmogorov et al. 1937; Loarie et al. 2009). In thermodynamic terms, an eco-inertial frame is one in which exergy and information gradients vanish to first order, so that production and dissipation are locally balanced (Jørgensen 1992).

On the Use of the Terms 'Biological' and 'Ecological'

Throughout this proposal, I refer variously to biological and ecological dimensions. While related, these terms are not interchangeable and some clarification is warranted. By biological, I refer to organisms in interaction, either with one another or with their environment. This relational stance is central: an organism does not act or evolve in isolation but through entanglements with trophic networks, competitors, symbionts and, importantly, abiotic shifts such as climate, the uplift of a mountain range or the opening of a seaway. In this context, biological is thus taken to be effectively equivalent to ecological in the broadest sense.

Also, the term ecomorphological, in turn, highlights those organismal traits that result from, or participate in, such ecological interactions, an approach that finds its roots in niche theory (Hutchinson, 1957 - also see Grinnell's 1917; Soberón, 2007

- abiotic niche for geographic distributions). Morphology is often the most visible and tractable proxy for ecological function, particularly in macroorganisms such as most animals and plants. However, it is by no means the only relevant axis: physiology, behaviour, life history traits and even biochemical pathways may equally define an organism's position in ecological space. This is especially true in the case of microorganisms, whose morphological simplicity belies their complex physiological and ecological versatility.

Also, genetic information is a first-class component that constrains and is shaped by eco-geometry. Concretely, lineages are embedded in a working phylogeny and use phylogenetically informed covariances to estimate the ecological metric g and its local normal coordinates, so that inheritance and shared history are explicit rather than tacit (Felsenstein 1985; Pagel 1999). Molecular features enter the matter fields Ψ either as slow variables or as latent modulators of the eco-coupling $\kappa(\phi(x,t))$, providing a transparent path from genotype–phenotype mappings to curvature-driven predictions (Hansen 1997; Maynard Smith & Szathmáry 1995). While a full genomic dynamics is not intended here, a tractable pipeline is committed: assemble a published tree for a focal clade, estimate g under phylogenetic dependence, then test whether geodesic curvature improves out-of-sample fit for longitudinal trajectories compared with flat models. This preserves the manuscript's mesoscopic focus while aligning it with molecular and comparative evidence.

It should be noted that scalar–tensor theories (e.g. Brans–Dicke) show how a coupling can be promoted to a field; analogously a scalar eco-coupling ϕ, with $\kappa = \kappa(\phi)$ (see §5(d) for the variational formulation) is adopted here (Brans and Dicke 1961).

2. From Spacetime to Ecomorphological Hyperspace/Evolutionary Time

In general relativity, matter and energy curve spacetime and this curvature governs the motion of objects through attraction. In an analogous way, organisms could be conceived as generating deformations in a hyperdimensional ecological space, causing other organisms to respond, whether through attraction (e.g., predation, mutualism) or repulsion (e.g., competition, avoidance). Crucially, interaction signals are scale dependent: what is repulsive at the level of a species pair (for example, exploitative competition) can, once coarse-grained over many interactions and environmental feedbacks, act attractively at the level of the community by promoting niche partitioning, modular cohesion and stable resource flows. In geometric terms, renormalising micro-interactions can flip the effective curvature so that local divergence between competitors aligns with convergence of assemblages toward a shared attractor basin. Repulsion and attraction (see 3) are therefore not fixed properties but scale-relative expressions of the same underlying field, mediated by cross-level couplings.

Following Reichenbach's (1956, 1958) distinction between mathematical geometry (a formal calculus) and physical geometry (an empirical structure fixed by measurement), the ecomorphological hyperspace is treated here as an example of the latter: its metric is inferred from data and its curvature co-determined by interacting organisms, rather than a fixed Euclidean background. This makes the geometry itself a testable object, not merely a convenient coordinate scaffold.

Formally, the axes of this ecological space can be estimated from a trait–environment matrix X (e.g. rows = taxa, columns = variables) using principal component analysis; the eigenvalues quantify the variance captured by each orthogonal axis, while distances are better computed with covariance-aware metrics to respect trait correlations and niche geometry (Hutchinson, 1957; Jolliffe, 2002; Etherington, 2021) such as Mahalanobis or the certainly more accurate Fisher-Rao Information Metric(Fisher, 1922; Rao, 1945), which does not imply a flat geometric space but defines a geometry on a space of probability distributions. Grinnell's (1917; Soberón, 2007) environmental niche underpins climatic axes in this geometry. Thus, each organism, population or species is represented by a vector in this space whose position and separation reflect both trait loadings and correlated environmental structure. At the interaction level, ecological structure is naturally encoded by $n \times n$ matrices (e.g. the community Jacobian, or the adjacency/Laplacian of a network), whose eigenvalues/eigenvectors govern local stability, modular cohesion and diffusion time-scales; thereby connecting classical community-matrix stability theory with spectral analyses of ecological networks

(May, 1972; Chung, 1997; Bascompte and Jordano, 2007). In the present framework, deformations of the space arise jointly from (i) covariance structure in X (axis scaling/rotation) and (ii) the spectral properties of the interaction matrix (field curvature across levels).

The ecological metric g is treated here as a data-fixed, physical geometry. Locally, two complementary estimators are adopted: (i) a covariance-aware metric (Mahalanobis) when states are represented as trait–environment vectors; (ii) the Fisher–Rao information metric when states are probability distributions, which directly yields a non-Euclidean geometry on the statistical manifold. In practice, (a) local charts are estimated with PCA or related low-distortion embeddings, (b) g is computed as Σ^{-1} (covariance) or as Fisher–Rao, and (c) uncertainty is propagated into curvature estimates. This makes the geometry testable and comparable across systems.

3. Attraction, Repulsion and Multiscale Interactions

Unlike gravity, which is only attractive, ecological interactions are both attractive and repulsive. The analogy here aligns more closely with the vectorial nature of electromagnetic fields (Maxwell 1865), where charges of opposite sign attract and like charges repel. However, it is worth noting that even in physics, the simplification of gravitational interaction as purely attractive holds only locally. On cosmological scales, observations since the late 1990s have revealed that the universe is undergoing accelerated expansion (Riess et al. 1998; Perlmutter et al. 1999). This suggests that the interplay between attraction and repulsion, and their scale-dependent dominance, is not unique to biological systems. In both physics and biology, large-scale organisation may arise from forces that are locally attractive but globally dispersive or expansive.

Undisputably, interactions in Biology are fairly more complex: mutualisms, parasitism and commensalism correspond to different types of ecological 'charges'. It may therefore be necessary to develop a theory of vector fields (analogous to electromagnetic fields) or complex tensor fields capable of representing multiple types of interactions simultaneously.

Additionally, biological systems are inherently hierarchical. Organisms are coded in genes, composed by molecules, cells, tissues and organs; they exist within populations, which constitute species, which participate in communities and ecosystems. This multilevel organisation implies that the geometry of the biological field must be able to represent scale-dependent and nested interactions. Each level could be understood as an additional dimension or as different scales within ecological space, requiring formalisms capable of moving fluidly across scales (renormalisation theories, hierarchical complex systems, fractals).

This shift from attraction to repulsion can be illustrated conceptually as a deformation of ecological space: a prey organism, initially creating a local depression that draws a predator toward it (Fig. 1A), may, following an evolutionary change such as increased body size, come to generate a convex, repelling curvature that inhibits predatory access (Fig. 1B). Note that the biological entities are in a continuous with the fabric of the hyperspace/evolutionary time. Such geometric transitions model the qualitative change in ecological interaction through alterations in local curvature. Moreover, the repelled organism may not simply remain isolated but could instead be displaced into the ecological domain of another species, triggering a cascade of indirect effects akin to a biological butterfly effect (Lorenz, 1963). In this way, local changes in one region of the ecomorphological hyperspace may propagate across the system at a maximum speed c_{eco} similar to c in Lorentz's equations, reshaping broader ecological dynamics.

Given g, the Levi-Civita connection (metric-compatible, torsion-free) is adopted as the canonical choice for inertial ecological motion (geodesics). This ensures a clean parallel transport and well-posed geodesic deviation. Extensions with torsion or non-metricity can encode history-dependence, but this subject is left for future work focused on irreversible memory effects.

4. Incorporating Evolution and Time's Arrow

The second difference is perhaps the most profound: there is a great difference between the physical and biological domains in the unidirectionality and historical contingency of evolution. Indeed, Gould (1987) distinguishes between cyclical time and the arrow of time and emphasises that historical processes, such as evolution and unlike planetary orbits, should be understood as irreversible, characterised by unique trajectories that cannot be replicated.

Eddington's cone and time's arrow.

In a Minkowski spacetime, the light-cone structure fixes what can causally influence what, even though the fundamental equations are largely time-reversal symmetric (Minkowski, 1908-09). Eddington (1928) argued that the arrow of time is selected by the growth of entropy, which picks out the future-directed branch of the cone and renders macroscopic evolution irreversible. For living systems, which are open and far from equilibrium, this asymmetry is amplified: they export entropy while maintaining organisation via flows and feedbacks, paradigmatic dissipative structures (Prigogine, 1977). In the present framework, causal influence is constrained by an ecological cone with slope c_{eco}. Beyond this kinematic bound, causation is shaped by a non-stationary ecological metric g_{ij} and by an evolving coupling $\kappa(\phi(x,\tau))$ mediated by the scalar eco-coupling field ϕ; when sensitivity to ecomorphological separation or evolutionary age is required, κ depends only on scalar invariants such as the geodesic distance d_g and ecological proper time τ. Together these features bias geodesics toward persistence.

The result is an anti-deterministic stance: given identical macrostates there remain many admissible microhistories, and which branch is realised depends on stochastic innovations, selection and cross-level feedbacks rather than geometry alone (Eddington, 1928; Prigogine, 1977).

The ecomorphological space itself is not static: it is constantly reshaped by evolutionary innovations, extinctions and environmental change from external sources. Evolution introduces irreversible directionality, unpredictability and historical contingency, in contrast to the relative stability and predictability of classical physical laws.

Whereas Fig. 1 shows the simplest case (i.e., similar to gravity interactions of general relativity), in Fig. 2, a single sphere moves forward in time (as in evolution) through a deformable ecological surface, simplified here again as a two-dimensional elastic mesh. It leaves behind a deep trailing depression and generating a forward elevation and lateral distortion. The shape and displacement of the surface illustrate the temporal dynamics of ecological interaction: the depression behind the sphere implies an attractive effect on other organisms or ecological entities, while the raised front region indicates repulsive or exclusionary effects due to the evolving position or properties of the focal organism. This asymmetric deformation represents how organisms and other biological entities, as they change over time, simultaneously reshape their ecological surroundings, attracting some components of the system while repelling others.

As a consequence of the above discussed issues, any generalised geometric theory for biology must go beyond the static manifolds of classical physics and embrace evolving topologies, singularities, bifurcations and path dependence. Concepts from dynamical systems theory, stochastic processes and non-equilibrium thermodynamics may be indispensable. This could be modelled using non-stationary dynamic geometries (spaces that change over time), which would require advanced mathematical theory such as dynamic differential geometry, chaos theory and non-linear dynamics, bifurcation theory (catastrophes), stochastic processes, Bayesian methods and, when needed, possibly new developments.

In line with Reichenbach's (1956, 1958) causal theory of time, temporal order is grounded in causal order. Within the present framework, evolutionary change causes deformations of the ecological geometry, and those deformations in turn constrain subsequent trajectories; the geometry thus acts as a record of causal history. The evolutionary arrow is therefore not imposed externally but emerges from the directed accumulation of causal constraints in the evolving metric–connection pair (g, Γ).

On the ecological speed bound. c_{eco} is treated as a system-dependent upper bound on the propagation of ecological reconfiguration, not as a universal constant. Operationally, it is bracketed by reaction–diffusion wave speeds and climate velocity in the focal system; estimates enter causal cones and delay kernels accordingly.

5. Toward a Formal Theory

Background

The conceptual framework proposed here seeks to extend the principle of relativity beyond its physical origins, offering a dynamic, geometric perspective on biological systems and their interactions. However, in doing so, it is essential to acknowledge and evaluate the theoretical landscapes that precede and, to some extent, inform this approach. Rather than reinvent existing frameworks, the present proposal aims to integrate and generalise their insights within a unified, relational geometry of life.

Several prior models are particularly relevant. First among them is ecological niche theory, particularly as formulated by Grinnell (1917) and Hutchinson (1957), who conceptualised species as occupying positions within a multidimensional ecological space. This notion of a hypervolume prefigures the idea of an ecomorphological hyperspace, albeit without a dynamic geometry or explicit interaction fields.

Darwin (1859) envisaged evolution as a historically contingent and branching process, comparable to a tree, where lineages diverge and cannot return to ancestral states. This vision anticipates the idea of adaptive landscapes as introduced by Wright (1932) and later expanded by Simpson (1944), in which fitness surfaces guide evolutionary trajectories. While powerful as a metaphor, these landscapes are typically scalar and static and lack the capacity to represent mutual deformation among interacting agents. In contrast, the present framework imagines a relational geometry in which all organisms are both shaping and being shaped by the hyperspace itself.

A related and independent attempt to import relativistic thinking into evolutionary theory is Greenstein's (2015) General Relativistic Biology, which recasts natural selection via an equality between statistical and causal perspectives on fitness and proposes a Newtonian-style relation $N = f s$ (where N is natural selection, f is fitness and s is struggle) under a general principle of biological relativity. The approach presented here differs in offering an explicit differential-geometric formalism: a dynamically deformable ecomorphological manifold with metric g_{ij}, curvature G_{ij}, interaction tensors T_{ij} and a context-dependent coupling

κ, alongside a special-relativistic analogue with c_{eco}, rest eco-mass and geodesic dynamics across hierarchical levels. In this sense, Greenstein (2015) motivates the relativistic reframing, whereas here a field-theoretic and tensorial structure aimed at analysis and testable modelling is provided.

For the action

$$S = \int \sqrt{|\, g \,|} \left[\frac{1}{2\kappa(\phi)} R + \mathcal{L}_{eco}(\Psi, \nabla\Psi; g) - \frac{1}{2} g^{ij} \partial_i \phi \partial_j \phi - U(\phi) \right] dV,$$

the scalar eco-coupling equation reads

$$\Box\phi - U'(\phi) - \frac{1}{2}\kappa(\phi)^{-2} R = 0.$$

Conservation. By diffeomorphism invariance and the contracted Bianchi identity, the total ecological–geometric current is covariantly conserved, so that any apparent non-conservation at the ecological sector is balanced by exchange with the ϕ-field; in closed, time-independent limits, Noether's theorem yields conservation of the eco-Hamiltonian.

The emerging fields of ecological network theory and complex network theory offer powerful tools for analysing the structure and dynamics of multispecies systems. These approaches highlight the importance of indirect effects, modularity and feedbacks, features that any relational geometry of life must ultimately accommodate.

In addition, the proposal resonates with the principles of complex adaptive systems theory, as developed in part through the work of the Santa Fe Institute (see Kauffman 1993; Levin 1998; Cowan, Pines & Meltzer 1994). Those models emphasise self-organisation, emergence and non-linear dynamics in hierarchical systems, key attributes of the biological world that a generalised theory of ecological relativity must be capable of capturing.

A complementary line of work is the Principle of Biological Relativity (Noble et al. 2019), which argues that, a priori, there is no privileged level of causation in multiscale biological systems, and that circular but asymmetric causality links levels: upward causation is captured by the dynamics (e.g., differential equations) of lower-level processes, whereas downward causation is best represented as the setting of initial and boundary conditions determined by higher-level organisation and environmental context. This view aligns with the present framework's emphasis on multilevel constraints and historical dependence. Again, this contribution differs by introducing an explicit differential-geometric formalism, a deformable ecological manifold with metric g_{ij}, curvature G_{ij}, interaction tensors T_{ij} and a state-dependent coupling $\kappa(\phi(x,t))$, together with a special-relativistic analogue c_{eco}, rest eco-mass) for modelling feasible trajectories.

Finally, the field of macroecology provides a large-scale empirical and theoretical context in which to situate this proposal. Macroecology seeks to understand broad patterns of diversity, distribution and interaction across space and time and may serve both as a testbed and as an arena for application of the proposed geometric formalism.

In sum, the framework outlined here is not constructed in opposition to these existing paradigms but seeks to unify and extend them through a novel mathematical and conceptual language inspired by the geometry of relativistic physics.

Mathematical Challenges

This theory aims to unify these approaches within a broader geometric-dynamic perspective, connecting evolution, ecology, morphology, and environment through an advanced mathematical formulation inspired by theoretical physics. At present, the proposal includes only the draft of a precise mathematical formalism. However, potential avenues include:

- Defining a dynamic manifold or fibre bundle for the ecological hyperspace.
- Constructing ecological interaction tensors analogous to the Einstein tensor.
- Describing interaction fields (e.g., attraction/repulsion) using vector calculus or differential geometry.

- Modelling species trajectories through geodesics influenced by other organisms' warping effects.
- Introducing evolutionary perturbations through stochastic deformation terms or field fluctuations.

Many classical models reappear as local or coordinate limits of a state space endowed with a time-varying metric g, connection Γ and context-dependent coupling κ. This provides a bridge between the proposed geometry and familiar dynamics and offers routes for parameter estimation.

Logistic growth: $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$

Local 1D flow on a flat metric. The vector field can be written as gradient flow in a suitable potential $\Phi(N)$ or as geodesic motion with non-zero drift.

Lotka–Volterra predator–prey (see Vallejo & Fariña 2025): $\frac{dx}{dt} = (\alpha - \beta y)x$, $\frac{dy}{dt} = (\delta x - \gamma)y$

Local 2D dynamics in normal coordinates (Γ= 0 at equilibrium, $g \approx \mathrm{I}$). Interaction coefficients map into entries of T_{ij} and the local Jacobian.

Generalised LV competition: $\dot{\mathbf{N}} = \mathbf{N} \odot (r - A\mathbf{N})$.

Near operating points the flow matches the proposed form with T capturing pairwise effects and g shaping anisotropic response.

MacArthur consumer–resource: $\dot{N_\iota} = N_i(\sum_\alpha c_{i\alpha} R_\alpha - m_i), \dot{R_\alpha} = s_\alpha - d_\alpha R_\alpha - \sum_i c_{i\alpha} N_i R_\propto$

Provides a mechanistic T via resources. After adiabatic elimination of R, an effective interaction tensor consistent with κ is obtained.

Replicator dynamics: $\dot{x_\iota} = x_i[(Ax)_i - \mathrm{x} \cdot A\mathrm{x}]$

Gradient flow under the Shahshahani metric on the simplex: a canonical example where the choice of metric determines the dynamics, aligning with the present emphasis on g.

Ornstein–Uhlenbeck trait evolution: $dv = - - \beta v dt + \sqrt{2D} dW_t$

Linear well in a flat metric with stochastic forcing; local SDE limit for traits.

Fisher–KPP reaction–diffusion: $\frac{\partial p}{\partial t} = k\frac{\partial^2 p}{\partial x^2} + mp(1-p)$

Spatiotemporal extension where diffusion acts as Δ_g if the substrate carries a metric g; front speed can bound c_{eco}.

Wright–Fisher diffusion / Fokker–Planck: $\frac{\partial P}{\partial t} \;=\; -\sum_i \frac{\partial}{\partial x_i}(A_i\, P) + \frac{1}{2}\sum_{i.j} \frac{\partial^2}{\partial x_i \partial x_j}\left(B_{ij}P\right)$

Links to information geometry and natural gradients, again highlighting the role of a non-Euclidean metric. Geometric fitness is a functional of the interaction metric g_{ij}, curvature G_{ij}, interaction tensors T_{ij} and a state-dependent coupling $\kappa(\phi(x,t))$, together with a replicator/Fokker–Planck–type equation for the state distribution P, where f increases with growing ρ_e and with the curvature of the space such that desirable trajectories (resources/allies) converge while threats diverge.

Leslie matrix / McKendrick–von Foerster: $\mathbf{n}_{t+1} = \mathbf{L}\mathbf{n}_{\mathrm{t}}; \quad \partial_t n \,+\, \partial_a n \,=\, -\mu(a)\; n$

provide structured axes and constraints that can be encoded in block metrics and cross-level couplings $C^{\{LM\}}$.

Networked reaction–diffusion on graphs: $\mathrm{x} = \dot{\mathrm{F}}(x) - \mathcal{L}x$ with graph Laplacian $\mathcal{L}$.

A special case of Δ_g on discrete geometries. Useful to estimate anisotropy and effective distances induced by g.

Technical box: Eco-inertial frames and geodesic motion

Definition. An eco-inertial frame at (x^*,t^*) is a local chart of the ecological state manifold $(\mathcal{M}, g)$ that is co-moving with the system's mean drift, such that normal coordinates satisfy $\Gamma^i_{jk}\, x^* = 0$ and the free motion of the focal system is geodesic:

$$\frac{Du^\mu}{d\tau} = \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda}\frac{dx^\nu}{d\tau}\frac{dx^\lambda}{d\tau} = 0,$$

with $u^\mu = \frac{dx^\mu}{d^\tau}$. Here g is a data-derived ecological metric.

Operational detection.

(i) Construct the state vector

z=(abundances, resources, traits, information, habitat)
(ii) Estimate g from an ecological similarity kernel or as the Hessian of a potential function.
(iii) Compute normal coordinates via the Riemannian exponential–logarithm around $(z^{\backslash *}, t^{\backslash *})$
(iv) Choose the co-moving chart that minimises $\parallel \frac{\mathrm{Du}}{d\tau} \parallel$ on short time windows.
(v) Diagnose ecoevolutionary forces as departures from geodesic motion in that chart.

Local validity. The frame is valid while the ecological curvature scale $\rho \sim \parallel \mathrm{R} \parallel^{-1/2}$ greatly exceeds the observational scale L, and while reconfiguration speeds are below a context-dependent bound. Two practical bounds are:

- **Reaction-diffusion propagation:** Fisher-KPP dynamics $u_\tau = D\nabla^2 u + ru(1-u)$, where $u_{(x,\tau)}$ is the local population density, τ denotes time, D is the diffusion coefficient representing dispersal and r is the intrinsic growth rate; the minimum wave speed is $c_* = 2\sqrt{rD}$.
- **Climate velocity**: the instantaneous speed required to track isotherms provides an empirical upper bound on environmental forcing speed.

See Carroll (2004) for normal coordinates in General relativity, Fisher (1937) and Kolmogorov et al. (1937) for wave speeds and Loarie et al. (2009) for climate velocity.

Thermodynamic characterisation. In an eco-inertial frame, net exergy and information fluxes cancel to first order at the focal scale, yielding zero local gradient of these potentials. This follows the exergy formalism in ecosystems (Jørgensen 1992).

Box 1. Classical models as local limits of the geometric framework

Given the conceptual ambition of the framework, it may ultimately require not only the application of existing mathematical tools but also the creation of new ones. Newton developed his method of fluxions, a form of differential calculus, to describe motion and change, and this formed the mathematical foundation for his formulation of classical mechanics in the *Principia* (1687). Hence, a genuinely extended theory of biological relativity may demand a corresponding expansion of mathematical language, capable of handling systems in which causality is distributed, feedback is ubiquitous and the geometry itself evolves under the influence of historical interactions. Such a development would lie beyond the scope of this initial proposal, but its necessity should not be excluded a priori. Apart from such unpredictable needs, this theory will require a robust and sophisticated mathematical formulation, which may involve:

a) Formal definition of ecological space (n-dimensional hyperspace):

- A vector space or differentiable manifold of dimension n.
- Each species i is described by a state vector whose coordinates correspond to ecological variables.

b) Tensor fields representing interactions:

- An ecological metric tensor that describes how each organism locally modifies the ecological space.
- Vector or tensor fields to represent attraction–repulsion interactions (analogous to the electromagnetic tensor).

Interaction fields would need to capture not only the presence of relationships among organisms but also their magnitude, directionality and context-dependence. These could take the form of ecological interaction tensors (e.g. predation, competition, mutualism), analogous in structure to the energy-momentum tensor in general relativity. However, unlike in the physical case, the coupling between interaction and curvature is unlikely to be governed by a universal constant. Instead, the ecological analogue of Einstein's field equation may require a context-sensitive coupling mechanism (possibly a dynamic operator or a nested system of equations – see c) modulated by trophic asymmetries, environmental feedbacks, local densities and historical constraints. This reflects the plastic and historically contingent nature of ecological interactions.

c) Porous boundaries and distributed sources

Unlike point sources, biological entities are open systems with fuzzy interfaces (Levin 1992; Allen & Starr 1982; Deacon 2011); we therefore represent identity as distributed fields (phase-/level-set), letting T_{ij} and g_{ij} depend on smoothed densities and evolving interfaces.

Formally, the porous, partly continuous nature of biological entities suggests replacing point sources by distributed identities. Each entity *i* can be represented by a smooth phase-field, with membership constrained to the unit interval; alternatively, a level-set field may be used whose zero-level gives the moving interface.

Phase-field range

$$\chi_\mathrm{i}(\mathrm{x},\mathrm{t}) \in [0{,}1]$$

Level-set interface

$$\text{Interface: } \{\mathrm{x}\colon\ \phi_\mathrm{i}(\mathrm{x},\mathrm{t}) = 0\}$$

Interaction and curvature depend on smoothed densities and interface geometry:

Smoothed density

$$\tilde{\rho}_\mathrm{i}(\mathrm{x},\mathrm{t}) = (\mathrm{K}_\sigma * \chi_\mathrm{i})(\mathrm{x},\mathrm{t}) = \int \mathrm{K}_\sigma(\mathrm{x}-\mathrm{y})\ \chi_\mathrm{i}(\mathrm{y},\mathrm{t})\,\mathrm{dy}$$

Interaction tensor

$$T_{ij}(\boldsymbol{x},\tau) = \sum_i w_i(\mathbf{x},t)\ \partial_i\chi_i\,\partial_j\chi_i + \sum_{i,k}\beta_{ik}(\mathbf{x},\tau)\ (K_\sigma * \chi_i)\,(K_\sigma * \chi_k)$$

Metric response

$$g_{ij}(x,t) = g^0_{ij}(\mathrm{x}) + \sum_\mathrm{i} \mathrm{a_i}(x,t)\ \partial_\mathrm{i}\chi_\mathrm{i}\ \partial_\mathrm{j}\chi_\mathrm{i}$$

The effect of distributed sources is represented through a scalar eco-coupling field φ\phiφ and write the coupling as $\kappa(x,\tau)=\kappa\,(\phi(x,\tau))$. The field ϕ aggregates non-local inputs via a kernel defined on $(\mathcal{M}, g)$,

$$\phi(x,\tau) = \phi_0(x) + \int_M K_\sigma\left(d_g(x,y)\right) F\big(\chi(y,\tau), \nabla^g \chi(y,\tau)\big)\mathrm{d}V_y,$$

so that

$$\kappa(x,\tau) = \kappa(\phi(x,\tau))$$

with d_g the geodesic distance, ∇^g the metric gradient, and $\mathrm{d}V_{\{y\}}$the Riemannian volume element.

Interfaces evolve under Allen–Cahn/Cahn–Hilliard-type dynamics:

$$\frac{\partial_\eta}{\partial_t} = -M\frac{\delta F}{\delta \eta} + \varsigma(x,t)$$

Free energy functional in analogy to classical density functional theory (Mermin 1973; Evans 1979):

$$\mathcal{F}_{bio}\big[\rho(x,\tau); g_{ij}(x,\tau), \phi(x,\tau)\big] = \mathcal{F}_{ent}[\rho] + \mathcal{F}_{int}[\rho, g] + \int_M V_{eco}\,(x,\tau)\rho(x,\tau)\,dV$$

Here $\rho(x,\tau)$ denotes the density of biological or ecological states (such as abundances, biomass or genetic diversity), $g_{ij}(x,t)$ is the eco-relativistic metric that curves the hyperspace of possibilities,and $\kappa(\phi(x,t))$ represents coupling factors dependent on spatial position, time and the phase of the life or evolutionary cycle. The term $\mathcal{F}_{ent}(\rho)$ accounts for entropic contributions associated with the diversity of states:

$$\mathcal{F}_{ent}(\rho) = k_{bio}T_{eco}\int dx\,\rho(x,t)\left[ln\left(\Lambda^d_{eco}\rho(x,t)\right) - 1\right]$$

Where T_{eco} is an effective ecological temperature reflecting environmental variability and Λ_{eco} a characteristic scale such as generation time or dispersal amplitude. Interactions between entities are encoded in

$$\mathcal{F}_{int} = \iint dx\,dx'\rho(x)W\big(x,x'; g_{ij},\kappa\big)\rho(x'),$$

where W is an eco-relativistic interaction kernel (competition, predation, symbiosis). Finally, $V_{eco}(x,t)$ plays the role of an external potential, representing the abiotic template (climate, soils, hydrology) and external perturbations such as invasions or anthropogenic impacts. Minimisation of this functional yields the stable distributions of ecological states, while fluctuations around the minimum may be described by Fokker–Planck- or replicator-type equations (cf. Hofbauer & Sigmund 1998; Traulsen & Hauert 2009), where the metric g_{ij} enters as a diffusion tensor in niche hyperspace. This construction highlights how the irreversibility and contingency of evolutionary trajectories can be formulated in a variational framework formally analogous to free energy but constrained by biorelativistic geodesics (Prigogine & Stengers 1984; Ulanowicz 1997).

d) Action principle and field equations:

In scalar–tensor extensions of general relativity (e.g. Brans–Dicke), the gravitational coupling becomes dynamical via a scalar field ϕ (Brans and Dicke 1961). In the present framework, the ecological coupling plays the analogous role: we write $\kappa = \kappa(\phi)$, with ϕ mediating how interactions deform the ecological geometry.

Coordinative definitions. In Reichenbach's (1956, 1958) sense, we adopt explicit coordinative definitions linking theory to observation: (i) axes of the ecological state space are fixed by a declared statistical procedure (e.g. trait–environment PCA with covariance-aware distances), and (ii) eco-inertial frames are defined operationally by removing mean drift and minimising covariant acceleration in local charts. These conventions are not empirical claims by themselves; they license the empirical content of the theory by making subsequent predictions falsifiable within a transparent measurement protocol.

Ecological matter Lagrangian. On the probability simplex Δ_{n-1} with Shahshahani metric $g_{ab}(p) = \frac{\delta_{ab}}{p_a} - 1$, it is taken

$$\mathcal{L}_{eco}(\Psi, \Psi; g\overset{\cdot}{)} = \frac{1}{2} g_{ab}(p)\dot{p}^a \dot{p}^b - V(p),$$

where V encodes interaction structure. For frequency-dependent selection with payoff matrix A, choose $V(p) = -\dot{f}(p)$ with $\dot{f}(p) = p^T A p$. Euler–Lagrange with the simplex constraint (plus Rayleigh dissipation) yields the replicator dynamics; thus V captures competition/mutualism/games and g captures inheritance/information geometry. Analogous forms obtain for Lotka–Volterra and consumer–resource models by taking Ψ as densities and V as a smooth eco-potential on a Riemannian chart.

Remark. Other ecological systems (e.g. resource–consumer or Lotka–Volterra – to be discussed elsewhere: Vallejo & Fariña 2025) admit analogous forms by choosing Ψ as densities and V as a smooth eco-potential (e.g. logistic or MacArthur resource potentials) on a Riemannian chart of M; dissipation accounts for open-system fluxes.

Conservation.

From diffeomorphism invariance of the total action and the Bianchi identity we obtain covariant conservation of the total current

$$\nabla^i \left\{ \kappa(\phi)(T^{eco}_{ij} + T^{(\theta)}_{ij}) \right\} = 0,$$

so that apparent non-conservation at the ecological level is balanced by exchange with the coupling field ϕ. In closed, time-independent conditions (no external drive, fixed g, fixed V), Noether's theorem for time translations implies conservation of the eco-Hamiltonian

$$\mathcal{E} = \frac{1}{2}\, g_{ab}\, \dot{p}^a\, \dot{p}^b + V(p),$$

interpretable as kinetic plus potential eco-energy. Biologically, this yields a precise balance: in the absence of external fluxes the total eco-energy is conserved, whereas in open systems deviations $\nabla^i T^{eco}_{ij} \neq 0$ quantify regulated exchange of biomass, diversity or information with ϕ or the environment.

An ecological state manifold $(\mathcal{M}, g)$ is considered on which "ecological matter" fields Ψ (e.g., densities, frequencies, trait fields) evolve. Let ϕ be a scalar eco-coupling field that modulates the geometry–matter interaction. The total action is

$$S[g, \Psi, \phi] = \int |g| \left[\frac{1}{2\kappa(\Phi)} R + \mathcal{L}_{eco}(\Psi, \nabla\Psi; g) - \frac{1}{2} g^{ij} \partial_i \phi \partial_j \phi - U(\phi) \right] dV.$$

Stationarity of S under independent variations yields:

(i) variation in g_{ij}:

$$G_{ij} = \kappa(\phi)\big(T_{ij}^{eco} + T_{ij}^{\phi}\big) + \nabla_i \nabla_j \left(\frac{1}{\kappa(\phi)}\right) - g_{ij} \Box \left(\frac{1}{\kappa(\phi)}\right),$$

with

$$T_{ij}^{eco} := -\frac{2}{\sqrt{|g|}} \frac{\delta}{\delta g^{ij}} \left(\sqrt{|g|} \mathcal{L}_{eco}\right) \text{ and } T_{ij}^{eco} := \partial_i \phi \partial_j \phi - \frac{1}{2} g_{ij} (\partial \phi)^2 - g_{ij} U(\phi);$$

ii) variation in Ψ: the ecological Euler–Lagrange equations;

(iii) variation in ϕ:

$$\phi - U'(\phi) + \frac{1}{2} \kappa'(\phi) \kappa(\phi)^{-2} R = 0.$$

Because $\nabla^i G_{ij} = 0$ (Bianchi identity), the total ecological-geometric current is covariantly conserved:

$$\nabla^i \left\{\kappa(\phi)\big(T_{ij}^{eco} + T_{ij}^{\phi}\big)\right\} = 0,$$

so that any apparent non-conservation at the ecological level is balanced by exchange with the coupling field ϕ. In the limit $\kappa(\phi) \equiv \kappa_0$ and $U'(\phi) = 0$, these equations reduce to the standard Einstein-type relation $G_{ij} = \kappa_0 T_{ij}$. Free motion in eco-inertial frames follows geodesics of g; non-geodesic terms represent external or cross-scale interactions encoded in $\mathcal{L}_{eco}$.

Let n = 2 with payoffs

$A = [[\alpha, \beta], [\gamma, \delta]]$, and $\bar{f}(p) = p^T A\, p$,

and p = (p, 1−p). With g(p) = 1/(p(1−p)) and V(p) = − $\bar{f}$(p), the matter Lagrangian reads

$$\mathcal{L}_{eco} = ½ \cdot (\dot{p})^2 / (p(1-p)) + \bar{f}(p).$$

The field equations with dissipation recover $\dot{p} = p(1-p)[(A\,p)_1 - (A\,p)_2]$. Embedding this subsystem in $(\mathcal{M}, g)$, the sectional curvature near a fixed point p* is governed by the Hessian of V pulled back by g. Negative curvature (destabilising games) predicts divergence of nearby ecological trajectories, while positive curvature (stabilising interactions) predicts convergence, providing a concrete, falsifiable link between interaction structure and geodesic behaviour. This operationalises the curvature–trajectory prediction without leaving the action-based framework.

Coupling structure. The geometric–matter coupling is carried by a scalar field ϕ so that κ=$\kappa(\phi(x,\tau))$ enters the action as $\frac{1}{2\kappa(\phi)}R$ and via the ϕ-sector $\frac{1}{2}(\partial\theta)^2 + U(\theta)$. Any apparent $\kappa(x,t,\phi)$ dependence is thus mediated through the dynamics of $\phi(x,\tau)$, which preserves diffeomorphism invariance and a clean Noether interpretation of conservation. When sensitivity to ecomorphological separation or evolutionary age is required, κ may depend only on scalar invariants, e.g. the geodesic distance d_g and ecological proper time τ; in practice through U(ϕ) sources or boundary data for ϕ rather than as explicit coordinate dependence.

e) Toy model (replicator).

f) Dynamical evolution equations:

A generalised form analogous to Einstein's field equation might initially be written as:

$$G_{\mu\nu} = \kappa\, T_{\mu\nu}$$

or

$$G_{ij} = \kappa\, T_{ij}(x,t)$$

where:

- $G_{ij}(x,t)$ is a curvature tensor of the ecological space (analogous to the Einstein tensor),
- $T_{ij}(x,t)$ represents the distribution and flow of species or other biological entities in ecological space (analogous to the energy-momentum tensor),
- κ is a constant relating ecological curvature to ecological interaction (see Coupling structure in 5d).

with T_{ij} and g_{ij} defined on smoothed densities as in (**c**).

Yet, due to the context-dependence discussed in b), a more appropriate form may be:

$$G_{ij}(x,t) = \kappa(x,t,\phi)\, T_{ij}(x,t)$$

where κ is a function or operator dependent on local ecological conditions, temporal dynamics and possibly evolutionary state variables ϕ. Alternatively, this could take the form of a nested or multi-level equation in which the coupling evolves along with the system. A transformation of this kind is not without precedent. In general relativity, Einstein introduced the constant $\kappa=8\pi G/c^4$ as a fixed proportionality factor linking the geometry of spacetime to the energy-momentum content of the universe. However, in 1917, when attempting to apply his equations to the cosmos as a whole, under the prevailing assumption of a static universe, he added a new term: the cosmological constant, modifying the field equations to counteract gravitational collapse. After Hubble's (1929) observations stated that the universe is in fact expanding, the need for such a constant diminished, and Einstein is famously (and perhaps apocryphally) alleged to have called to its introduction as the "greatest blunder" of his life (Gamow, 1956). Later developments in relativistic cosmology (Friedmann, 1922; Lemaître, 1927; Hubble, 1929) reinstated the idea of an evolving, dynamic geometry, where the field equations themselves must accommodate large-scale historical transformations. The analogy here is both instructive and cautionary: in a biological context, ecological coupling may likewise appear constant only under restricted assumptions, but may in fact vary systematically across time, scale and context. The proposed shift from a constant κ to a functional or nested formulation is thus in keeping with a broader move toward historically responsive, dynamically modulated field equations.

g) Incorporation of evolutionary processes:

A theory with the characteristics of the one dealt with here would need to be flexible enough to accommodate both deterministic and contingent aspects of biological evolution and robust enough to scale from individual (or even lower-level) interactions to ecosystem-level patterns. The framework must account for the temporal and historical character of biological systems. Evolution introduces both directional change and structural novelty, making the geometry of ecological space itself an evolving entity. Accordingly, the curvature–interaction coupling cannot be fixed. Evolutionary innovations, extinctions, migrations and environmental perturbations alter the effective geometry of interaction space and, in turn, the very rules by which such geometry responds to biological forces. This reinforces the need for a variable or nested form of κ, one that captures not only spatial heterogeneity but also temporal asymmetry and path-dependence. Suitable mathematical tools may involve non-linear partial differential equations (PDEs), integro-differential equations, Fokker–Planck formulations or stochastic differential equations (SDEs). These tools offer the flexibility needed to model systems in which local interactions, feedback loops and external perturbations produce complex, history-dependent dynamics. For instance, non-linear PDEs may be used to describe the propagation of trait distributions across ecological space, where the diffusion term accounts for phenotypic variability and the reaction term incorporates local adaptation or selective pressures. Integro-differential equations could formalise long-range ecological interactions or evolutionary memory effects, such as niche construction or environmental inheritance, by integrating over extended spatial or trait domains. Fokker–Planck equations would allow modelling the time evolution of probability densities in systems with stochastic influences, capturing the effects of drift, mutation and random dispersal. Finally, stochastic differential equations may be suitable for simulating the trajectories of populations or species through the ecological hyperspace under conditions of environmental noise, demographic stochasticity or contingency-driven transitions such as speciation or extinction. These formalisms may not be mutually exclusive, and a full treatment of ecological relativity might require hybrid or multiscale approaches in which deterministic structure and stochastic variation are jointly represented. In this light, the variability of κ and the interface dynamics introduced in **(c)** are the natural conveyors of historical contingency across scales.

6. Falsifiable predictions, empirical tests and cautions

6.1. Geometric framework

From the variational structure. The predictions below follow from the action in §5(d) via g, Γ, $\kappa(\phi)$ and the matter Lagrangian rather than from analogy alone. Curvature, anisotropy and finite propagation emerge as Euler–Lagrange consequences and are therefore testable.

1) Trajectory deflection by local curvature

Prediction. If two populations or traits move under comparable environmental gradients, positive sectional curvature induces convergence of trajectories whereas negative curvature induces divergence.

Observable. Time evolution of the geodesic distance $\Gamma_{1,2}$ between trajectories in eco-space.

Data/design. Manipulated gradients in laboratory microcosms or field mesocosms; time series of trait or abundance vectors.

Falsification. Systematic absence of convergence/divergence when curvature estimates are non-zero within confidence bounds.

2) Hysteresis from history-dependent connection

Prediction. If Γ depends on system history (e.g. niche construction), a round trip along the same external gradient produces a loop with non-zero enclosed area.

Observable. Loop area in eco-space and phase lags on return paths.

Data/design. Cyclic perturbations in microcosms or mesocosms; reciprocal translocation or press–release experiments.

Falsification. Robust loop closure (area ≈ 0) after controlling for noise and drift.

3) Directional anisotropy of response

Prediction. If g is anisotropic, the magnitude and timescale of response depend on the direction of perturbation in eco-space.

Observable. Ratios of displacement and characteristic time constants as a function of perturbation direction.

Data/design. Factorial perturbations applied along orthogonal axes of the trait or state space.

Falsification. Sustained isotropy after local estimation of g near the operating point.

4) Local reduction to classical models

Prediction. In sufficiently small neighbourhoods, the dynamics reduce to established formulations (e.g. Lotka–Volterra, Leslie) under appropriate changes

of variables.
Observable. Local equivalence of first-order predictions between the proposed flow and classical models.
Data/design. Classical population datasets or replicated experiments near equilibria; local linearisation in normal coordinates.
Falsification. Systematic local discrepancies despite reparameterisation and coordinate normalisation.

A comprehensive comparison of biorelativity-based versus classic Lotka–Volterra models will be published separately (Vallejo & Fariña 2025). Results show improved performance of the biorelativity model under external environmental and climatic perturbations, despite the model-selection penalty for additional parameters (e.g. AIC/BIC).

5) Finite-speed propagation across weakly coupled subspaces
Prediction. With weakly coupled subspaces, disturbances propagate with delays that scale with effective distance induced by g, defining a finite c_{eco}.
Observable. Arrival times and attenuation profiles as functions of effective geodesic distance.
Data/design. Spatial metacommunities on laboratory landscapes or natural gradients; interaction networks with measured coupling strengths.
Falsification. Apparent instantaneous transmission or lack of dependence on effective distance.

Note. These tests naturally interface with the formalism in §5: local curvature and geodesic deviation (1), path dependence via state- or history-dependent Γ (2), anisotropy encoded in g (3), consistency with classical limits in normal coordinates (4) and constraints set by a finite biological speed limit c_{eco} (5).

6) Common-cause heuristic (Reichenbach). If two lineages display correlated directional changes in eco-space without direct interaction, we expect an unobserved common cause (empirically, a hidden source of curvature or a shared driver. like a limiting resource, a keystone mutualist, or an abiotic factor). Test: correlations should weaken or vanish after conditioning on the inferred curvature field or on measured proxies of the putative driver; failure to do so refutes the common-cause explanation.

Box 2. Reframing natural selection in Biorelativity

Definition.

Natural selection is the statistical tendency for genotype–phenotype configurations that increase their effective ecological coupling, quantified as eco-mass-energy that deforms the ecological hyperspace in favourable ways (attracting resources and allies, repelling threats and costs), to accrue more ecological proper time (greater persistence and reproduction) and to expand their effective volume in eco-space relative to alternatives under the same constraints.

An observable eco-mass-energy density.

Define a composite density from measurable components:

$$\rho_e(x,t) = w_R J_R + w_I I + w_C C - w_K K,$$

where J_R is resource/biomass flux (local ecological power), I is organisation/information (e.g. negentropy, functional redundancy/modularity), C is network centrality/robustness (trophic, mutualistic, habitat graphs), and K is aggregated cost/vulnerability; w are normalised weights. The eco-mass-energy over a domain Ω is

$$Me_{eco}(\Omega,t) = \int_{\Omega} \rho_e\,(\mathbf{x},\mathrm{t})\,\mathrm{d}V$$

Along a lineage trajectory $\gamma(\tau)$, selection implies (on average, relative to competitors in the same environment)

Geometric–dynamic form.

Let fitness be a functional of deformation and coupling,

$$f(x,t) = \mathcal{F}\big[\rho_e(\mathbf{x},t), g_{ij}(x,t), \kappa(\mathbf{x},t,\phi)\big],$$

and let the distribution of states $P(\mathbf{x},t)$ evolve as

$$\partial_t P = -\nabla \cdot (\mathbf{b}, P) + \frac{1}{2}\nabla \cdot (D\nabla P) + P\big(f - \dot{f}\big),$$

so increases in ρ_e and favourable curvature (via g_{ij},κ) bias drift **b** and growth.

Falsifiable predictions / empirical tests.

1. **Curvature–success link.** Lineages with persistently higher $\Delta\rho_e > 0$ exhibit **locally more attractive curvature** towards resources/mutualists and **divergent curvature** from predators/competitors; geodesic deviation between focal lineages and resource manifolds decreases through time.
2. **Finite influence speed.** Adjustments in interaction structure propagate with an upper bound c_{eco} ; shocks in one guild show delayed, distance-dependent responses in others.
3. **Local reduction.** In regimes where $g_{ij} \approx \delta_{ij}$ and κ is constant, the framework reduces to classical fitness (e.g. Malthusian growth, Lotka–Volterra).
4. **Trade-off realism.** Increases in I or C that raise costs K can lower ρ_e ; simplification (e.g. parasitic reduction) can increase ρ_e ; by cost minimisation.

Measurement notes.

Choose the organisational level (individual, population, species, guild) before computing I,C,K; harmonise units and scale weights $w\,\cdot$ by cross-validation. Report sensitivity of results to $w\,\cdot$ and to the spatial/temporal windows used. Where possible, estimate c_{eco} from lagged cross-correlations in time-series or from reaction–diffusion fits on ecological networks.

6.2. Scope and caution: lessons from applied catastrophe theory

The aim of the present framework is methodological: to provide a minimal geometric notation that makes constraints, couplings and memory explicit, and that yields falsifiable predictions. It does not propose a biological cosmology. The history of applied Catastrophe Theory illustrates the risks of extrapolating mathematical elegance without solid empirical validation. Classic critiques exposed conceptual and empirical weaknesses in social and biological applications

(Zahler and Sussmann, 1977; Sussmann and Zahler, 1978; Sussmann, 1978). In this work, evaluation will rest on the tests in the subsection *Falsifiable predictions and empirical tests* and on the ability to recover established models locally (see Box 1).

7. Not a Final Theory, but a Theoretical Invitation

This is not a complete theory, nor is it intended to replace existing ecological or evolutionary models. Rather, it is a conceptual invitation or a thought experiment meant to stimulate discussion and foster interdisciplinary collaboration. An extended general theory of relativity in biology would not constitute a literal analogue of physics, but rather a metaphorically inspired, mathematically framed attempt to capture the dynamic, emergent and interactive character of life in its many forms and scales.

Beyond its theoretical ambition, such a framework may also hold practical value. At large temporal scales, it may offer a novel lens through which to interpret major transitions in Earth's biological history, such as the appearance of the biosphere. At shorter timescales, in an era of pronounced ecological agency on the part of *Homo sapiens* and an unprecedented anthropogenic reshaping of the biosphere (to be discussed elsewhere), this framework might contribute to more rational ecosystem management by helping to identify, model and perhaps even quantify the consequences of ecological interventions. A geometry of biological interaction could, in principle, inform not only scientific understanding but also environmental governance.

It is worth acknowledging, on a more personal note, that while Einstein published his General Theory of Relativity as a solitary author, the complexity of contemporary science renders such individual endeavours increasingly rare. The theoretical ambition outlined here, as modest as it may be in comparison, requires a collaborative spirit from the very outset. If this framework is to mature into something formally viable, it will need the combined insights of mathematicians, physicists, ecologists, philosophers and other scholars willing to engage with ideas that fall between disciplinary boundaries. Einstein's extraordinary genius reminds us not only of the heights to which a single mind can ascend, but also of the shoulders upon which we must collectively stand if we are to be worthy of that legacy.

8. Conclusion

Could life be understood not merely as a set of reactions in space, but as a sculptor of its own multidimensional field? If so, the geometry of life must be richer than any static Euclidean space, but one that bends, shifts and co-evolves with its inhabitants. Developing such a framework is a monumental task, requiring insights from physics, mathematics, philosophy and the life sciences.

This essay marks only a conceptual starting point. I invite readers from diverse disciplines to help formalise and challenge this idea, so that we may together imagine a geometry worthy of the biosphere. Moreover, while this proposal has been developed in the context of biological systems, the underlying approach may be extensible to other domains of knowledge. As said above, any field in which agents interact, modify their environment and alter the trajectories of others, whether in the geology, human sciences, environmental (and otherwise) politics, psychology, linguistics, economics, climate science, collective sports, all transdisciplines, epistemology itself or any communicative endeavour could, in principle, be modelled as a system of local deformations in a multidimensional relational space given the appropriate hyperspace and the relevant variables. The generalised relativistic framework envisioned here thus invites a broader reflection on how structure, interaction and historical contingency might coalesce in a unified, geometrically informed theory of complex systems.

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Table 1. Comparison of coupling formulations in Biorelativity, showing the two main alternatives for the coupling term κ in the proposed theory.

Formulation	**Equation**	**Coupling term (κ)**	**Biological interpretation**	**Flexibility**	**Analytical implications**
Fixed coupling (Einstein analogue)	$G_{ij}(x,t) = \kappa\ T_{ij}(x,t)$	Universal constant (disputed in modern cosmology)	Assumes uniform relationship between ecological interaction and curvature	Low - does not accommodate contingency or feedback	Simpler to analyse but biologically rigid.
Dynamic coupling (proposed ecological extension)	$G_{ij}\ (x,t) = \kappa\tau\ T_{ij}\ (x,\tau)$	Context-dependent function or operator	Allows for variation due to evolutionary history, local densities, trophic asymmetries and environmental feedback	High- accounts for plasticity and historical contingency	Mathematically complex; nested or multi-scale formulation required.

Figures

Figure 1. Curvature and trajectory deviation. A) convergence due to local positive curvature. B) divergence due to negative curvature or effective gradients that induce avoidance. In both cases the underlying manifold remains continuous; attraction and repulsion are encoded in the metric g and the connection Γ, not in changes to the topology of the substrate.

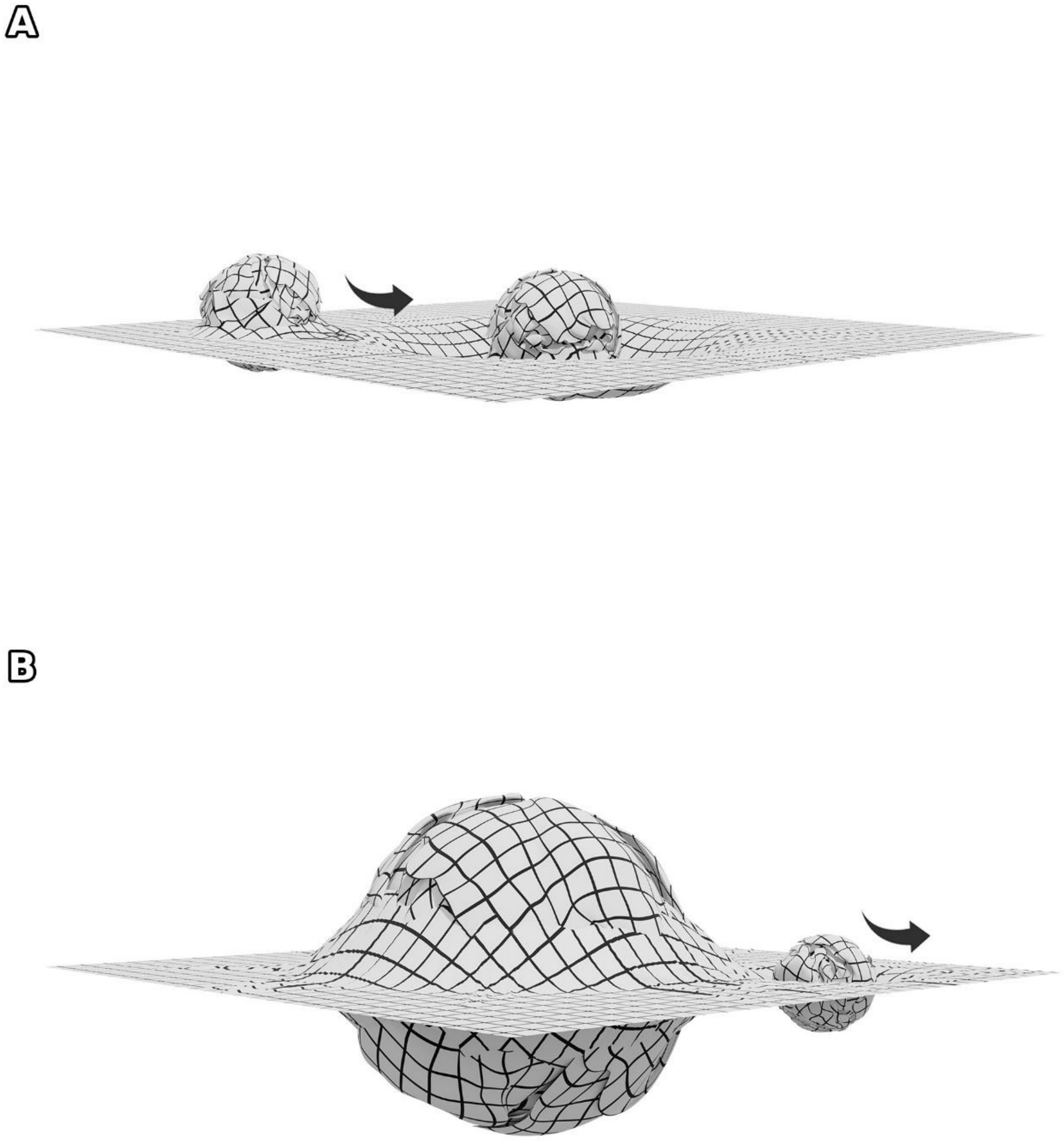
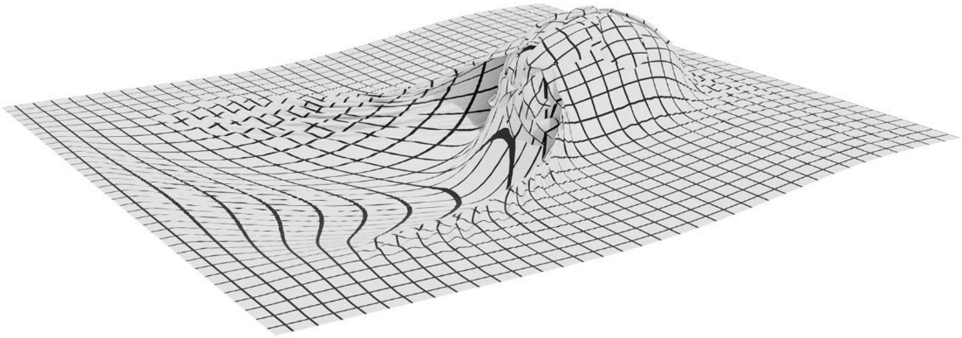


Figure 2. A single sphere moves forward in time (as in evolution) through a deformable ecological surface, simplified here again as a two-dimensional elastic mesh. It leaves behind a deep trailing depression and generating a forward

elevation and lateral distortion. The shape and displacement of the surface illustrate the temporal dynamics of ecological interaction: the depression behind the sphere implies an attractive effect on other organisms or ecological entities, while the raised front region indicates repulsive or exclusionary effects due to the evolving position or properties of the focal organism. This asymmetric deformation represents how organisms and other biological entities, as they change over time, simultaneously reshape their ecological surroundings, attracting some components of the system while repelling others.



Appendix A. Minimal Illustrative Model of Ecological Curvature Dynamics

To demonstrate the potential behaviour of ecological curvature fields as a function of evolving organismal traits, a simplified one-dimensional model is proposed. Consider a prey species S_1 with a single evolving trait $m_1(t)$ (e.g. body size), and a predator species S_2 that responds to ecological curvature $G(x,t)$ induced by S_1 in ecomorphological space.

The curvature field is defined as:

$$G(x,t) = \kappa(x,t) \cdot T(x,t)$$

where:

- x is a position along an ecological trait axis,
- *T(x,t)* is a constant interaction intensity (*T*=1),
- $\kappa(x,t) = e^{-\alpha(x-m_i(t))^2}$ is a Gaussian-like field that reflects the local influence of the evolving trait.

This formulation creates a deformable ecological landscape where the curvature 'well' induced by the prey changes location and steepness as its trait evolves. As shown in Fig. A1, increases in m1 shift the attractor in ecological space, potentially repelling or decoupling the predator's effective interaction.

Appendix B. Eco-hyper spacetime with hierarchical dimensions (Minkowski-style formalism)

This appendix outlines a minimal Minkowski-style formalism for an evolutionary eco-hyper spacetime with hierarchical levels. It provides working definitions and equations in a compact form suitable for subsequent formal development and for Word equation entry.

B.1 Coordinates and state variables

Let t denote oriented evolutionary time and let the ecomorphological hyperspace be stratified into L_{max} hierarchical levels (e.g. organism, population, community and ecosystem). Let ξ^(L) ∈ $\mathbb{R}^{n_L}$ be the coordinates at level *L*. Define the extended coordinate vector:

$$X^A = \left(c_{\text{eco}} \cdot t, \xi^{(1)}, \xi^{(2)}, \dots, \xi^{(L_max)}\right).$$

B.2 Minimal eco-interval and speed limit

A biological speed limit c_{eco} is postulated that bounds feasible rates of change in eco-space. A minimal eco-relativistic interval with block-diagonal structure is:

$$s_{eco}^2 = \mathrm{c}_{eco}^2 \cdot \mathrm{dt}^2 - \sum_{L=1}^{L_{max}} w_{L(X)} \cdot || \, \mathrm{d}\xi^{(\mathrm{L})}||^2.$$

Here $w_{L(X)} \geq 0$ are hierarchical weights that encode characteristic response times, for example $w_L = \tau_L^{-2}$ with τ_L the typical timescale of level L. The proper eco-time along a trajectory is given by $s_{eco}^2 = c_{eco}^2 \cdot d\tau^2$

The corresponding four-velocity and normalisation read:

$U^A = \frac{\mathrm{dX^A}}{\mathrm{d\tau}}, \; g_{(AB)} U^A U^B = c_{eco}^2$

A Lorentz-type factor emerges as:

$$\gamma_{\mathrm{eco}} = \left(1 - \frac{1}{c_{eco}^2} g_{ij} \dot{\xi^i} \dot{\xi^j}\right)^{-1/2}$$

B.3 Coupled metric with multilevel interactions

To represent cross-level interactions and historical deformation, allow the metric to depend on state and include off-diagonal blocks:

$$s_{eco}^2 = \alpha(X)\, c_{eco}^2 dt^2 - \sum_{L} d\xi^{(L)\top} \mathbf{M}^{(L)}(X)\, d\xi^{(L)} - 2 \sum_{L<M} d\xi^{(L)\top} \mathbf{C}^{(LM)}(X)\, d\xi^{(M)}$$

Here α(X) > 0 sets temporal orientation, $M^{(L)(X)}$ are symmetric positive matrices that weight directions of change within level L and $C^{(LM)(X)}$ are cross-level coupling blocks that transmit feedbacks between levels.

B.4 Geodesics (inertial change) and ecological forces

In the absence of external drivers, inertial trajectories follow geodesics of $g_{\{AB\}}$:

$$\frac{d^2 X^A}{d\tau^2} + \Gamma^A{}_{BC}(X) \frac{dX^B}{d\tau} \frac{dX^C}{d\tau} = 0$$

Selection, management and stochastic drivers enter as forces F^A:

$$\frac{d^2 X^A}{d\tau^2} + \Gamma^A{}_{BC}(X) \frac{dX^B}{d\tau} \frac{dX^C}{d\tau} = F^A(X, t)$$

A potential-driven representation uses $F = -\nabla_X \Phi(X, t) + \eta(t)$, where η denotes demographic or environmental noise.

B.5 Special-relativistic quantities in eco-space

Define eco-momentum and eco-force as $p_{eco} = m0_{eco} \cdot \gamma_{eco} \cdot \left(\frac{dX}{dt}\right)$ and $F_{eco} = \frac{dp_{eco}}{dt}$, where $m0_{eco}$ quantifies intrinsic resistance to change, arising from developmental and genetic constraints, network embedding and energetic costs of maintaining low local entropy. The eco-energy reads $E_{eco} = \gamma_{eco} \cdot m0_{eco} \cdot c_{\{eco\}}^2 + \Phi(X, t)$. In the slow-change limit $\left\|\frac{dX}{dt}\right\| \ll c_{eco}$ one recovers a Newtonian-type relation $F_{eco} \approx m0_{eco} \cdot \frac{d^2X}{dt^2}$.

B.6 Notes on calibration and hierarchy

- Calibrating c_{eco}: use empirical upper bounds of phenotypic rates (e.g. haldanes) across clades and timescales as lower-bound estimates, with expected dependence on generation time and metabolic scaling.

- Weights and matrices: w_L or M^L implement temporal hierarchy, making higher levels costlier to move. Cross-level couplings C^{LM} encode feedbacks, producing state-dependent geometry and path dependence.

- Directionality: irreversibility can be represented by explicit time dependence $g_{\{AB\}}(t)$) or by introducing torsion or non-metricity to reflect history dependence.

B.7 Summary of Novel Formulations Proposed by the Biorelativity theory

The following equations constitute the specific, novel definitions required by the Biorelativity theory framework.

Eco-morphological coordinates

$$X^A = (c_{eco} \cdot \tau, \xi^{(1)}, \xi^{(2)}, \dots, \xi^{(L_{max})})$$

X^A: Represents a point (coordinate) in the proposed eco-morphological hyperspace, which combines evolutionary time and spatial/trait axes across different biological levels. The index A ranges over all dimensions.

c_{eco}: A postulated biological speed limit, analogous to the speed of light c. It bounds the maximum rate of propagation of ecological reconfigurations or influences.

τ: Oriented evolutionary time.

$\xi^{(L)}$: Represents the coordinates within a specific hierarchical level L (e.g., L=1 for organism traits, L=2 for population densities). Each $\xi^{(L)}$ can be a vector.

L_{max}: The maximum hierarchical level considered in the model.

Implication: This equation defines the arena. It fuses time, traits and population levels into a single geometric object (a manifold) upon which the dynamics will unfold.

Ecological Interval ("Eco-Interval")

$$s_{eco}^2 = c_{eco}^2 \cdot d^2 - \sum_{L=1}^{L_{max}} w_L(X) \cdot \|d\xi^L\|^2$$

s_{eco}^2: The square of the infinitesimal eco-interval, analogous to the spacetime interval ds^2 in relativity. It defines the metric structure (geometry) of the eco-spacetime.

$w_L(X)$: Hierarchical weights that may depend on position X in the hyperspace. They encode the relative importance or characteristic response times (τ_L) of different levels.

$\|d\xi^L\|^2$: Square of the infinitesimal displacement (distance) within the coordinate space of level L.

Implication: This defines a simplified (block-diagonal) metric. It measures the distance between infinitesimally close states, combining temporal separation with weighted spatial/trait separation. It implies that proper time $\tau\,(d\tau^2 = ds^2/c_{eco}^2)$ is affected by movement through the trait/population space.

Ecological Gamma Factor ("Eco-Lorentz factor")

$$\gamma_{eco} = \frac{1}{\sqrt{1 - \frac{1}{c_{eco}^2} \sum_L w_L |d\xi^{(L)}/dt|^2}}$$

γ_{eco}: The ecological Lorentz factor, analogous to γ in special relativity.

$\left|d\xi^{(L)}/dt\right|^2$: The squared velocity of change at level L with respect to evolutionary time τ.

Implication: A direct consequence of the interval. If a system changes very fast (denominator approaches 0), γ_{eco} becomes large. This would imply "time dilation" (proper time τ passes slower than coordinate time t) and mass-energy increases (see E_{eco}). It is a relativistic correction for rapid ecological change.

Coupled Hierarchical Eco-Interval

$$s_{eco}^2 = \alpha(X)c_{eco}^2 dt^2 - \sum_{L} d\xi^{(L)T} M^{(L)}(X)\, d\xi^{(L)} - 2\sum_{L<M} d\xi^{(L)T} C^{(LM)}(X)\, d\xi^{M}$$

$\alpha(X)$: A position-dependent temporal scaling factor (> 0), allowing for local variations in the flow of evolutionary time (analogous to g_{00} in GR).

$M^{(L)}(X)$: Position-dependent matrices defining the metric structure within each level L. (Note: T denotes transpose).

$C^{(LM)}(X)$: Position-dependent cross-level coupling matrices. They explicitly represent interactions and feedbacks between different hierarchical levels L and M.

Implication: This provides a more general and flexible metric. The geometry is complex, non-diagonal, and state-dependent (X). It is designed to explicitly model hierarchical feedbacks (e.g., how population density affects trait selection) as part of the geometry itself.

Ecological Lagrangian

$$\mathcal{L}_{eco}(\Psi, \dot{\Psi}; g) = \frac{1}{2} g_{ab}(p)\dot{p}^a \dot{p}^b - V(p)$$

$\mathcal{L}_{eco}$: The Lagrangian density for ecological matter fields Ψ. This defines the dynamics of the system's state variables.

$g_{ab}(p)$: The ecological metric on the state space, exemplified here by the Shahshahani metric for frequencies p^a.

$\frac{1}{2}g_{ab}(p)\dot{p}^a\dot{p}^b$: The kinetic energy term, as defined using the ecological metric.

$V\,(p)$: A potential function representing ecological interactions (e.g., related to average payoff in game theory).

Implication: This recasts standard evolutionary dynamics as a classical mechanics problem: a particle (the system state) moving on a curved manifold (the simplex with the Shahshahani metric) under a potential (V). This Lagrangian is the source term L_{eco} for the Hilbert stress-energy tensor $T_{\mathrm{ij}}^{\mathrm{eco}}$.

Eco-Hamiltonian

$$\varepsilon = \frac{1}{2}g_{ab}\dot{p}^a\dot{p}^b + V(p)$$

ε: The ecological Hamiltonian or eco-energy, corresponding to the Lagrangian $\mathcal{L}_{eco}$.

Implication: Represents the total conserved eco-energy (Kinetic + Potential) for closed, time-independent systems described by $\mathcal{L}_{eco}$.

Geodesic Equation with "Ecological Force"

$$\frac{d^2X^A}{d\tau^2} + \Gamma^A_{BC}(X)\frac{dX^B}{d\tau}\frac{dX^C}{d\tau} = F^A(X,t)$$

Γ^A_{BC}: Christoffel symbols derived from the eco-metric g_{AB}. This term encapsulates the fictitious force (like gravity) that comes from the curvature of the eco-spacetime.

F^A: An ecological force term representing non-inertial influences like selection, external drivers or stochasticity not already encoded in the metric.

Implication: This is the equation of motion. It states that an organism's trajectory deviates from a straight line for two reasons: 1) the eco-spacetime is curved (the Γ term, the equivalent of gravity), and 2) other ecological forces are pushing it (the F^A term).

Ecological Energy (Eco-energy)

$E_{eco} = \gamma_{eco} \cdot m0_{eco} c_{eco}^2 + \Phi(X,t)$

E_{eco}: Total Ecological Energy.

mo_{eco}: Ecological rest mass, the intrinsic inertia of a biological entity.

$\gamma_{eco} \cdot m0_{eco} c_{eco}^2$: The relativistic energy term.

$\Phi(X,t)$: An ecological potential energy, analogous to electric potential.

Implication: Defines the energy of an entity. Rapid change (high γ_{eco}) increases this energy, just as rapid movement increases relativistic mass-energy.

Eco-mass-energy Density

$$\rho_e(x,t) = w_R J_R + w_I I + w_C C - w_K K$$

ρ_e: The proposed observable density of eco-mass-energy. This is the source term for the curvature.

J_R: Flux of resources or biomass (ecological power).

I: Measure of organisation or information (e.g., negentropy).

C: Measure of network centrality or robustness.

K: Measure of aggregated cost or vulnerability.

$w_{\cdot}$.. Normalised weights.

Implication: This is a key hypothesis. It states that curvature is not just from mass/biomass, but also from information, network structure and costs. A highly organised node in a food web curves the eco-spacetime around it more than an isolated organism of the same mass.

Eco-coupling Field Definition

$$\phi(x,\tau) = \phi_0(x) + \int_M K_\sigma\left(d_g(x,y)\right) F\big(\chi(y,\tau), \nabla^g\, \chi(y,\tau)\big) dV_g$$

$\phi(x;\tau)$: The scalar eco-coupling field that determines the strength $\kappa(\phi)$ of the geometry-interaction coupling.

$\phi_o\,(x)$: A background value of the field.

$K_o(d_g(x,y))$: A kernel function depending on the geodesic distance $d_g(x,y)$, modelling non-local influence that decays with distance.

$F(\chi, \nabla^g \chi$: A source function dependent on the state (χ) and gradient ($\nabla^g \chi$) of biological entities.

Implication: This equation is highly non-local. It states that the strength of physics (the coupling κ) at one point x depends on an integral of the state of the entire rest of the ecosystem, weighted by geometric distance.

Bio-Free Energy Functional

$$\mathcal{F}_{bio}[\rho(x,\tau); \dots] = \mathcal{F}_{ent}[\rho] + \iint dx\, dx'\, \rho(x) W\big(x, x' : g_{ij}, \kappa\big) \rho(x') + \int_M V_{eco} \dots \rho dV$$

$\mathcal{F}_{bio}$: The Bio-Free Energy functional for the ecological system.

$\mathcal{F}_{ent}[\rho]$: The entropic component (favours diversity/spread).

$W\big(x, x' : g_{ij}, \kappa\big)$: An eco-relativistic interaction kernel between states at x and x_o. It depends on the eco-metric g_{ij} and coupling κ.

V_{eco}: An external potential representing abiotic factors.

Implication: This provides a statistical-mechanical view. The system will evolve to minimise this free energy. This connects the geometric framework (g_{ij}, κ) to thermodynamic principles and population-level dynamics (ρ).